What Do Nominal Rigidities and Monetary Policy Tell Us about the Real Yield Curve?

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Abstract

We provide a theoretical analysis of the implications of monetary policy on the term premia in real bonds and the inflation risk premia in nominal bonds. Monetary policy has real effects in an economy characterized by recursive preferences, nominal wage and price rigidities, and a nominal interest-rate policy rule. Positive monetary policy shocks increase the marginal utility to consume while making long-term real bonds cheap, leading to positive real term premia. However, inflation is low due to the positive policy shocks, and the inflation risk premia in nominal bonds are negative. Productivity growth shocks in conjunction with wage rigidity generate a positive covariance between consumption growth and the price of real long-term bonds, resulting in an upward sloping real yield curve and positive real term premia. At the same time, inflation is high following the negative productivity growth shocks when consumption growth is low, and the inflation risk premia in nominal bonds are positive. Overall, the effects of the productivity growth shocks dominate those of the policy shocks, and the real term premia and the inflation risk premia are positive, on average. Finally, more responsive monetary policies reduce the government's cost of borrowing when issuing nominal debt, and reduce the diversification benefits of real bonds.

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1 Introduction

Default-free real interest rates of short and long maturities are of fundamental importance in macroeconomics, finance, and policymaking. These rates can help us understand the willingness to substitute consumption over time in the economy, represent a benchmark to compare financial asset expected returns, and may play an important role in the transmission mechanism of monetary policy, among others. This importance radically contrasts with our understanding of the economic drivers of these rates. While the empirical literature has been limited by data availability, the theoretical literature suffers from the lack of a satisfactory model connecting asset prices to economic dynamics. In this paper, we explore an equilibrium model characterized by nominal price and wage rigidities to (i) understand how these rigidities affect several properties of real bond yields and, (ii) gain insights into how monetary policy can affect these properties through nominal rigidities.

Treasury inflation protected securities (TIPS) have been issued by the United States government since 1997. We use TIPS as a proxy for real bond yields and compute different descriptive statistics of TIPS yields to make comparisons with U.S. nominal government bond yields.[1] Our empirical analysis for the 1999-2008 period shows that long-term TIPS yields have been higher than short-term TIPS yields on average, the spreads between long- and short-term TIPS yields are smaller than comparable spreads for nominal bond yields, the TIPS yields are as volatile as nominal bond yields, and are highly correlated for comparable maturities. In addition, a principal component analysis shows that nominal and TIPS yields are mainly driven by two common factors. We link these properties of bond yields to nominal rigidities and monetary policy in our theoretical model.

Wage and price rigidities and monetary policy play an important role in our model capturing

[1]Strictly speaking, TIPS are not exactly comparable to real bond yields given their particular inflation indexation procedure and their embedded optionality. The comparison of TIPS and nominal government bond yields can be affected by the difference in liquidity in the two bond markets. These differences are not captured in our theoretical model.

the empirical properties above. First, wage rigidities generate positive average spreads between long and short-term real yields, which are lower than comparable spreads for nominal yields. It results from positive term and inflation risk premia for permanent productivity shocks. Second, the volatility of real bond yields is increased by monetary policy shocks in the presence of rigidities, but not by shocks to an inflation target. Third, wage rigidities increase the correlations between real and nominal bonds. Fourth, changes in the response of monetary policy to economic conditions affect the magnitudes of these findings.

Our model is based on Li and Palomino (2012) and contains four important ingredients. First, a representative household with Epstein and Zin (1989) recursive preferences over consumption and labor. As shown by Tallarini (2000), it allows us to keep a low level of elasticity of intertemporal substitution to match macroeconomic dynamics and a high degree of risk aversion to match high compensations for risk in financial assets. Second, Calvo (1983) rigidities on nominal wages and prices. The representative household has market power to set its wages from supplying labor, but at each point of time faces the probability of not being able to adjust these wages optimally. Firms have market power to set their product prices, but at each point of time face the probability of not being able to adjust these prices optimally. Third, a Taylor (1993) interest rate rule describes monetary policy. As a result of nominal rigidities, monetary policy has effects on real economic activity and then real bond yields. Fourth, the economy is affected by three types of shocks: permanent and productivity shocks, policy shocks, and inflation target shocks. Equilibrium in the economy implies that real and nominal yields are driven by these shocks and depend on preference, production, and policy parameters. While the first ingredient is common in asset pricing models, the last three ingredients are standard in New Keynesian models for the analysis of monetary policy. Our model calibration matches important first and second moments of macroeconomic variables and the high Sharpe ratio observed in the stock market. The implied average nominal yield curve is upward sloping, but the spread between long- and short-term yields and yield volatility are lower than in the data.

The average spread between long and short-term real bond yields captures real term premia and provides a measure of the difference in borrowing costs for the government of issuing long- vs. short-term bonds. The average spread between nominal and real bonds adjusted by inflation captures inflation risk premia and provides a measure of the difference in borrowing cost for the government of issuing nominal vs. real bonds. The model ability to capture positive real term and inflation risk premia crucially depends on wage rigidities and permanent productivity shocks. Both premia are negative in the absence of this rigidity.

Wage rigidities induce a procyclical but mean reverting labor demand and countercyclical inflation with respect to permanent productivity shocks. Bad news for productivity growth is bad news for consumption growth and labor demand, and generates inflation. Labor demand decreases since wages (marginal costs) are higher. However, expected future labor demand increases since wages will adjust downwards and generates positive expected consumption growth. Simultaneously, since wages are higher, producers increase their product prices to restore their markup, generating inflation. The positive effect on expected consumption growth increases interest rates and decreases real bond returns. Therefore, real term premia are positive since real bond returns are low and marginal utility is high at the same time. The increase in prices reduces the real return of nominal bonds. Therefore, inflation risk premia are positive since real returns on nominal bonds are low and marginal utility is high at the same time.

The unconditional volatilities of nominal v.s. real yields and long-term v.s. short-term yields are examined as an application of the model. In general, the shock to the inflation target has significant impact on the volatilities of nominal long-term bonds. This is due to the fact that the inflation target shocks generate volatility in inflation, making nominal yields more volatile, and are highly persistent, making long-term bonds more volatile. Monetary policy, on the other hand, has significant impact on the volatility ratios of nominal yields over real yields. When the policy rule is very responsive to inflation, the volatility of inflation decreases resulting in more stable nominal yields. The effects of stronger response to the output gap by the policy rule are the opposite: volatility ratios of nominal yields over real yields increase with the reaction coefficient.

There are possible diversification benefits of investing in real bonds for a constrained investor's portfolio. As a first step to study the diversification benefits of real bonds, we calculate the unconditional correlations between real returns on real bonds and real returns, excess of inflation, on nominal bonds. We find that these return correlations are the lowest in a fully flexible economy without permanent productivity shocks. This means the diversification benefits of TIPS are the greatest in the absence of wage and price rigidities. When there are permanent productivity shocks, the return correlations are the lowest when only price is sticky, and this is especially true for long-term bonds. Under the benchmark specification, strong monetary policy response to inflation keeps inflation risk premium low, and returns between real and nominal bonds become more correlated. Persistence in monetary policies also has the same effect on returns of long-term bonds: inflation risk premium is low, and nominal bonds behave more like real bonds resulting in more correlated returns.

The paper is organized as follows. Section 2 presents some descriptive statistics for nominal government bonds and treasury inflation protected securities. The principal component analysis in this section shows that two factors capture most of the variability of nominal and TIPS yields. Section 3 describes the economic model and its equilibrium conditions. Section 4 presents the analysis and section 5 concludes. The appendix contains all proofs.

2 Some Descriptive Statistics

We use United States monthly data for real and nominal bond yields from 1999 to 2008. The term structure series was obtained from monthly data on bond yields for yearly maturities from 1 to 20 years. The nominal and real yields are obtained following the procedure in Gurkaynak, Sack and Wright (2006) and Gurkaynak, Sack and Wright (2008), respectively. These data are published on the Federal Reserve website. The short-term nominal interest rate is the 3-month T-bill from the Fama risk-free rates database. TIPS with maturities between 2 and 4 years are only available since 2004. Table 1 reports the average yields and the standard deviation of yields for TIPS and nominal bonds. We report statistics computed for the sample $1999-2008$ and $2004-2008$, given concerns about liquidity in the TIPS market in the early period. The table shows upward sloping average curves and similar volatilities for TIPS and comparable nominal bonds. The nominal yield curve is steeper than the TIPS curve, suggesting positive inflation risk premia in nominal bonds.

[**Table 1 Here**]

Table 2 shows the variability of nominal bond and TIPS yields captured by their three principal components when these components are computed for TIPS and nominal bonds separately and jointly. Two principal components in the joint analysis can capture most of the variability of TIPS and nominal bond yields. The first principal component explains a significantly larger fraction of yield variability than the second component.

[**Table 2 Here**]

Figures 1 and 2 present the loadings of TIPS and nominal bond yields for different maturities for the 1999-2008 and 2004-2008 periods, respectively. The two figures show that the loadings on the first principal component for TIPS and nominal bonds are very similar in magnitude across all maturities. However, the magnitude increases slightly with maturity for the 1999-2008 period, and decreases considerably with maturity for the 2004-2008 period. The difference can be driven by the fact that the 2004-2008 period includes TIPS with 2, 3, and 4 years to maturity. The second principal component has bond yield loadings that are significantly higher for nominal bonds than for TIPS for comparable maturities. The loadings are decreasing with maturity for the 1999-2008 period and increasing with maturity for the 2004-2008 period. Tables 3 and 4 complete the analysis showing regressions of TIPS and nominal bond yields on the first three principal components. The adjusted R^2's show that most of the variability of both TIPS and nominal bonds is captured by these components. The explanatory power for 20-year TIPS and 30 year nominal bond yields is lower, suggesting that the long-end of the two curves has an additional important driving factor.

[Tables 3 and 4 Here]

3 Economic Model

We model a production economy where households derive utility from the consumption of a basket of goods and disutility from supplying labor to the production sector. The labor market is characterized by nominal wage rigidities. The production sector is characterized by monopolistic competition and nominal price rigidities. Wage and price rigidities generate real effects of monetary policy. When nominal prices and/or wages are not adjusted optimally, price and wage inflation generates distortions that affect production decisions. Since inflation is determined by monetary policy, different policies have different implications for real activity, and then real and nominal interest rates for all maturities. We model monetary policy as an interest-rate policy rule that reacts to economic conditions. The model can be seen as an extension of the standard New-Keynesian framework (see Woodford (2003), for instance) and is based on Li and Palomino (2012). It incorporates recursive preferences for households and is solved using a second-order perturbation to capture and analyze bond pricing dynamics.

3.1 Household

The representative agent in this economy chooses consumption C_t and labor N_t to maximize the Epstein and Zin (1989) recursive utility function

$$V_t = (1-\beta)U(C_t, N_t^s)^{1-\psi} + \beta \mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\psi}} \right]^{\frac{1-\psi}{1-\gamma}}, \tag{1}$$

where $0 < \beta < 1$ is the subjective discount factor, and ψ and γ determine the elasticity of intertemporal substitution and the coefficient of relative risk aversion, respectively. The recursive utility formulation allows us to relax the strong assumption of $\gamma = \psi$ implied by constant relative risk aversion. The intratemporal utility of consumption, C_t, and aggregate labor supply, N_t^s, is

$$U(C_t, N_t^s) = \left(\frac{C_t^{1-\psi}}{1-\psi} - \frac{(N_t^s)^{1+\omega}}{1+\omega} \right)^{\frac{1}{1-\psi}},$$

where ω^{-1} captures the Frisch elasticity of labor supply. The consumption good is a basket of differentiated goods produced in a continuum of firms. Specifically, consumption of the final good is

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where $\theta > 1$ is the elasticity of substitution across differentiated goods, and $C_t(j)$ is the consumption of the differentiated good j. Labor supply is the aggregate of a continuum of different labor types supplied to the production sector. Specifically,

$$N_t^s = \int_0^1 N_t^s(k) dk,$$

where $N_t^s(k)$ is the supply of labor type k.

The representative agent is subject to the intertemporal budget constrain

$$\mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} P_{t+s} C_{t+s} \right] \leq \mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} P_{t+s} \left(LI_{t+s} + \Psi_{t+s} \right) \right], \tag{3}$$

where $M_{t,t+s}^{\$}$ is the nominal discount factor for cashflows at time $t+s$, P_t is the nominal price of a unit of the basket of goods, LI_t is the real labor income from supplying labor to the production sector and Ψ_t is the aggregate profits and other claims to the production sector.

Appendix A shows that the household's optimality conditions imply that the one-period real and nominal stochastic discount factors are

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\psi} \left(\frac{V_{t+1}}{\mathbb{E}_t [V_{t+1}^{1-\gamma}]^{1/(1-\gamma)}} \right)^{\psi-\gamma}, \quad \text{and} \quad M^{\$}_{t,t+1} = M_{t,t+1} \left(\frac{P_{t+1}}{P_t} \right)^{-1}, \tag{4}$$

respectively. They allow us to price real and nominal default-free bonds. In particular, a one-period nominal bond has the price

$$e^{-i_t} = \mathbb{E}_t \left[M^{\$}_{t,t+1} \right], \tag{5}$$

where the one-period nominal interest rate i_t is the instrument of monetary policy.

Wage Setting

As in Li and Palomino (2012), we model an imperfectly competitive labor market where the representative household monopolistically provides a continuum of labor types indexed by $k \in [0, 1]$[2]. Specifically, the supply of labor type k satisfies the demand equation

$$N_t^s(k) = \left(\frac{W_t(k)}{W_t} \right)^{-\theta_w} N_t^d, \tag{6}$$

where N_t^d is the aggregate labor demand of the production sector, $W_t(k)$ is the wage for labor type k, and W_t is the aggregate wage index given by

$$W_t = \left[\int_0^1 W_t^{1-\theta_w}(k)\, dk \right]^{\frac{1}{1-\theta_w}}.$$

The labor demand equation (6) is derived in the production sector section below. The household

[2]This approach is different from the standard heterogeneous households approach to model wage rigidities as in Erceg, Henderson and Levin (2000), where each household supplies a differentiated type of labor. In the presence of recursive preferences, this approach introduces heterogeneity in the marginal rate of substitution of consumption across households since it depends on labor. We avoid this difficulty and obtain a unique marginal rate of substitution by modeling a representative agent who provides all different types of labor.

chooses optimal wages $W_t(k)$ for all labor types k under Calvo (1983) staggered wage setting. Specifically, each period the household is only able to adjust wages optimally for a fraction $1-\alpha_w$ of labor types. A fraction α_w of labor types adjust their previous period wages by the wage indexation factor $\Lambda_{w,t-1,t}$. We choose the indexation factor to capture real growth in the economy and inflation. The optimal wage maximizes (1) subject to demand functions (6) and the budget constraint (3), where real labor income is given by

$$LI_t = \int_0^1 \frac{W_t(k)}{P_t} N_t^s(k)dk\,.$$

Since both the demand curve and the cost of labor supply are identical across different labor types, the household chooses the same optimal wage W_t^* for all the labor types subject to a wage change at time t. Appendix A shows that the optimal wage satisfies

$$\frac{W_t^*}{P_t} = \mu_w \kappa_t \left(N_t^s\right)^{\omega} C_t^{\psi} \frac{G_{w,t}}{H_{w,t}}\,, \tag{7}$$

where $\mu_w \equiv \frac{\theta_w}{\theta_w - 1}$,

$$\begin{aligned} H_{w,t} &= 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{N_{t+1}^d}{N_t^d}\right) \left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w} H_{w,t+1} \right], \\ \text{and} \quad G_{w,t} &= 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{P_{t+1}}{P_t}\right) \left(\frac{C_{t+1}}{C_t}\right)^{\psi} \left(\frac{N_{t+1}^d}{N_t^d}\right) \left(\frac{\kappa_{t+1}}{\kappa_t}\right) \right. \\ &\qquad \left. \times \left(\frac{N_{t+1}^s}{N_t^s}\right)^{\omega} \left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w} G_{w,t+1} \right]. \end{aligned}$$

In the absence of wage rigidities ($\alpha_\omega = 0$), the optimal wage is given by the markup-adjusted marginal rate of substitution between labor and consumption, with optimal markup μ_w. Wage rigidities imply the time-varying markup $\mu_w \frac{G_{w,t}}{H_{w,t}}$.

3.2 Firms

The production of differentiated goods is characterized by monopolistic competition and price rigidities. Producers have market power to set the price of their differentiated goods in a Calvo (1983) staggered price setting. That is, a producer is able to change the product price optimally at each point of time, with a probability $1-\alpha$. If the producer is not able to adjust the price optimally, the price is the previous period price adjusted by the price indexation factor $\Lambda_{p,t,t+1}$. We choose an indexation factor that captures the previous period inflation. When the producer is able to adjust the price optimally, the price is set to maximize the present value of profits. The maximization problem can then be written as

$$\max_{\{P_t(j)\}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \alpha^s M^{\$}_{t,t+s} \left[\Lambda_{p,t,t+s} P_t(j) Y_{t+s|t}(j) - W_{t+s|t}(j) N^d_{t+s|t}(j) \right] \right\},$$

subject to the production function

$$Y_{t+s|t}(j) = A_{t+s} N^d_{t+s|t}(j),$$

and the demand function

$$Y_{t+s|t}(j) = \left(\frac{P_t(j) \left(\Lambda_{p,t,t+1}\right)^s}{P_{t+s}} \right)^{-\theta}.$$

The producer takes into account the probability of not being able to adjust the price optimally in the future. The price in this case is adjusted to incorporate the indexation $\Lambda_{p,t,t+s}$, for $s>0$.

We assume that labor productivity contains permanent and transitory components. Specifically, $A_t = A^p_t Z_t$, where the permanent and transitory components follow processes

$$\Delta \log A^p_{t+1} = \phi_a \Delta \log A^p_t + \sigma_a \varepsilon_{a,t+1},$$

and

$$\log Z_{t+1} = \phi_z \log Z_t + \sigma_z \varepsilon_{z,t+1},$$

respectively, with Δ as the difference operator, and innovations $\varepsilon_{a,t}$ and $\varepsilon_{z,t} \sim \text{IID}\mathcal{N}(0,1)$.

Labor demand in production is a composite of a continuum of differentiated labor types indexed by $k \in [0,1]$ via the aggregator

$$N_t^d(j) = \left[\int_0^1 N_t^d(j,k)^{\frac{\theta_w - 1}{\theta_w}}\, dj\right]^{\frac{\theta_w}{\theta_w - 1}}, \tag{8}$$

where $\theta_w > 1$ is the elasticity of substitution across differentiated labor types.

The appendix shows that the solution to this maximization problem involves solving the equation

$$\left(\frac{P_t^*}{P_t}\right) H_t = \frac{\mu}{A_t}\frac{W_t}{P_t} G_t, \tag{9}$$

where $\mu = \frac{\theta}{\theta-1}$, and the processes H_t and G_t are described by the recursive equations

$$\begin{aligned} H_t &= 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{p,t,t+1}^{1-\theta} \left(\frac{Y_{t+1}}{Y_t}\right) \left(\frac{P_{t+1}}{P_t}\right)^{-\theta} H_{t+1} \right], \\ \text{and} \quad G_t &= 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{p,t,t+1}^{-\theta} \left(\frac{Y_{t+1}}{Y_t}\right) \left(\frac{P_t}{P_{t+1}}\right)^{-\theta} \left(\frac{W_{t+1}}{W_t}\right) \left(\frac{A_t}{A_{t+1}}\right) G_{t+1} \right], \end{aligned}$$

respectively. The product price is the markup-adjusted marginal cost of production. In the absence of price rigidities, the markup is constant, given by μ. Price rigidities imply the time-varying markup $\mu \frac{G_t}{H_t}$.

3.3 Monetary Policy

We model a monetary authority that sets the level of a short-term nominal interest rate. Monetary policy is described by the policy rule

$$i_t = \rho i_{t-1} + (1-\rho)\left[\bar{\imath} + \imath_\pi(\pi_t - \pi^\star_{t-1}) + \imath_x x_t\right] + u_t. \tag{10}$$

The one-period nominal interest rate, i_t, has an interest-rate smoothing component captured by ρ, and is set responding to aggregate inflation, the output gap, and a policy shock u_t. The ouput gap x_t is defined as the log deviation of total output, Y_t from the output in an economy under flexible prices and wages, Y_t^f. That is, $X_t \equiv \frac{Y_t}{Y_t^f}$, and $x_t \equiv y_t - y_t^f$. The coefficients $\imath_x$, and $\imath_\pi$ capture the response of the monetary authority to the output gap and inflation, respectively. The process $\pi^\star_t$ denotes the time-varying inflation target. The inflation target is time-varying, as in **?** and Rudebusch and Swanson (2010). Its process is

$$\pi^\star_t = (1-\phi_{\pi^\star})g_\pi + \phi_{\pi^\star}\pi^\star_{t-1} + \sigma_{\pi^\star}\varepsilon_{\pi^\star,t},$$

where g_π is the unconditional mean of inflation, $\phi_{\pi^\star}$ is the autoregressive coefficient, $\sigma_{\pi^\star}$ is the conditional volatility of the inflation target shock, and $\varepsilon_{\pi^\star,t} \sim \text{IID}\mathcal{N}(0,1)$. The policy shocks u_t follow the process

$$u_{t+1} = \phi_u u_t + \sigma_u \varepsilon_{u,t+1},$$

where $\varepsilon_{u,t} \sim \text{IID}\mathcal{N}(0,1)$.

3.4 The Term Structure of Interest Rates

The price of a real and nominal bonds with maturity at $t+n$ can be written as

$$\exp\left(-r_t^{(n)}\right) = \mathbb{E}_t[M_{t,t+n}], \quad \text{and} \quad \exp\left(-i_t^{(n)}\right) = \mathbb{E}_t[M_{t,t+n}^{\$}], \tag{11}$$

respectively, where $r_t^{(n)}$ and $i_t^{(n)}$ are the associated real and nominal bond yields, and $M_{t,t+n}$ and $M_{t,t+n}^{\$}$ are the real and nominal discount factors for payoffs at time $t+n$.

3.5 Equilibrium

4 Analysis

We study the bond pricing implications of the economic model in section 3. We analyze the short-term real rate, the term premia in real bonds, and the inflation risk premia in nominal bonds. We describe the baseline calibration and present different model specifications with and without nominal rigidities to understand their main effects. We complete the analysis with comparative statics for rigidity and monetary policy parameters, and present impulse responses to the model shocks to understand the implications of monetary policy on the real term structure and inflation risk premia. Finally, we provide an analysis of the effects of nominal rigidities on the volatility of real and nominal bond yields, the government borrowing costs associated to nominal vs. real bonds, and the diversification benefits of real bonds.

4.1 Calibration

We calibrate the model to match some properties of United States macroeconomic and financial quarterly data from 1987:Q1 to 2010:Q4. We focus on the Greenspan era to avoid potential changes in monetary policy and the interest-rate policy rule, as suggested by Clarida, Galí and Gertler

(2000). The consumption growth series was constructed using quarterly data on real per-capita consumption of nondurables and services from the Bureau of Economic Analysis. The inflation series was constructed following the methodology in Piazzesi and Schneider (2007) to capture inflation related to non-durables and services consumption only. The three-month T-bill and the bond yield data series are from the Federal Reserve Economic Data (FRED II) published by the Federal Reserve Bank of St. Louis.

Table 5 presents the parameter values used in the calibration. Panel A is a collection of preference and rigidity parameters. The values are standard in the macroeconomic literature, except for the parameter γ which is chosen to capture the high Sharpe ratio of financial assets. Under recursive preferences on consumption and labor, the average coefficient of risk aversion can be computed as in Swanson (2012). It is

$$\frac{\psi}{1+\frac{\psi}{\omega\mu}} + \frac{\gamma-\psi}{1-\frac{1-\psi}{1+\omega}} \approx 32\,.$$

The elasticities θ and θ_ω are set such that the markups in prices and wages are 20%, which is within the range found in the literature. The rigidity parameters α and α_w capture the average duration of prices and wages, respectively, in the data.

[Table 5 Here]

Panel B displays the parameters of the interest-rat monetary policy rule. The parameters satisfy the Taylor principle and then ensure a stable equilibrium. The baseline values of the Taylor rule coefficients are consistent with what Clarida, Galí and Gertler (2000) find for the Greenspan era. Panel C shows the autoregressive coefficients and the conditional volatilities of the policy shock to the Taylor rule, the permanent productivity shock and the transitory productivity shock. Following Li and Palomino (2012), these numbers are chosen to match some of the empirical results presented in Altig, Christiano, Eichenbaum and Linde (2011). Spcifically, the variance of inflation, de-trended consumption, and the nominal short-term rate attributed to productivity and policy shocks. Finally, panel D shows values for parameters controlling the time-varying inflation target. The unconditional mean of inflation is chosen to be roughly 2% annually. This helps us to match the level of the nominal short rate. The autoregressive coefficient and the conditional volatility of the shock to the inflation target are in line with values used by Rudebusch and Swanson (2010). The parameter σ_π is chosen to match the unconditional volatility of inflation.

Table 6 shows selected moments implied by the data and the model. The model does a good job matching the volatility of de-trended consumption, consumption growth, and inflation. The average levels of inflation and the short-term rate are also matched. The macroeconomic volatility in the model is not enough to capture the high volatility of the short-term nominal interest rate. While matching the average level of the short end of the nominal yield curve, the model fails to generate the observed average slope of the nominal curve between the 3-month yield and the 5-year yield. Increasing the risk aversion parameter, γ, does not seem to make a noticeable difference. The time-varying inflation target has the greatest impact on the average volatility of long-term yields.

[**Table 6 Here**]

4.2 Model Specification

To understand how each component of the model contributes to the results, we reproduce summary statistics under different model specifications by shutting down exogenous shocks and nominal rigidities in turn. We focus our attention on unconditional means and volatilities of macroeconomic and financial variables implied by the model. The variable definitions are as follow: $\tilde{c}$ is de-trended real consumption, x is the output gap, w is the real wage, $log(\mu)$ is the price markup due to monopolistic power, Δc is real consumption growth, Δw is real wage growth, π is inflation, r is the real interest rate and i is the nominal interest rate. $IRP^{(n)}$ stands for the inflation risk premium calculated as the difference between the nominal yield and the real yield for n-quarter bonds and subtracting expected inflation.

Table 7 compares the benchmark specification to the model specification when only one exogenous shock is turned on. Examining the unconditional inflation risk premium and the average slope of the nominal curve, the permanent productivity shock is the most important shock to bond risk premia because no other shock by itself can generate the same level of IRP. This is consistent with the fact the permanent productivity shock contributes the most to the average volatility of real consumption growth, which leads to higher volatility on the pricing kernel. Looking at the effects of the inflation target shock, it contributes the most to the second moments of real output, inflation and interest rates. The shock to the inflation target has an especially strong impact on the volatility of the long-term nominal interest rate that no other shock can reproduce. On an interesting note, the transitory productivity shock seems to have a strong effect on the volatility of the real wage. This can be due to the presence of wage rigidities.

[Table 7 Here]

Besides the different shock specifications, the model also allows for comparisons with and without price and wage rigidities. Table 8 presents summary statistics of the model under four scenarios: no rigidities[3], only wage rigidity (WR)[4], only price rigidity (PR) and both rigidities (benchmark). The significance of WR on bond risk premia is striking. In the absence of WR, columns (2) and (4), the unconditional IRP is negative, and the nominal term structure is downward sloping, on average. A closer examination of the level of real yields shows that the average real term structure is also downward sloping when WR is turned off. This observation implies that WR makes long-term bonds risky instruments since their cashflows are low during periods of high marginal utility.

[Table 8 Here]

[3]In the absence of both price and wage rigidities, there is no output gap and markup.

[4]In the absence of price rigidities, there is no resulting markup for the firm.

Since the permanent productivity shock is the major source of bond risk premia, we want to see how the model would behave under different rigidity specifications in the absence of these shocks. Table 9 presents summary statistics of the model under five scenarios with no permanent productivity shocks: no rigidities, only wage rigidity (WR), only price rigidity (PR) , both WR and PR, and the benchmark specification. Similar to table 8, we see that the unconditional IRP is negative without WR in columns (2) and (4). However, by turning the permanent productivity shock off, PR alone is enough to generate an upward sloping nominal yield curve, on average. This is consistent with what the existing literature using DSGE models to study the term structure has found[5]. The average real yield curve is almost flat across the five different specifications in table 9, and then negligible term premia. This evidence suggests that the permanent productivity shock not only affects the magnitude of bond risk premia, as shown in table 7, but it also drives the hedging properties of long-term bonds. Without WR, permanent shocks make long-term bonds hedging instruments against high marginal utility, resulting in negative risk premia.

[Table 9 Here]

4.3 Dynamic Responses

We study the impulse response functions of the individual shocks to understand how the interaction between shocks and rigidities affects the dynamic behavior of the endogenous variables in the model. From a steady state, we perturb the system with a positive one standard deviation permanent productivity shock, transitory productivity shock, policy shock and inflation target shock, individually. For each shock, we examine the impulse response of the endogenous variables under four rigidity specification: the benchmark, no rigidities, wage rigidity only (No PR) and price rigidity only (No WR).

Figure 3 presents the impulse response of macroeconomic variables and interest rates following a positive one standard deviation permanent productivity shock. When prices and wages are fully

[5]See Rudebusch and Swanson (2010) and Hsu (2012) for example

flexible, real output does not respond to the permanent productivity shock while inflation and interest rates show very strong reactions. The opposite is true when prices and wages are sticky. Interest rates in general increase immediately following the permanent shock in the absence of rigidities, while they decrease first when prices and wages are rigid.

Figure 4 show the impulse responses following a positive one standard deviation transitory productivity shock. Inflation increase following shock and is very persistent in the presence of price rigidity. Interest rates generally decrease after the shock with the exception of the real short rate. When prices are rigid, the real short rate increase first then decrease before reverting back to the steady state. The slope of the nominal term structure has a strongly positive reaction to the transitory shock when wages are rigid. Finally, the response of the real yields is much weaker following the transitory productivity shock than following the permanent productivity shock.

The dynamic response of the endogenous variables due to a positive one standard deviation policy shock are displayed in figure 5. Not surprisingly, when prices and wages are fully flexible, the policy shock solicit almost no reaction from the economy except inflation. With rigidities, the policy shock generates strong reactions in real and nominal yields, especially at the short-end of the yield curves. The is consistent with the fact that the nominal short rate is the policy instrument. In addition, the policy shock has a flattening effect on the nominal term structure under price and wage rigidities.

Finally, we examine the impulse responses following a positive one standard deviation shock to the inflation target in figure 5. Similar to the policy shock, the inflation target shock does not generate much reaction from the macroeconomy in the absence of rigidities. It has a big impact on inflation, however, when prices and wages are fully flexible. Furthermore, nominal yields increase following the positive shock and stay elevated due to the persistence of the inflation target shock. This is purely due to the inflation effect since the shock has no impact on real yields. When prices and wages are sticky, the inflation target shock lowers productivity and consumption growth, which leads to lower real interest rates after the shock. The inflation target shock also has a steepening effect on the nominal term structure in the presence of sticky price and sticky wage.

4.4 Comparative Statics

We conduct comparative statics on the model by perturbing selected parameters to gain further insight into the mechanism delivering the results. Table 10 contains summary statistics of the model when we vary six parameters, one at a time, while keeping all other parameters at their baseline values. Under each parameter, the middle column shows the summary statistics under the benchmark specification. The column to the right of the benchmark shows how the model responds when the parameter value is increased, and the column to the left of the benchmark shows the response when the parameter value is decreased.

[**Table 10 Here**]

In panel A, we focus on the parameters governing price rigidity (PR), α, and wage rigidity(WR), α_w. As α and α_w increase, the degree of rigidity goes up in the economy. This means price and wage, respectively, become stickier. Immediately, it is apparent that the impact of WR on bond risk premia is more pronounced than the impact of PR on bond risk premia, and the two frictions work in opposite directions. Higher PR leads to lower unconditional IRP and lower average slope of the nominal curve, while the opposite is true for WR. A 5% decrease in WR translates to a 15 basis points decrease in the unconditional IRP, or roughly 40% of the benchmark value.

In panel B, we vary the autoregressive coefficient and the conditional volatility of the shock to the inflation target. Not surprisingly, when the inflation target shock becomes more persistent or more volatile, the unconditional IRP increases, and the nominal term structure becomes steeper, on average. The impact of these parameters is particularly striking on the unconditional volatility of the long-term nominal interest rate. When $\sigma_{\pi^\star}$ is zero, equivalent to shutting down the inflation target shock, the long-term yield becomes very stable, and the model has trouble matching the data.

[**Table 11 Here**]

Table 9 presents comparative statics on the Taylor rule coefficients. The coefficients $\imath_\pi$ and $\imath_x$ capture the reaction of the nominal short rate on inflation and output gap, respectively. The coefficient ρ captures interest-rate smoothing. Consistent with the existing literature on monetary policy, as the central bank tightens monetary policy by increasing $\imath_\pi$, the unconditional volatility of inflation decreases, leading to lower average IRP and a flatter nominal yield curve. In addition, a stronger response to inflation also has a stabilizing effect on the economy, evident from lower unconditional standard deviations of output gap, consumption growth, inflation and long-term nominal rates. On the other hand, the effects of a stronger response to the output gap by the central bank through increasing $\imath_x$ are completely opposite. It generates inflation and associated IRP while making the economy more volatile in general. Lastly, when the the nominal short rate is more persistent, high ρ, monetary policy is more stable. Therefore, even though inflation is higher, on average, the unconditional IRP is actually lower, and the nominal yield curve is flatter.

4.5 Applications

Term Premia

The average spread between long- and short-term bonds contains the average compensation for risk required by investors to hold long-term bonds over short-term bonds. It can be shown that average spreads for real bonds satisfy

$$n\mathbb{E}\left[r_t^{(n)} - r_t\right] + \frac{1}{2}\sum_{k=1}^{n-1}(n-k)^2\mathbb{E}\left[\mathrm{var}_t\left(r_{t+1}^{(n-k)}\right)\right] = \sum_{k=1}^{n-1}(n-k)\mathbb{E}\left[\mathrm{cov}_t\left(m_{t,t+1}, r_{t+1}^{(n-k)}\right)\right].$$

Therefore, long-term real bonds contain compensations for permanent and transitory productivity shocks, policy shocks and inflation target shocks. The average real term structure is upward sloping in the model, implying positive risk premia for long-term real bonds. Following a negative permanent productivity shock, consumption growth is low meaning marginal utility is high. In the presence of sticky wages, real interest rates increase after the negative shock, and prices of real bonds decrease. This means that in the bad state of the world when marginal utility is high, long-term real bonds are cheap, making them bad hedges against consumption risk. Therefore, ex ante, investors require positive term premia in exchange for holding real bonds. Wage rigidity is crucial to generate the positive covariance between the marginal utility and yields after a permanent productivity shock is realized. If prices and wages are fully flexible or if only prices are sticky, then this covariance is negative meaning risk premia on long-term real bonds are negative.

In the case of transitory productivity shocks, the dynamics between marginal utility and bond prices are similar but less quantitatively important. Figure 4 shows that consumption growth is low following a negative transitory productivity shock resulting in higher real marginal utility. However, the response of real yields is weaker compared to the permanent productivity shock, and the price of long-term real bonds decreases regardless of the rigidity specification. The policy shock only affects the term premium in the presence of rigidities, price or wage. In figure 5, a positive shock to the nominal short rate lowers consumption growth making the real marginal rate of consumption substitution higher. At the same time, real yields increase following the shock if price or wage or both are sticky. Therefore, the policy shock by itself can generate the positive covariance between the real marginal utility and real interest rates, leading to positive term premia for long-term real bonds.

The impact of wage rigidity on bond risk premia in conjunction with the inflation target shock is actually negative. Like the policy shock, the inflation target shock does not affect real consumption growth in a fully flexible economy and has no impact on bond risk premium. From the impulse responses in figure 6, when wages are sticky, a positive inflation target shock raises the marginal utility of consumption while decreasing real yields making real bonds more expensive. Thus, in the bad state of the world when consumption growth is low, long-term real bonds payoff higher and are good hedging instruments against consumption risk in the presence of wage rigidity. The resulting term premium is negative.

Inflation Risk Premia

Nominal bond yields can be decomposed into real bond yields and an inflation compensation as

$$ni_t^{(n)} = nr_t^{(n)} + \mathbb{E}_t[\pi_{t,t+n}] - \frac{1}{2}\text{var}_t(\pi_{t,t+n}) + \text{cov}_t(m_{t,t+n}, \pi_{t,t+n}),$$

where

$$\pi_{t,t+n} = \sum_{s=1}^{n} \pi_{t+s}$$

is the inflation observed between t and $t+n$ and

$$m_{t,t+n} = \sum_{s=1}^{n-1} \log M_{t+s,t+s+1}$$

is the marginal rate of substitution of consumption between t and $t+n$. The difference between nominal and real yields contains the expected inflation during the life of the bond and compensations for inflation risk or inflation risk premia. These premia capture the expected excess real return for investing in nominal n-period bonds over real n-period bonds for n periods. Investors require a compensation for holding nominal bonds over real bonds because the marginal utility of consumption is correlated with inflation and the return of nominal bonds is affected by inflation. When consumption and inflation are uncorrelated, the expected real returns on real and nominal bonds with the same maturity are the same. Then, understanding the sources of covariance between consumption and inflation helps us to understand the determinants of the inflation risk premia. The covariance of the real discount factor and inflation, expressed as a function of their sensitivities to the exogenous shocks, determines the inflation risk premia.

Following a negative permanent productivity shock, consumption growth is low and the marginal rate of consumption substitution is high. Without rigidities, inflation is low, and low inflation increases the payoff on nominal bonds in real terms. The negative covariance between the marginal utility and inflation generates negative inflation risk premium for the investors who hold nominal bonds. However, when wages are sticky, inflation is high after the negative permanent productivity shock. High inflation erodes the payoff on nominal bonds in real terms making them bad hedges against inflation in the bad state of the world. As a result, investors demand a positive inflation risk premium in exchange for holding nominal bonds. Figure 3 outlines the dynamics of consumption growth and inflation due to a permanent productivity shock.

The dynamics of consumption growth and inflation in the aftermath of a transitory productivity shock are shown in figure 4. Following a negative transitory shock, marginal rate of consumption substitution is high while inflation is also high in the presence of wage rigidity. The positive covariance between the real marginal utility and inflation means nominal bonds are risky and inflation risk premium is positive. The mechanism is the same here as for the permanent productivity shock, but the magnitudes are different. A one standard deviation permanent productivity shock generates a greater response in consumption growth than a one standard deviation transitory productivity shock does.

Figure 5 helps us understand the impact of the policy rule shock on inflation risk premium. In a fully flexible economy, marginal utility is not affected by the positive policy shock. This means that even though inflation is low, inflation risk premium is actually zero. Price rigidity makes the policy shock non-neutral to the real economy, and the marginal utility increases after the positive shock is realized. However, inflation is lower following the same shock in the case of only sticky prices. This means nominal bonds are good hedging instruments, and inflation risk premium is negative. Sticky wages neutralize the reaction of inflation due to the policy shock, and the negative inflation risk premium is mitigated.

The inflation target shock and inflation risk premium are also connected through consumption growth and inflation. Similar to the policy shock, figure 6 shows that the inflation target shock is neutral with respect to consumption growth when prices and wages are fully flexible leading to zero inflation risk premium. However, wage rigidity changes the dynamics by allowing the inflation target shock to affect consumption growth. Following a positive inflation target shock, marginal rate of consumption substitution is high when wages are sticky. Inflation is also high following the same shock, eroding the payoff on nominal bonds. The positive covariance between marginal utility and inflation means wage rigidity helps the inflation target shock to generate positive inflation risk premium.

We can link the inflation risk premium to government borrowing costs. When inflation and output are correlated the return required by investors to hold comparable real and nominal bonds differ. The government then can reduce the cost of issuing debt by choosing between real or nominal debt. Since the sign of inflation risk premium depends on the presence of wage rigidity in this economy, issuing nominal bonds involves a lower financing cost for the government than issuing comparable real bonds if wages are fully flexible. The savings become larger for weaker responses to economic conditions in the policy rule.

Volatility of Real and Nominal Yields

Table 1 shows that the volatility of TIPS has been similar or even higher than the volatility of comparable nominal government bonds. It is reasonable then to ask whether different monetary policies can have different implications on the volatility of real and nominal bonds. We compute the ratio of the unconditional volatilities of nominal and real bonds implied by the model. Table 12 is the comparative statics of the ratios of unconditional volatilities of nominal yields over real yields at different maturities (top two rows in each panel) as well as the ratios of unconditional volatilities of long-term yields over short-term yields (bottom two rows in each panel). Panel A shows the effects of price and wage rigidities on the volatility ratios. Overall, the rigidities do not have a significant impact on the volatility ratios. Generally speaking, higher rigidities increase the ratios with the exception of wage rigidity on long-term interest rates. Higher wage rigidity decrease the ratio of volatilities slightly. There is minimal impact on the volatility ratios between long-term and short-term bonds from price and wage rigidities.

[Table 12 Here]

Panel B in table 12 presents the comparative statics by varying the coefficients governing the inflation target shock. Since the persistence of the inflation target shock as well as its conditional volatility both increase the volatility of inflation, it is straight forward to explain the increase in the volatility ratios at both ends of the term structure. Because inflation volatility only affects nominal yields and not real yields, the volatility ratios increase dramatically with $\phi_{\pi^\star}$ and $\sigma_{\pi^\star}$ at all maturities. An interesting observation is that the only incidence where the unconditional volatility ratio of nominal over real is less than one is in the absence of the inflation target shock ($\sigma_{\pi^\star} = 0$) for short-term rates. Without the shock to the inflation target, nominal interest rates become very stable. The model requires another potential source of variations to generate observed volatilities on real yields.

Panel B also demonstrates the importance of the inflation target shock in generating volatility on long-term nominal yields. When the conditional volatility of the inflation target shock is zero, the long-term nominal yield is not volatile compared to the short-term yield. However, when the inflation target shock is switched on, the volatility ratio of long-term over short-term nominal yields gets closer to one.

Panel C shows that more aggressive responses to economic conditions in the policy rule reduces the volatility of nominal yields with respect to real yields. The reason is that more aggressive policies reduce the volatility of output and inflation. As an extreme case, when inflation is constant, real and nominal rates move one to one. Monetary policy also affects the volatility of long-rates with respect to short-term rates. As the response of monetary policy to economic conditions increases the rate of decay in volatility across maturities increases for both nominal and real curves.

The Correlation Between Nominal and Real Bonds and the Diversification Benefits of Real Bonds

An interesting question to ask is whether real bonds provide investors with additional diversification benefits. The complete-market environment that characterizes the economic model in this paper does not allow us to obtain a satisfactory answer to this question. However, we can provide some insights into the benefits of diversification of a real bond and how it might be valuable to a constrained investor who only has access to a nominal bond with a particular maturity. Given that there are four sources of risk affecting the marginal utility of consumption, this investor faces an incomplete market and could be benefited by the existence of a real bond. We try to capture the risk sharing benefits of a real bond for this investor by computing the unconditional correlation between the realized real return of the nominal bond and the realized return of the real bond with the same maturity.

[**Table 13 Here**]

Table 13 panel A presents the unconditional correlations of returns on real bonds and real returns (excess of inflation) on nominal bonds under different rigidity specifications. Realized returns are calculated over one quarter holding horizons. Return correlation is a decreasing function of time to maturity. The correlation is the weakest when only prices are sticky. On the other hand, the correlation is the strongest when only wages are sticky.

The returns of long-term real and nominal bonds are the least correlated when only prices are rigid. This implies greater benefits of diversification for the constrained investor. In the absence of price and wage rigidities, the return correlations are high even for long-term bonds. By examining the impulse responses, we see that both real and nominal interest rates react the strongest and in the same direction following a one standard deviation permanent productivity shock when prices and wages are flexible. Whereas the interest rate responses are mild or none following a one standard deviation transitory productivity shock, a policy shock or a inflation target shock when prices and wages are flexible. The permanent productivity shock seems to drive the high correlations in a fully flexible economy.

Panel B in table 13 confirms this intuition by showing the return correlations under different rigidity specifications when the permanent productivity shock is switched off. Immediately, we notice that return correlations under no rigidities are much smaller in panel B than panel A, and they are monotonically increase with maturity. Wage rigidity still drives large portions of the correlations across maturities. Finally, in the absence of the permanent productivity shock, returns on real bonds and real returns on nominal bonds are the least correlated under no rigidities, making real bonds desirable for portfolio diversification purposes.

[**Table 14 Here**]

Table 14 presents the comparative statics of the real and nominal return correlations as functions of monetary policy rule variables. Real returns for real and nominal bonds become more correlated, unconditionally, under tight monetary policy regimes when $\imath_\pi$ is high or when $\imath_x$ is low. This is consistent with the fact that inflation risk premium is low when monetary policy is tight, and nominal yields behave more like real yields once inflation is subtracted. Furthermore, when the nominal short rate is persistent, ρ is high, the correlations are also high. This is also due to the fact that inflation risk premium is low under high persistence, and nominal yields act like real yields.

5 Conclusion

This paper explores the implications of a monetary policy with real economic effects on the compensations for risk in long-term real bonds and the inflation risk premium in nominal bonds. The analysis shows that productivity growth shocks in the presence of wage rigidity implies positive compensations for risk in long-term real bonds and positive inflation risk premia. Stronger monetary policy reaction to inflation increases the consumption hedging properties of nominal bonds implying lower inflation risk premia, while decreases the consumption hedging properties of long-term real bonds implying higher real term premia. Stronger monetary policy response to the output gap has the opposite outcome, inflation risk premia go up and real term premia go down. Persistence in the nominal short rate decreases bond risk premia as a result of stable monetary policy.

In an economy where productivity shocks can be permanent, wage rigidity is crucial to generate positive average inflation risk premium and positive average real term premium in the model. A negative permanent productivity shock lowers consumption growth and raises the marginal utility. When wages are rigid, inflation is high following the permanent productivity shock leading to positive inflation risk premium, and long-term TIPS are cheap after the same shock resulting in positive real term premium. If wages are flexible, then the direction of response of inflation and TIPS price is flipped generating negative average risk premia, regardless of price rigidity.

This implication of wage rigidity on bond risk premia does not hold when productivity shocks are always transitory. Without permanent productivity shocks, the unconditional real term premium is always positive regardless of the rigidity specification, while the unconditional inflation risk premium is positive in the presence of rigidities, wage or price. This is largely driven by the transitory productivity shock and the monetary policy shock, both of which have significant impact on inflation and long-term real yields in comparison to the inflation target shock.

Despite the friction presented by wage and price rigidities, the calibrated model fails to capture the average yield spreads of the real and nominal term structures. Specifically for nominal bonds, the unconditional slope of the model-implied yield curve is roughly one quarter of what is observed in the data from 1982 to 2010. This "bond premium puzzle" is particular prevalent in models with production economies where the representative agent can further smooth consumption through labor supply. As the next step, we aim to incorporate other source of friction, such as heterogeneous households, in the model to better capture the dynamics of the yield curve and resolve the bond premium puzzle.

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A Household's Utility Maximization under Wage Rigidities (from Li and Palomino (2012))

The household's problem is

$$\max_{\{C_t, N_t^s, W_t^*\}} \quad V_t = U_t + \beta Q_t^{\frac{1-\psi}{1-\gamma}}$$

where

$$U_t = \frac{C_t^{1-\psi}}{1-\psi} - \kappa_t \frac{(N_t^s)^{1+\omega}}{1+\omega}, \quad \text{and} \quad Q_t = \mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\psi}} \right],$$

subject to the budget constraint

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} C_{t+\tau} \right] \leq \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} \left(LI_{t+\tau} + D_{t+\tau} \right) \right],$$

where LI_t and D_t are aggregate labor income and firm profits, respectively. The Lagrangian associated with this problem is

$$\mathcal{L} \;=\; \frac{C_t^{1-\psi}}{1-\psi} - \kappa_t \frac{(N_t^s)^{1+\omega}}{1+\omega} + \beta Q_t^{\frac{1-\psi}{1-\gamma}} + \lambda \, \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau}^{\$} P_{t+\tau} \left(LI_{t+\tau} + D_{t+\tau} - C_{t+\tau} \right) \right].$$

It can be shown that utility maximization implies $\lambda = \frac{C_t^{-\psi}}{P_t}$, and

$$\begin{aligned} M_{t,t+1}^{\$} &= \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t} \frac{P_t}{P_{t+1}} = \beta \frac{\frac{\partial Q_t}{\partial C_{t+1}} \frac{\partial V_t}{\partial Q_t}}{C_t^{-\psi}} \frac{P_t}{P_{t+1}} \\ &= \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\psi} \left(\frac{V_{t+1}^{1/(1-\psi)}}{Q_t^{1/(1-\gamma)}} \right)^{\psi-\gamma} \frac{P_t}{P_{t+1}}. \end{aligned}$$

The τ-period nominal pricing kernel is

$$M_{t,t+\tau}^{\$} = \prod_{s=1}^{\tau} M_{t,t+s}^{\$}.$$

The household cannot change wages for α_w fraction of labor types. For the remaining $1-\alpha_w$ fraction of labor types k, the household chooses wages $W_t^*(k)$ to maximize V_t. We assume that the wage choice for one labor type has negligible effects on the aggregate wage index and the aggregate labor demand. To see the impact of $W_t^*(k)$

on the household's utility, we rewrite the labor supply at $t+\tau$ as

$$N^s_{t+\tau} = \int_0^1 N^s_{t+\tau}(k)\, dk = N^d_{t+\tau} \int_0^1 \left(\frac{W_{t+\tau}(k)}{W_{t+\tau}} \right)^{-\theta_w} dk,$$

and the aggregate labor income at $t+\tau$ as

$$LI_{t+\tau} = \int_0^1 \frac{W_{t+\tau}}{P_{t+\tau}}(k) N^s_{t+\tau}(k)\, dk = \frac{N^d_{t+\tau} W_{t+\tau}}{P_{t+\tau}} \int_0^1 \left(\frac{W_{t+\tau}(k)}{W_{t+\tau}} \right)^{1-\theta_w} dk.$$

For the wage of type k labor at $t+\tau$, there are $\tau+2$ possible values:

$$W_{t+\tau}(k) = \begin{cases} W^*_{t+\tau-s}(k), & \text{with prob} = (1-\alpha_w)\alpha_w^s \text{ for } s = 0, 1, \cdots, \tau \\ W_{t-1}\Lambda_{w,t-1,t+\tau}, & \text{with prob} = \alpha_w^{\tau+1}. \end{cases}$$

We obtain derivatives

$$\begin{aligned} \frac{\partial N^s_{t+\tau}}{W^*_t(k)} &= N^d_{t+\tau}(1-\alpha_w)\alpha_w^\tau \left(\frac{-\theta_w}{W^*_t(k)} \right) \left(\frac{W^*_t(k)\Lambda_{w,t,t+\tau}}{W_{t+\tau}} \right)^{-\theta_w}, \\ \frac{\partial LI_{t+\tau}}{\partial W^*_t(k)} &= \frac{N^d_{t+\tau}}{P_{t+\tau}}(1-\alpha_w)\alpha_w^\tau (1-\theta_w) \left(\frac{W^*_t(k)\Lambda_{w,t,t+\tau}}{W_{t+\tau}} \right)^{-\theta_w}. \end{aligned}$$

The first-order condition of the Lagrangian with respect to $W^*_t(k)$ is given by

$$\frac{\partial \mathcal{L}}{\partial W^*_t(k)} = \frac{\partial V_t}{\partial W^*_t(k)} + \lambda\, \mathbb{E}_t \left[\sum_{\tau=0}^\infty M^\$_{t,t+\tau} P_{t+\tau} \frac{\partial LI_{t+\tau}}{\partial W^*_t(k)} \right] = 0\,,$$

where

$$\frac{\partial V_t}{\partial W^*_t(k)} = -\mathbb{E}_t \left[\sum_{\tau=0}^\infty M^\$_{t,t+\tau} \frac{P_{t+\tau}}{P_t} \left(\frac{C_{t+\tau}}{C_t} \right)^\psi \kappa_{t+\tau} (N^s_{t+\tau})^\omega \frac{\partial N^s_{t+\tau}}{\partial W^*_t(k)} \right].$$

Rearranging terms, we get

$$\mathbb{E}_t \left[\sum_{\tau=0}^\infty M^\$_{t,t+\tau} \Lambda_{w,t,t+\tau} \alpha_w^\tau W^{\theta_w}_{t+\tau} N^d_{t+\tau} \frac{W^*_t(k)}{P_t} C_t^{-\psi} \right] = \mathbb{E}_t \left[\sum_{\tau=0}^\infty M^\$_{t,t+\tau} \Lambda_{w,t,t+\tau} \alpha_w^\tau \left(\frac{P_{t+\tau}}{P_t} \right) W^{\theta_w}_{t+\tau} N^d_{t+\tau} \mu_w \kappa_{t+\tau} (N^s_{t+\tau})^\omega \left(\frac{C_{t+\tau}}{C_t} \right)^\psi \right].$$

Since all labor types face the same demand curve, we have $W^*_t(k) = W^*_t$ for all k. We can write the left-hand side of the equation as

$$LHS = C_t^{-\psi} W_t^{\theta_w} N^d_t H_{w,t} \frac{W^*_t}{P_t},$$

where

$$H_{w,t} = 1 + \alpha_w \mathbb{E}_t \left[M^{\$}_{t,t+1} \Lambda_{w,t,t+1} \left(\frac{N^d_{t+1}}{N^d_t} \right) \left(\frac{W_t}{W_{t+1}} \right)^{-\theta_w} H_{w,t+1} \right].$$

Similarly, the right-hand side of the first-order condition can be written as

$$RHS = \mu_w W_t^{\theta_w} N_t^d (N_t^s)^{\omega} G_{w,t} = \mu_w W_t^{\theta_w} N_t^d \kappa_t \left(N_t^s\right)^{\omega} G_{w,t}$$

where

$$G_{w,t} \quad = \quad 1 + \alpha_w \, \mathbb{E}_t \left[M^{\$}_{t,t+1} \Lambda_{w,t,t+1} \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{C_{t+1}}{C_t} \right)^{\psi} \left(\frac{N^d_{t+1}}{N^d_t} \right) \left(\frac{\kappa_{t+1}}{\kappa_t} \right) \left(\frac{N^s_{t+1}}{N^s_t} \right)^{\omega} \left(\frac{W_t}{W_{t+1}} \right)^{-\theta_w} G_{w,t+1} \right].$$

The optimal real wage and the optimal wage markup $\mu_{w,t}$ are then given by

$$\frac{W_t^*}{P_t} = \mu_{w,t} C_t^{\psi} \kappa_t \left(N_t^s\right)^{\omega} \quad \text{and} \quad \mu_{w,t} = \mu_w \frac{G_{w,t}}{H_{w,t}}.$$

B Profit Maximization under Price Rigidities (from Li and Palomino (2012))

Consider the Dixit-Stiglitz aggregate (2) as a production function, and a competitive "producer" of a differentiated good facing the problem

$$\max_{\{C_t(j)\}} P_t C_t - \int_0^1 P_t(j) C_t(j) dj$$

subject to (2). Solving the problem, we find the demand function

$$P_t(j) = P_t \left(\frac{C_t(j)}{C_t} \right)^{-1/\theta} \tag{12}$$

The zero-profit condition implies

$$P_t C_t = \int_0^1 P_t(j) C_t(j) dj = \int_0^1 P_t C_t \left(\frac{P_t(j)}{P_t} \right)^{-\theta} dj.$$

Solving for P_t, it follows that

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}, \tag{13}$$

which can be written as the demand function for each differentiated good

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} C_t\,. \tag{14}$$

Therefore, when prices are flexible, prices of all differentiated goods are the same.

The profit maximization problem is

$$\max_{\{P_t(j)\}} \quad \mathbb{E}_t\left[\sum_{\tau=0}^{\infty} M^{\$}_{t,t+\tau}\alpha^{\tau}\left[\Lambda_{p,t,t+\tau}P_t(j)Y_{t+\tau|t}(j) - W_{t+\tau|t}(j)N^d_{t+\tau|t}(j)\right]\right]$$

subject to

$$Y_{t+\tau|t}(j) = Y_{t+\tau}\left(\frac{P_t(j)}{P_{t+\tau}}\right)^{-\theta}, \quad \text{and} \quad Y_{t+\tau|t}(j) = A_t N^d_{t+\tau|t}(j)\,.$$

The first-order condition of this problem with respect to $P_t(j)$ is

$$\mathbb{E}_t\left[\sum_{\tau=0}^{\infty} M^{\$}_{t,t+\tau}\alpha^{\tau}Y_{t+\tau|t}(j)\Lambda_{p,t,t+\tau}P^*_t(j)\right] = \mathbb{E}_t\left[\sum_{\tau=0}^{\infty} M^{\$}_{t,t+\tau}\alpha_I^{\tau}Y_{t+\tau|t}(j)\mu\frac{W_{t+\tau|t}(j)}{A_{t+\tau}}\right].$$

The left-hand side (LHS) of the equation can be written recursively as

$$LHS = P^*_t\left(\frac{P^*_t}{P_t}\right)^{-\theta} Y_t H_t,$$

where

$$H_t = 1 + \alpha\mathbb{E}_t\left[M^{\$}_{t,t+1}\Lambda^{1-\theta}_{p,t,t+\tau}\left(\frac{Y_{t+1}}{Y_t}\right)\left(\frac{P_t}{P_{t+1}}\right)^{-\theta} H_{t+1}\right].$$

Similarly, the right-hand side (RHS) of the equation can be written as

$$RHS = \frac{\mu}{A_t}Y_t\left(\frac{P^*_t}{P_{I,t}}\right)^{-\theta}\frac{W_t}{P_t}P_t G_t$$

where

$$G_t = 1 + \alpha\mathbb{E}_t\left[M^{\$}_{t,t+1}\Lambda^{-\theta}_{p,t,t+\tau}\left(\frac{Y_{t+1}}{Y_t}\right)\left(\frac{P_t}{P_{t+1}}\right)^{-\theta}\left(\frac{W_{t+1}}{W_t}\right)\left(\frac{A_t}{A_{t+1}}\right)G_{t+1}\right].$$

The optimal price is hence given by

$$\left(\frac{P^*_t}{P_t}\right)H_t = \frac{\mu}{A_t}\frac{W_t}{P_t}G_t\,.$$

Here, $P_t^*(j) = P_t^*$ because all firms changing prices face the same demand curve and hence the same optimization problem. Based on the definition of markup, the optimal time-varying product markup is given by

$$\mu_t = \mu \frac{G_t}{H_t} \quad \text{and} \quad P_t^* = \mu_t \frac{W_t}{A_t}\,.$$

Price inflation is given by

$$1 = (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{1-\theta} + \alpha\, \Lambda_{p,t-1,t}^{1-\theta} \left(\frac{P_{I,t+1}}{P_{I,t}}\right)^{-(1-\theta)}.$$

C Equilibrium Conditions

This appendix provides a summary of the equilibrium equations for the model. These conditions need to be expressed in terms of de-trended variables. In order to obtain balanced growth, we make $\kappa_t = (A_t^p)^{1-\psi}$. This condition ensures that Y_t, W_t, and W_t^* are growing at the same rate. Therefore, the equations can be written in terms of $\hat{Y}_t = \frac{Y_t}{A_t^p}$, $\hat{W}_t = \frac{W_t}{A_t^p}$, and $\hat{W}_t^* = \frac{W_t^*}{A_t^p}$.

Wage Setting

$$\begin{aligned}
\frac{W_t^*}{P_t} &= \mu_w \kappa_t \left(N_t^s\right)^{\omega} C_t^{\psi} \frac{G_{w,t}}{H_{w,t}}\,. \\
H_{w,t} &= 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{N_{t+1}^d}{N_t^d}\right) \left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w} H_{w,t+1} \right], \\
\text{and} \quad G_{w,t} &= 1 + \alpha_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \Lambda_{w,t,t+1}^{-\theta_w} \left(\frac{P_{t+1}}{P_t}\right) \left(\frac{C_{t+1}}{C_t}\right)^{\psi} \left(\frac{N_{t+1}^d}{N_t^d}\right) \left(\frac{\kappa_{t+1}}{\kappa_t}\right) \right. \\
&\quad \left. \times \left(\frac{N_{t+1}^s}{N_t^s}\right)^{\omega} \left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w} G_{w,t+1} \right].
\end{aligned}$$

Price Dispersion

$$F_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta} dj = (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{-\theta} + \alpha \Lambda_{p,t-1,t}^{-\theta} \left(\frac{P_{I,t-1}}{P_{I,t}}\right)^{-\theta} F_{I,t-1}.$$

Wage Dispersion

$$F_{w,t} = \int_0^1 \left(\frac{W_t(k)}{W_t}\right)^{-\theta_w} dk = (1-\alpha_w)\left(\frac{W_t^*}{W_t}\right)^{-\theta_w} + \alpha_w \Lambda_{w,t-1,t}^{-\theta_w} \left(\frac{W_{t-1}}{W_t}\right)^{-\theta_w} F_{w,t-1}\,.$$

Wage Aggregator

$$\left(\frac{W_t}{P_t}\right)^{1-\theta_w} = \int_0^1 \left(\frac{W_t(k)}{P_t}\right)^{1-\theta_w} dk = (1-\alpha_w)\left(\frac{W_t^*}{P_t}\right)^{1-\theta_w} + \alpha_w \Lambda_{w,t-1,t}^{1-\theta_w}\left(\frac{P_{t-1}}{P_t}\right)^{1-\theta_w}\left(\frac{W_{t-1}}{P_{t-1}}\right)^{1-\theta_w},$$

Price Setting

$$\begin{aligned}
\left(\frac{P_t^*}{P_t}\right) H_t &= \frac{\mu}{A_t}\frac{W_t}{P_t} G_t\,, \\
H_t &= 1 + \alpha \mathbb{E}_t\left[M_{t,t+1}^{\$}\Lambda_{p,t,t+1}^{1-\theta}\left(\frac{Y_{t+1}}{Y_t}\right)\left(\frac{P_{t+1}}{P_t}\right)^{-\theta} H_{t+1}\right], \\
\text{and}\quad G_t &= 1 + \alpha \mathbb{E}_t\left[M_{t,t+1}^{\$}\Lambda_{p,t,t+1}^{-\theta}\left(\frac{Y_{t+1}}{Y_t}\right)\left(\frac{P_t}{P_{t+1}}\right)^{-\theta}\left(\frac{W_{t+1}}{W_t}\right)\left(\frac{A_t}{A_{t+1}}\right) G_{t+1}\right].
\end{aligned}$$

Price Aggregator

$$1 = (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{1-\theta} + \alpha \Lambda_{p,t-1,t}^{1-\theta}\left(\frac{P_{t-1}}{P_t}\right)^{1-\theta}.$$

Aggregate Labor Supply and Demand

$$N_t^s = F_{w,t} N_t^d, \qquad N_t^d = \frac{Y_t}{A_t} F_t.$$

Markup

$$\mu_t = \frac{Y_t}{LI_t} = \frac{A_t}{F_t}\left(\frac{W_t}{P_t}\right)^{-1}.$$

Pricing Kernel

$$\begin{aligned}
M_{t,t+1} &= \left[\beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-\psi}\right]^{\frac{1-\gamma}{1-\psi}}\left(\frac{1}{R_{YL,t+1}}\right)^{1-\frac{1-\gamma}{1-\psi}}, \\
R_{YL,t+1} &= (1-\nu_t) R_{C,t+1} + \nu_t R_{LI^*,t+1}, \\
R_{Y,t+1} &= \frac{C_{t+1} + S_{Y,t+1}}{S_{Y,t}}, \qquad R_{LI^*,t+1} = \frac{LI_{t+1}^* + S_{LI^*,t+1}}{S_{LI^*,t}}, \\
\nu_t &= \frac{\bar{\nu} S_{LI^*,t}}{\bar{\nu} S_{LI^*,t} - S_{Y,t}}.
\end{aligned}$$

Real and Nominal Bond Yields

$$
\begin{aligned}
\exp\left(-nr_t^{(n)}\right) &= \mathbb{E}_t\left[M_{t,t+1}\exp\left(-(n-1)r_{t+1}^{(n-1)}\right)\right], \\
\exp\left(-ni_t^{(n)}\right) &= \mathbb{E}_t\left[M_{t,t+1}^{\$}\exp\left(-(n-1)i_{t+1}^{(n-1)}\right)\right],
\end{aligned}
$$

Indexation

$$
\begin{aligned}
\Lambda_{p,t,t+1} &= \Pi_t^\star, \\
\Lambda_{w,t,t+1} &= e_a^g\Pi_t^\star,
\end{aligned}
$$

D Tables

	1999 - 2008		2004 - 2008	
	Real	Nominal	Real	Nominal
Average Yields				
2 years		3.79	1.22	3.64
5 years	2.30	4.32	1.60	3.98
10 years	2.66	4.97	2.00	4.54
15 years	2.78	5.31	2.15	4.86
20 years	2.81	5.41	2.17	4.95
Standard Deviations				
2 years		1.50	1.06	1.09
5 years	1.11	1.05	0.63	0.65
10 years	0.87	0.73	0.34	0.33
15 years	0.77	0.62	0.25	0.29
20 years	0.75	0.58	0.22	0.32
Correlations				
2 years	N.A.		0.934	
5 years	0.906		0.948	
10 years	0.938		0.918	
15 years	0.942		0.837	
20 years	0.928		0.809	

Table 1: **Average Levels (%), Standard Deviations (%), and Correlations for U.S. Government TIPS and Nominal Bond Yields**

	1999 - 2008			2004 - 2008		
	TIPS	Nominal	All	TIPS	Nominal	All
1st	98.04	78.46	81.88	90.41	70.63	75.63
2nd	1.92	20.89	15.50	9.09	28.33	21.2
3rd	0.03	0.44	2.05	0.44	0.53	2.19
Total	99.99	99.79	99.43	99.94	99.49	99.02

Table 2: **Variability (%) explained by the first three principal components for U.S. Government TIPS and Nominal Bond Yields**
"All" refers to columns when principal components are computed using both TIPS and nominal yields.

	1999 - 2008				2004 - 2008			
Maturity	1st	2nd	3rd	Adj. R^2	1st	2nd	3rd	Adj. R^2
1	-0.25	0.51	-0.03	0.986	0.41	-0.21	0.19	0.990
	-66.729	60.474	-1.283		69.43	-18.64	5.45	
2	-0.24	0.40	0.07	0.998	0.35	-0.12	0.30	0.999
	-219.232	158.798	10.583		211.56	-37.54	30.96	
5	-0.19	0.16	0.14	0.995	0.21	0.04	0.22	0.996
	-146.233	51.844	16.326		108.68	9.79	19.44	
7	-0.17	0.07	0.14	0.994	0.15	0.08	0.14	0.994
	-134.069	23.476	17.734		88.32	26.48	13.87	
10	-0.14	-0.02	0.14	0.994	0.09	0.13	0.06	0.981
	-134.394	-6.529	21.223		40.88	31.89	5.07	
15	-0.11	-0.09	0.14	0.996	0.03	0.17	0.03	0.975
	-148.815	-52.296	28.737		14.09	43.33	2.66	
20	-0.10	-0.13	0.13	0.996	0.00	0.20	0.04	0.992
	-147.321	-84.696	31.555		2.23	79.09	5.03	
25	-0.09	-0.15	0.12	0.992	-0.01	0.21	0.04	0.986
	-96.283	-68.27	20.97		-4.36	61.09	3.60	
30	-0.09	-0.15	0.12	0.974	-0.01	0.22	0.02	0.938
	-51.581	-39.313	10.683		-2.70	28.23	0.90	

Table 3: Regression of nominal yields on the three principal components for U.S. Government TIPS and nominal bond yields. T-statistics are reported below each regression coefficient.

	1999 - 2008				2004 - 2008			
Maturity	1st	2nd	3rd	Adj. R^2	1st	2nd	3rd	Adj. R^2
2	N.A.	N.A.	N.A.	N.A	0.34	-0.12	-0.42	0.991
	N.A.	N.A.	N.A.	N.A	75.06	-14.16	-16.17	
5	-0.21	0.02	-0.27	0.991	0.21	0.02	-0.17	0.995
					102.68	5.59	-14.56	
7	-0.19	-0.03	-0.23	0.995	0.15	0.06	-0.14	0.994
	-109.086	4.654	-22.219		87.77	17.27	-13.18	
10	-0.17	-0.07	-0.18	0.999	0.10	0.09	-0.13	0.994
	-149.327	-10.806	-28.902		82.81	36.57	-17.74	
15	-0.14	-0.09	-0.14	0.996	0.05	0.11	-0.12	0.969
	-420.088	-72.67	-73.086		27.13	28.58	-10.65	
20	-0.14	-0.12	-0.13	0.989	0.03	0.11	-0.11	0.907
	-164.719	-47.266	-25.316		10.10	19.60	-5.94	

Table 4: Regression of TIPS yields on the three principal components for U.S. Government TIPS and nominal bond yields. T-statistics are reported below each regression coefficient.

Parameter	Description	Value
	Panel A: Preferences	
β	Subjective discount factor	0.992
ψ	Inverse of elasticity of intertemporal substitution	4.5
γ	Risk aversion parameter	120
ω	Inverse of Frisch labor elasticity	0.5
	Panel B: Rigidities	
α	Price rigidity parameter	0.66
θ	Elasticity of substitution of differentiated goods	6
λ	Wage rigidity parameter	0.82
θ_w	Elasticity of substitution of labor types	6
	Panel C: Interest Rate Rule	
ρ	Interest-rate smoothing coefficient in policy rule	0.76
$\bar{\imath} \times 10^2$	Constant in the policy rule	1.01
$\imath_\pi$	Response to inflation in the policy rule	1.6
$\imath_x$	Response to output gap in the policy rule	0.125
	Panel D: Exogenous Shocks	
ϕ_u	Autocorrelation of policy shock	0.15
$\sigma_u \times 10^2$	Conditional vol. of policy shock	0.175
ϕ_a	Autocorrelation of permanent productivity shock	0.275
$\sigma_a \times 10^2$	Conditional vol. of permanent productivity shock	0.246
ϕ_z	Autocorrelation of transitory productivity shock	0.957
$\sigma_z \times 10^2$	Conditional vol. of transitory productivity shock	0.19625
	Panel E: Inflation Target	
$g_{\pi^\star} \times 10^2$	Unconditional Mean of Inflation Target	0.55
$\phi_{\pi^\star}$	Autocorrelation of Inflation Target	0.99
$\sigma_{\pi^\star} \times 10^2$	Conditional vol. of Inflation Target	0.015

Table 5: **Calibrated Parameter Values**

	(1) Data from 1982 to 2010	(2) Benchmark
$\sigma(y)$	2.90	2.37
$\sigma(\Delta c)$	1.60	1.63
$\mathbb{E}[\pi_t]$	3.07	3.01
$\mathbb{E}[i_t]$	4.89	4.61
$\mathbb{E}[i_t^{(20)}]$	6.26	4.93
$\mathbb{E}[i_t^{(20)} - i_t]$	1.38	0.32
$\sigma(\pi_t)$	1.68	1.67
$\sigma(i_t)$	2.86	1.93
$\sigma(i_t^{(20)})$	2.86	1.52
$\sigma(i_t^{(20)} - i_t)$	0.94	0.88

Table 6: **Model Calibration**

σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. y is output. Δc is consumption growth. π is inflation. r denotes the real yield while i denotes the nominal yield.

	(1) Benchmark	(2) Only A^p	(3) Only Z	(4) Only u	(5) Only $\pi^\star$
$\sigma(y)$	2.3730	0.5675	1.1126	0.8324	1.8380
$\sigma(x)$	2.1179	0.5675	0.3040	0.8324	1.8380
$\sigma(w)$	2.5944	0.8529	2.4037	0.2364	0.4119
$\sigma(log(\mu))$	1.3243	0.9614	0.8202	0.3328	0.2145
$\sigma(\Delta c)$	1.6299	1.4575	0.3041	0.6492	0.1352
$\sigma(\Delta w)$	0.9785	0.9545	0.2098	0.0450	0.0183
$\mathbb{E}[\pi_t]$	3.0134	2.9366	6.8019	6.8183	6.9473
$\mathbb{E}[IRP^{(20)}]$	0.3082	0.2360	0.0168	-0.0001	0.0555
$\mathbb{E}[rTP^{(20)}]$	0.1190	0.1183	0.0067	0.0056	-0.0116
$\mathbb{E}[r_t]$	1.4858	1.5251	3.1908	3.2068	3.2018
$\mathbb{E}[r_t^{(20)}]$	1.6045	1.6430	3.1971	3.2120	3.1898
$\mathbb{E}[i_t]$	4.6089	4.5621	9.9976	10.024	10.154
$\mathbb{E}[i_t^{(20)}]$	4.9257	4.8151	10.015	10.030	10.192
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3177	0.2539	0.0186	0.0063	0.0389
$\sigma(\pi_t)$	1.6701	0.3294	0.4266	0.2274	1.5643
$\sigma(r_t)$	1.2128	0.2074	0.2453	1.1457	0.2348
$\sigma(r_t^{(20)})$	0.2573	0.0583	0.1481	0.1884	0.0734
$\sigma(i_t)$	1.9268	0.2760	0.5044	0.9628	1.5668
$\sigma(i_t^{(20)})$	1.5189	0.0703	0.3266	0.1491	1.4742
$\sigma(i_t^{(20)} - i_t)$	0.8842	0.2131	0.1870	0.8152	0.1921

Table 7: **Model Summary Statistics for Different Shock Specifications**

The baseline parameter values are presented in Table 5. "Benchmark" indicates an economy with both price and wage rigidities plus all four exogenous shocks. "Only A^p" indicates only permanent productivity shocks ($\sigma_z = \sigma_u = \sigma_{\pi^\star} = 0$). "Only Z" indicates only transitory productivity shocks ($\sigma_a = \sigma_u = \sigma_{\pi^\star} = 0$). "Only u" indicates only policy shocks ($\sigma_a = \sigma_z = \sigma_{\pi^\star} = 0$). "Only $\pi^\star$" indicates only shocks to inflation target ($\sigma_a = \sigma_z = \sigma_u = 0$). σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. y is output. x is the output gap. w is wage. $log(\mu)$ is the markup charged by the producers. Δc is consumption growth. Δw is wage growth. π is inflation. $IRP^{(20)}$ is the inflation risk premium for a 5-year to maturity bond. $rTP^{(20)}$ is the real term premium for a 5-year to maturity TIPS. r denotes the real yield while i denotes the nominal yield. Annualized percentages are used in all appropriate cells.

	(1) Benchmark	(2) No Rig.	(3) Only WR	(4) Only PR
$\sigma(y)$	2.3730	0.8118	2.2158	0.9821
$\sigma(x)$	2.1179	$-$	1.9351	0.5760
$\sigma(w)$	2.5944	2.7061	2.7061	3.9183
$\sigma(log(\mu))$	1.3243	$-$	$-$	3.1058
$\sigma(\Delta c)$	1.6299	1.0508	1.7855	1.3785
$\sigma(\Delta w)$	0.9785	1.0301	1.0301	2.5638
$\mathbb{E}[\pi_t]$	3.0134	3.5176	2.9950	3.3688
$\mathbb{E}[IRP^{(20)}]$	0.3082	-0.2183	0.4106	-0.0607
$\mathbb{E}[rTP^{(20)}]$	0.1190	-0.6272	0.3900	-0.1117
$\mathbb{E}[r_t]$	1.4858	2.3714	1.2030	1.8276
$\mathbb{E}[r_t^{(20)}]$	1.6045	1.7438	1.5926	1.7154
$\mathbb{E}[i_t]$	4.6089	5.3210	4.5460	5.1082
$\mathbb{E}[i_t^{(20)}]$	4.9257	5.0434	4.9980	5.0238
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3177	-0.2775	0.4527	-0.0843
$\sigma(\pi_t)$	1.6701	2.8705	1.9288	1.3897
$\sigma(r_t)$	1.2128	1.2762	1.2955	1.0145
$\sigma(r_t^{(20)})$	0.2573	0.1380	0.2612	0.1520
$\sigma(i_t)$	1.9268	1.2464	1.8997	1.2403
$\sigma(i_t^{(20)})$	1.5189	0.9910	1.4434	1.0064
$\sigma(i_t^{(20)} - i_t)$	0.8842	0.5489	0.9330	0.5256

Table 8: **Model Summary Statistics for Different Rigidity Specifications**
"No Rig." indicates no price and wage rigidities ($\alpha = \alpha_w = 0$). "Only WR" indicates no price rigidities ($\alpha = 0$). "Only PR" indicates no wage rigidities (α_w=0). σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. y is output. x is the output gap. w is wage. $log(\mu)$ is the markup charged by the producers. Δc is consumption growth. Δw is wage growth. π is inflation. $IRP^{(20)}$ is the inflation risk premium for a 5-year to maturity bond. $rTP^{(20)}$ is the real term premium for a 5-year to maturity TIPS. r denotes the real yield while i denotes the nominal yield. Annualized percentages are used in all appropriate cells.

	(1) Benchmark	(2) No Rig.	(3) Only WR	(4) Only PR	(5) WR and PR
$\sigma(y)$	2.3730	0.8118	2.0875	0.9343	2.3041
$\sigma(x)$	2.1179	–	1.7868	0.4901	2.0405
$\sigma(w)$	2.5944	2.7061	2.7061	3.6082	2.4502
$\sigma(log(\mu))$	1.3243	–	–	2.6875	0.9108
$\sigma(\Delta c)$	1.6299	0.2381	0.7711	0.5041	0.7296
$\sigma(\Delta w)$	0.9785	0.1164	0.1164	1.1060	0.2153
$\mathbb{E}[\pi_t]$	3.0134	5.8424	6.8007	5.8798	6.9071
$\mathbb{E}[IRP^{(20)}]$	0.3082	−0.0037	0.0602	0.0020	0.0722
$\mathbb{E}[rTP^{(20)}]$	0.1190	0.0044	0.0144	0.0057	0.0008
$\mathbb{E}[r_t]$	1.4858	3.2008	3.1677	3.1991	3.1736
$\mathbb{E}[r_t^{(20)}]$	1.6045	3.2047	3.1816	3.2045	3.1740
$\mathbb{E}[i_t]$	4.6089	9.0406	9.9867	9.0770	10.090
$\mathbb{E}[i_t^{(20)}]$	4.9257	9.0437	10.042	9.0865	10.153
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3177	0.0032	0.0562	0.0096	0.0638
$\sigma(\pi_t)$	1.6701	2.4034	1.7422	1.3727	1.6373
$\sigma(r_t)$	1.2128	0.1571	1.2167	1.0028	1.1950
$\sigma(r_t^{(20)})$	0.2573	0.1068	0.2451	0.1503	0.2506
$\sigma(i_t)$	1.9268	1.1137	1.8360	1.2337	1.9069
$\sigma(i_t^{(20)})$	1.5189	0.9902	1.4412	1.0063	1.5173
$\sigma(i_t^{(20)} - i_t)$	0.8842	0.1726	0.8389	0.5136	0.8582

Table 9: **Model Summary Statistics under No Permanent Shocks**
Other than the Benchmark, columns (2) through (5) reports model output when $\sigma_a = 0$ jointly with different rigidity specifications. "No Rig." indicates no price and wage rigidities ($\alpha = \alpha_w = 0$). "Only WR" indicates no price rigidities ($\alpha = 0$). "Only PR" indicates no wage rigidities (α_w=0). "WR and PR" indicates both rigidities are turned on. σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. y is output. x is the output gap. w is wage. $log(\mu)$ is the markup charged by the producers. Δc is consumption growth. Δw is wage growth. π is inflation. $IRP^{(20)}$ is the inflation risk premium for a 5-year to maturity bond. $rTP^{(20)}$ is the real term premium for a 5-year to maturity TIPS. r denotes the real yield while i denotes the nominal yield. Annualized percentages are used in all appropriate cells.

Panel A: Rigidities						
Parameters:		α			α_w	
	0.61	0.66	0.71	0.77	0.82	–
$\sigma(x)$	2.0396	2.1179	2.2762	1.2255	2.1179	–
$\sigma(\Delta c)$	1.6440	1.6299	1.6160	1.5794	1.6299	–
$\sigma(\Delta w)$	0.9581	0.9785	1.0035	0.9672	0.9785	–
$\mathbb{E}[\pi_t]$	3.0221	3.0134	2.9831	3.3114	3.0134	–
$\mathbb{E}[IRP^{(20)}]$	0.3261	0.3084	0.2892	0.1886	0.3083	–
$\mathbb{E}[rTP^{(20)}]$	0.1465	0.1190	0.0891	0.0931	0.1190	–
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3415	0.3174	0.2893	0.1994	0.3174	–
$\sigma(\pi_t)$	1.6673	1.6701	1.6967	1.4047	1.6701	–
$\sigma(i_t^{(20)})$	1.4928	1.5189	1.5692	1.2269	1.5190	–

Panel B: Inflation Target						
Parameters:		$\phi_{\pi^\star}$			$\sigma_{\pi^\star} \times 10^2$	
	0.985	0.990	0.995	0	0.015	0.052
$\sigma(x)$	1.7446	2.1179	2.9703	1.0523	2.1179	6.4581
$\sigma(\Delta c)$	1.6287	1.6299	1.6313	1.6243	1.6299	1.6906
$\sigma(\Delta w)$	0.9784	0.9785	0.9785	0.9783	0.9785	0.9804
$\mathbb{E}[\pi_t]$	2.9737	3.0134	3.1302	2.8963	3.0134	4.3032
$\mathbb{E}[IRP^{(20)}]$	0.2924	0.3078	0.3341	0.2523	0.3078	0.9192
$\mathbb{E}[rTP^{(20)}]$	0.1223	0.1190	0.1132	0.1306	0.1190	−0.0084
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3059	0.3168	0.3352	0.2780	0.3168	0.7450
$\sigma(\pi_t)$	1.3930	1.6701	2.2961	0.5850	1.6701	5.4545
$\sigma(i_t^{(20)})$	1.2091	1.5189	2.1890	0.3658	1.5189	5.1236

Table 10: **Comparative Statics I**

Model output by perturbing one parameter at a time while keeping all other parameters at the baseline values. Please see table 5 for parameter definitions. Under each parameter, the middle column represents the baseline calibration. For α_w, the wage rigidity parameter, the highest allowable value by the model is used for the basement calibration. σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. x is the output gap. Δc is consumption growth. Δw is wage growth. π is inflation. $IRP^{(20)}$ is the inflation risk premium for a 5-year to maturity bond. $rTP^{(20)}$ is the real term premium for a 5-year to maturity TIPS. i denotes the nominal yield.

	$\imath_\pi$		
	1.5	1.6	1.7
$\sigma(x)$	2.1179	2.1179	1.5378
$\sigma(\Delta c)$	1.6299	1.6299	1.6281
$\sigma(\Delta w)$	0.9785	0.9785	0.9726
$\mathbb{E}[\pi_t]$	3.0134	3.0134	3.0742
$\mathbb{E}[IRP^{(20)}]$	0.3082	0.3084	0.2638
$\mathbb{E}[rTP^{(20)}]$	0.1190	0.1190	0.1395
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3177	0.3174	0.2987
$\sigma(\pi_t)$	1.6701	1.6701	1.3662
$\sigma(i_t^{(20)})$	1.5189	1.5190	1.2112
	$\imath_x$		
	0	0.125	0.20
$\sigma(x)$	1.4916	2.1179	3.5325
$\sigma(\Delta c)$	1.6591	1.6299	1.6400
$\sigma(\Delta w)$	0.9708	0.9785	0.9849
$\mathbb{E}[\pi_t]$	3.0995	3.0134	2.3708
$\mathbb{E}[IRP^{(20)}]$	0.2442	0.3082	0.4278
$\mathbb{E}[rTP^{(20)}]$	0.1618	0.1190	0.0868
$\mathbb{E}[i_t^{(20)} - i_t]$	0.3050	0.3172	0.3914
$\sigma(\pi_t)$	1.2694	1.6701	2.3196
$\sigma(i_t^{(20)})$	1.1103	1.5189	2.1805
	ρ		
	0.61	0.76	0.91
$\sigma(x)$	2.1118	2.1179	2.6696
$\sigma(\Delta c)$	1.5984	1.6299	1.9928
$\sigma(\Delta w)$	0.9791	0.9785	0.9863
$\mathbb{E}[\pi_t]$	2.9442	3.0134	3.2478
$\mathbb{E}[IRP^{(20)}]$	0.3525	0.3078	0.2283
$\mathbb{E}[rTP^{(20)}]$	0.1735	0.1190	0.0287
$\mathbb{E}[i_t^{(20)} - i_t]$	0.4083	0.3168	0.1618
$\sigma(\pi_t)$	1.7190	1.6701	1.8468
$\sigma(i_t^{(20)})$	1.5853	1.5189	1.2984

Table 11: **Comparative Statics II**

Model output by perturbing one monetary policy parameter at a time while keeping all other parameters at the baseline values. Please see table 5 for parameter definitions. σ denotes the unconditional volatility while $\mathbb{E}$ denotes the unconditional mean. x is the output gap. Δc is consumption growth. Δw is wage growth. π is inflation. $IRP^{(20)}$ is the inflation risk premium for a 5-year to maturity bond. $rTP^{(20)}$ is the real term premium for a 5-year to maturity TIPS. i denotes the nominal yield.

Panel A: Rigidities						
Parameters:		α			α_w	
	0.61	0.66	0.71	0.77	0.82	–
$\frac{\sigma(i_t)}{\sigma(r_t)}$	1.5649	1.5887	1.6312	1.3635	1.5887	–
$\frac{\sigma(i_t^{(20)})}{\sigma(r_t^{(20)})}$	5.8222	5.9032	6.0331	5.5541	5.9036	–
$\frac{\sigma(r_t^{(20)})}{\sigma(r_t)}$	0.2106	0.2122	0.2152	0.1859	0.2122	–
$\frac{\sigma(i_t^{(20)})}{\sigma(i_t)}$	0.7836	0.7883	0.7958	0.7571	0.7884	–
Panel B: Inflation Target						
Parameters:		$\phi_{\pi^\star}$			$\sigma_{\pi^\star} \times 10^2$	
	0.985	0.990	0.995	0	0.015	0.052
$\frac{\sigma(i_t)}{\sigma(r_t)}$	1.3926	1.5887	2.0542	0.9425	1.5887	3.8471
$\frac{\sigma(i_t^{(20)})}{\sigma(r_t^{(20)})}$	4.6828	5.9032	8.5877	1.4834	5.9032	14.461
$\frac{\sigma(r_t^{(20)})}{\sigma(r_t)}$	0.2130	0.2122	0.2101	0.2072	0.2122	0.2458
$\frac{\sigma(i_t^{(20)})}{\sigma(i_t)}$	0.7163	0.7883	0.8784	0.3262	0.7883	0.9238
Panel C: Policy Rule						
Parameters:		$\imath_\pi$			$\imath_x$	
	1.5	1.6	1.7	0	0.125	0.20
$\frac{\sigma(i_t)}{\sigma(r_t)}$	1.5887	1.5887	1.3855	1.2674	1.5887	2.0847
$\frac{\sigma(i_t^{(20)})}{\sigma(r_t^{(20)})}$	5.9032	5.9036	4.9905	4.3439	5.9032	7.6589
$\frac{\sigma(r_t^{(20)})}{\sigma(r_t)}$	0.2122	0.2122	0.2033	0.2034	0.2122	0.2337
$\frac{\sigma(i_t^{(20)})}{\sigma(i_t)}$	0.7883	0.7884	0.7324	0.6972	0.7883	0.8585

Table 12: **Volatility Ratios**

The ratio of the unconditional standard deviations of real and nominal yields by perturbing one parameter at a time while keeping all other parameters at the baseline values. Please see table 5 for parameter definitions. Under each parameter, the middle column represents the baseline calibration. For α_w, the wage rigidity parameter, the highest allowable value by the model is used for the basement calibration. σ denotes the unconditional volatility. r denotes the real yield while i denotes the nominal yield.

	Panel A: With Permanent Productivity Shock			
	Benchmark	No Rig.	Only WR	Only PR
$\rho_r^{(4)}$	0.8174	0.7175	0.8233	0.5818
$\rho_r^{(8)}$	0.7493	0.6903	0.8050	0.4673
$\rho_r^{(12)}$	0.6543	0.6565	0.7383	0.3604
$\rho_r^{(16)}$	0.5666	0.6226	0.6704	0.2946
$\rho_r^{(20)}$	0.4982	0.5906	0.6122	0.2524
	Panel B: Without Permanent Productivity Shock			
	WR and PR	No Rig.	Only WR	Only PR
$\rho_r^{(4)}$	0.8503	0.1845	0.8352	0.5714
$\rho_r^{(8)}$	0.7571	0.2872	0.7784	0.4430
$\rho_r^{(12)}$	0.6458	0.3367	0.6919	0.3347
$\rho_r^{(16)}$	0.5515	0.3586	0.6148	0.2714
$\rho_r^{(20)}$	0.4814	0.3659	0.5540	0.2319

Table 13: **Real and Nominal Return Correlations**

Unconditional correlations between returns on real bonds and real returns on nominal bonds (excess of inflation). Realized returns are calculated over 1-Quarter holding horizon. "No Rig." indicates no price and wage rigidities ($\alpha = \alpha_w = 0$). "Only WR" indicates no price rigidities ($\alpha = 0$). "Only PR" indicates no wage rigidities (α_w=0). Values in the parentheses denote the number of quarters to maturity on the bond.

	$\imath_\pi$		
	1.55	1.6	1.65
$\rho_r^{(4)}$	0.7751	0.8174	0.8383
$\rho_r^{(8)}$	0.6816	0.7493	0.7836
$\rho_r^{(12)}$	0.5682	0.6543	0.7001
$\rho_r^{(16)}$	0.4732	0.5666	0.6186
$\rho_r^{(20)}$	0.4057	0.4982	0.5518
	$\imath_x$		
	0	0.125	0.175
$\rho_r^{(4)}$	0.8686	0.8174	0.7395
$\rho_r^{(8)}$	0.8332	0.7493	0.6252
$\rho_r^{(12)}$	0.7686	0.6543	0.4997
$\rho_r^{(16)}$	0.6997	0.5666	0.4024
$\rho_r^{(20)}$	0.6383	0.4982	0.3377
	ρ		
	0.61	0.76	0.91
$\rho_r^{(4)}$	0.7937	0.8174	0.7981
$\rho_r^{(8)}$	0.7259	0.7493	0.7859
$\rho_r^{(12)}$	0.6431	0.6543	0.7232
$\rho_r^{(16)}$	0.5745	0.5666	0.6427
$\rho_r^{(20)}$	0.5251	0.4982	0.5639

Table 14: **Return Correlation Comparative Statics**
Unconditional correlations between returns on real bonds and real returns on nominal bonds (excess of inflation) calculated by varying monetary policy parameters. The middle column represents the benchmark calibration. Values in the parentheses denote the number of quarters to maturity on the bond.

E Figures

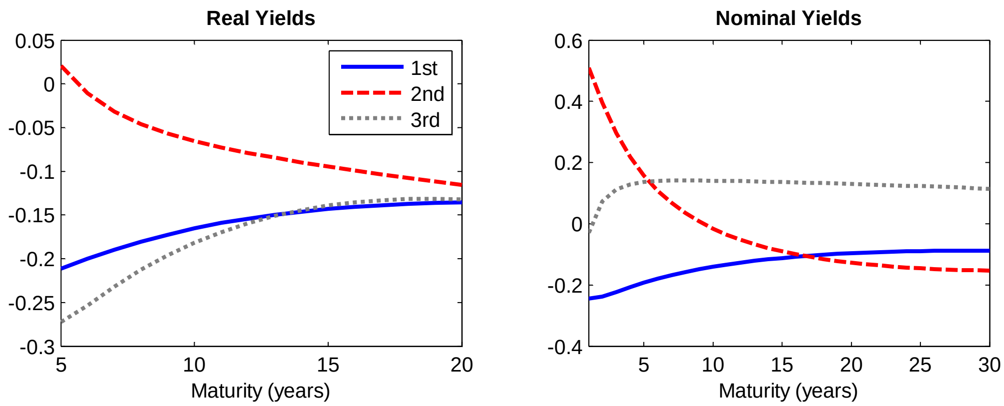

Figure 1: Loadings on the first three principal components for U.S. Government TIPs and Nominal Bond Yields. 1999 - 2008

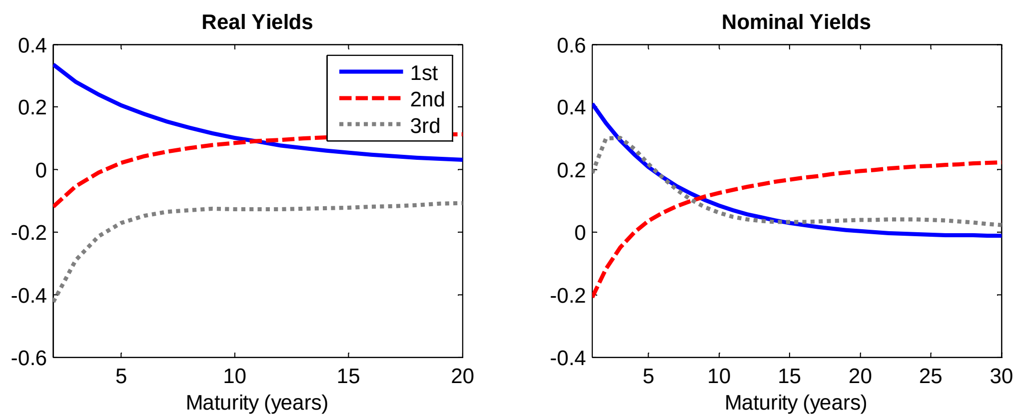

Figure 2: Loadings on the first three principal components for U.S. Government TIPs and Nominal Bond Yields. 2004 - 2008

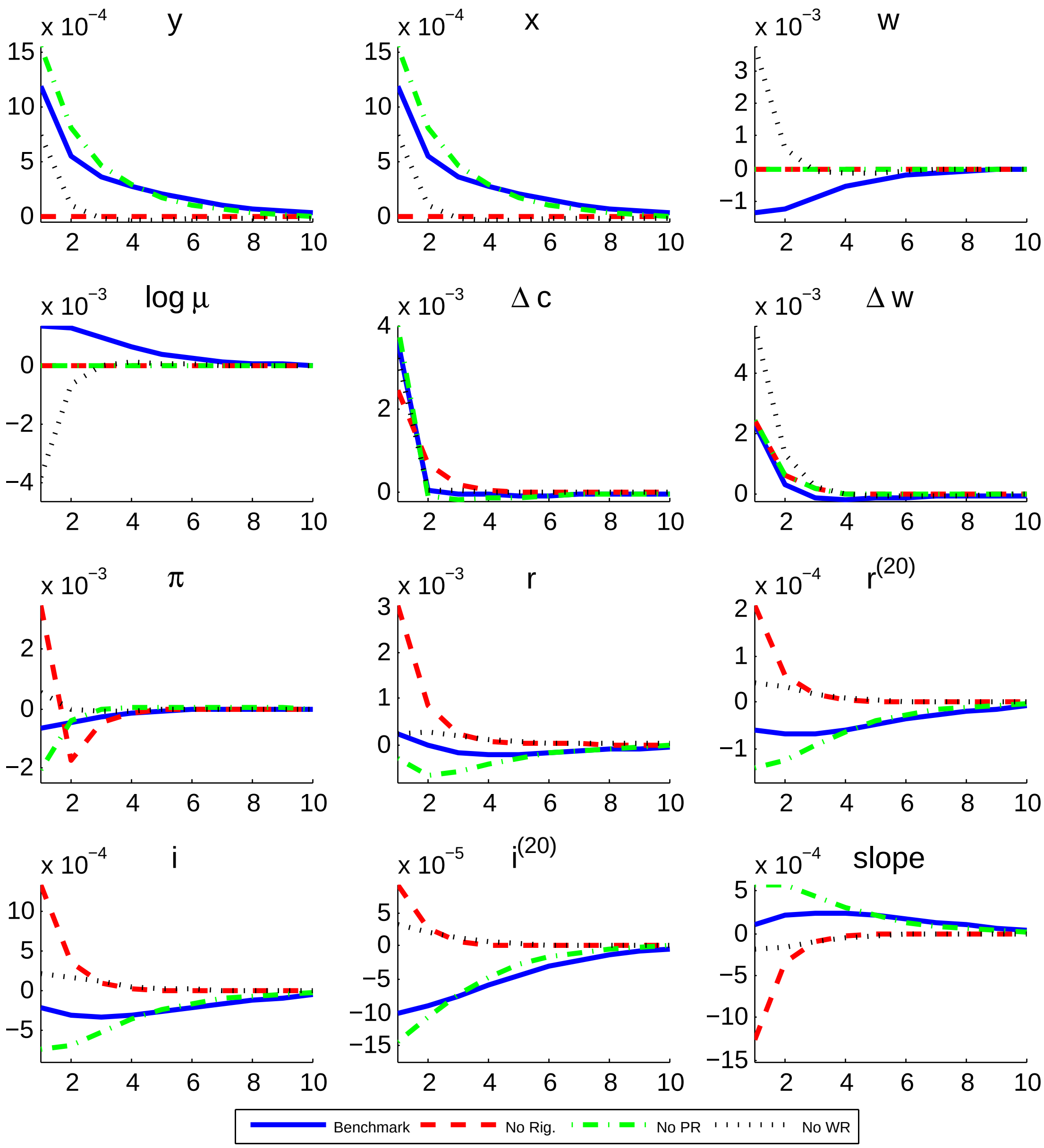

Figure 3: Impulse responses to a one-standard deviation positive permanent productivity shock for different macroeconomic variables and asset returns. The parameter values are presented in Table 5.

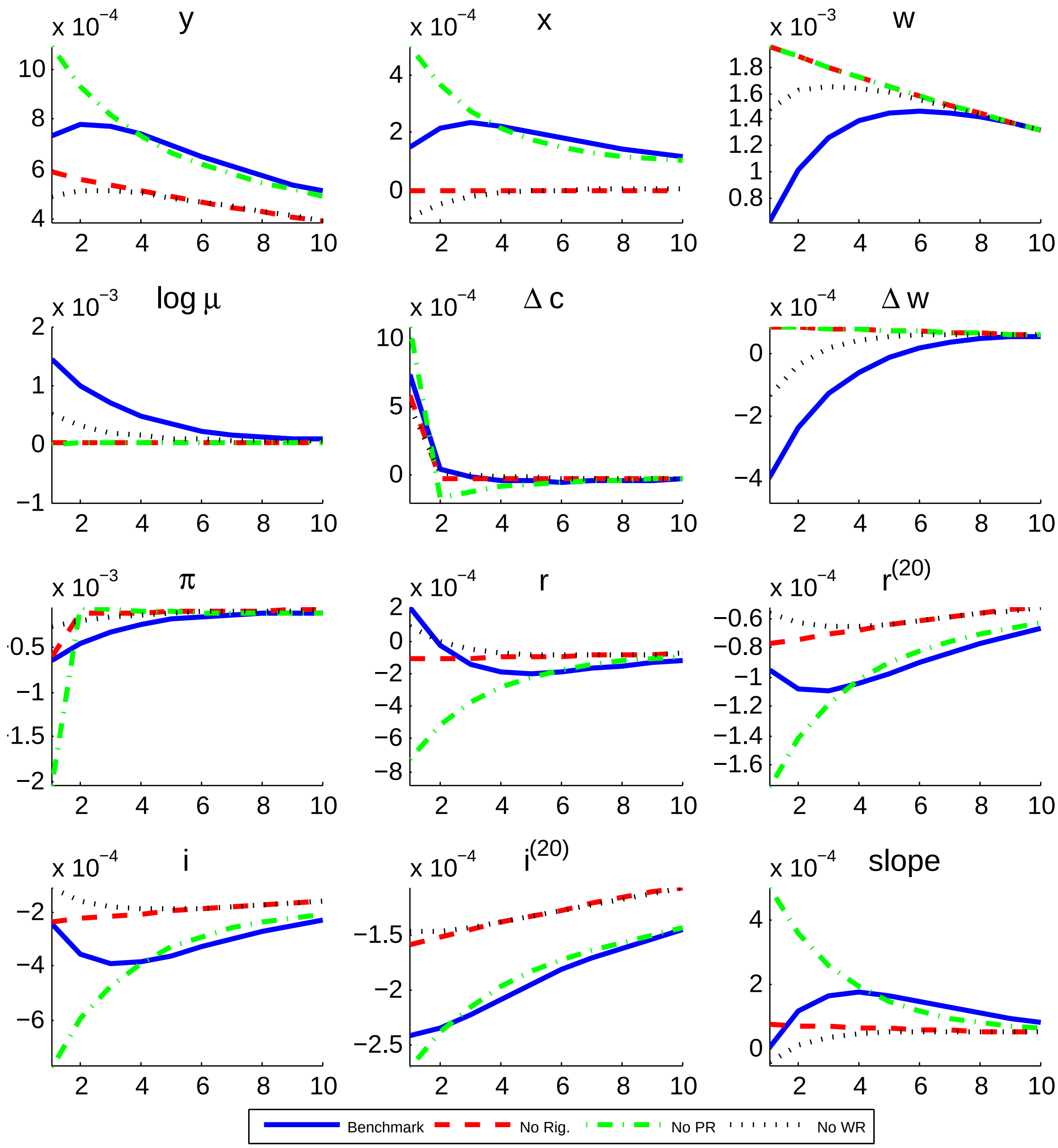

Figure 4: Impulse responses to a one-standard deviation positive transitory productivity shock for different macroeconomic variables and asset returns. The parameter values are presented in Table 5.

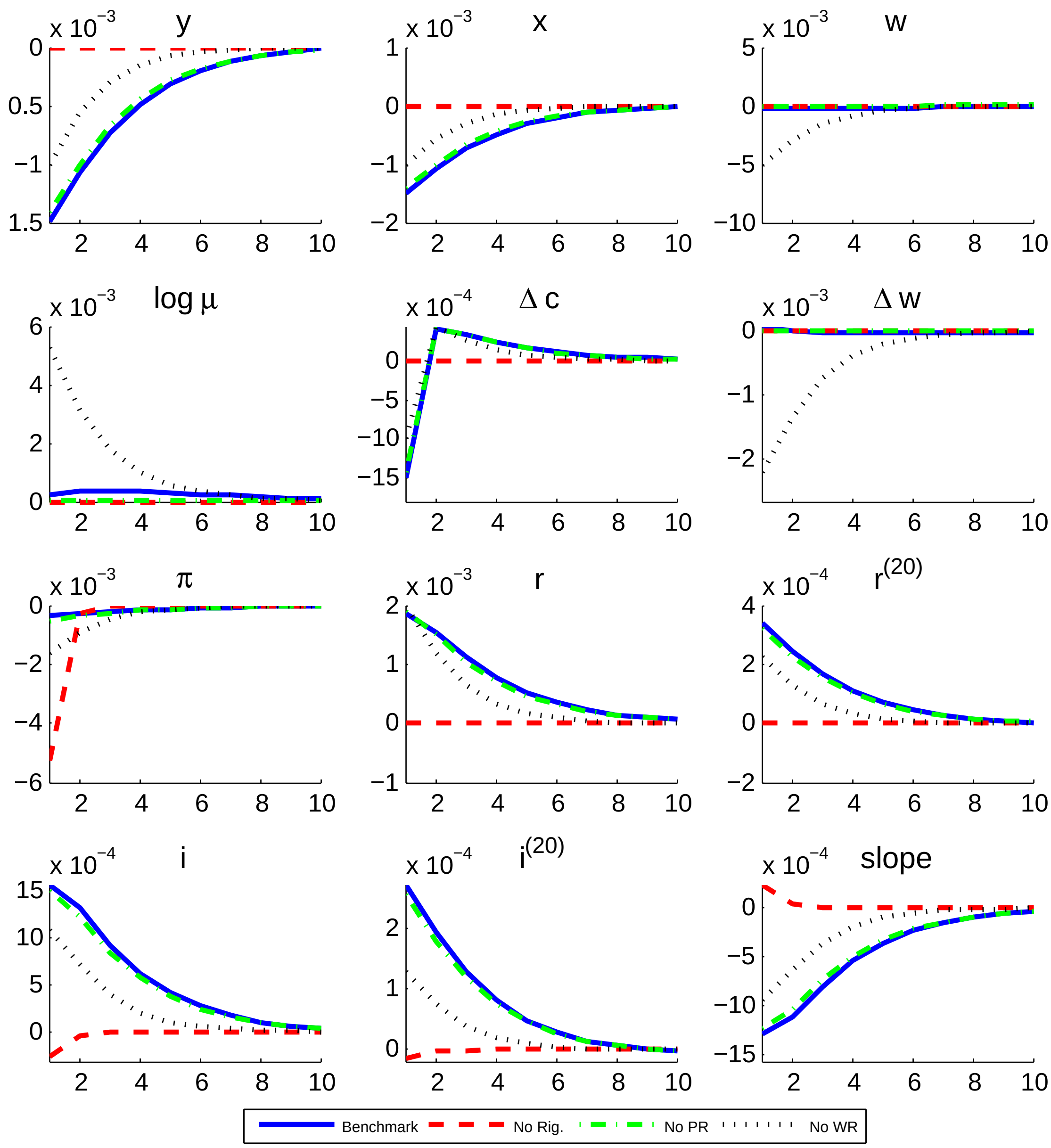

Figure 5: Impulse responses to a one-standard deviation positive policy shock for different macroeconomic variables and asset returns. The parameter values are presented in Table 5.

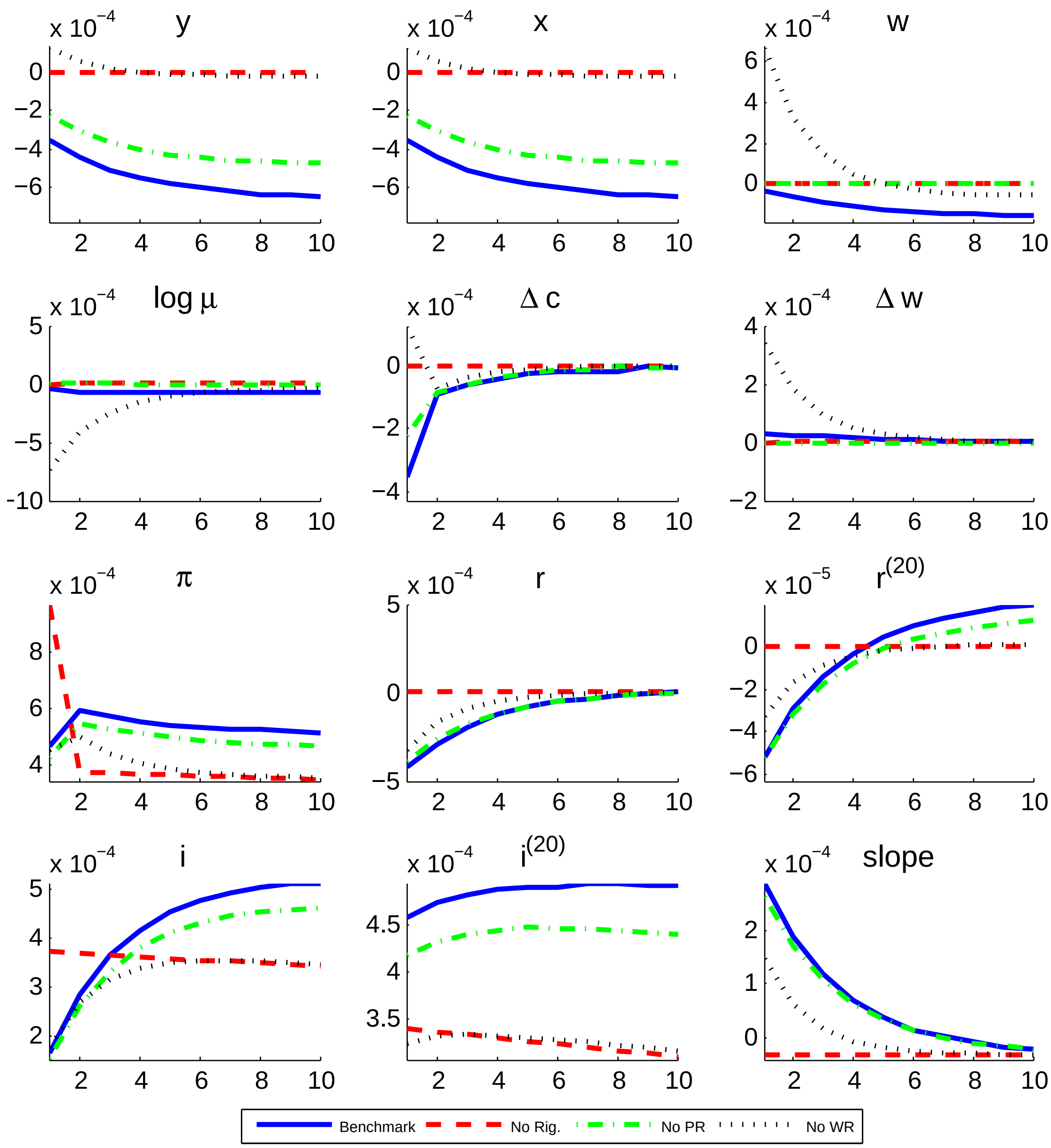

Figure 6: Impulse responses to a one-standard deviation positive inflation target shock for different macroeconomic variables and asset returns. The parameter values are presented in Table 5.