Assessing Credit Risk in the Colombian Financial System

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Introduction

The recurrent financial crises in the developed and developing economies during the last two decades show how costly these events can be[1]. They also underscore the importance of financial stability as an economic policy consideration. Among the authorities responsible for overseeing financial stability, central banks play an extremely important role, one that is gaining ground among monetary authorities, since most economies have made important progress towards reducing inflation.

According to Large (2005)[2] , there are at least three things central banks must do to encourage financial stability. They first must assess the threats to the financial system. Secondly, they must monitor stability of the payment system and take actions to reduce any risk on this front. Finally, as lenders of last resort, they must be able to inject liquidity, even in times of crisis.

Stress test exercises have become a fundamental tool for evaluating the soundness of the financial system and the threats it faces. They involve a number of ways and means to assess the financial system's vulnerability to exceptional but plausible shocks. The use of these exercises is now common practice within financial institutions and in multilateral organizations, supervisory bodies, central banks and private financial institutions[3].

The purpose of this article is to determine, based on exercises of this type, how sound the Colombian financial system is in the face of credit risks that could

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1 For example, the fiscal cost alone for a sample of 40 countries was estimated by Honohan and Klingebiel (2000) at 12.8% of GDP, on average.

2 Large (2005) bases his discussion on the Bank of England case.

3 The financial sector assessment programs (FSAPs) of the IMF, Bank of England, Central Bank of Chile and others are an example.

materialize. Although, in the short term, the major threats to the country's financial system come from the market risks associated with the domestic public debt[4], the last crisis was generated largely by an increase in credit risk. This is the reason for examining the financial system's current stability in the event of macroeconomic changes that could affect debtors' ability to pay (i.e. households and corporations).

The present article includes three sections, in addition to this introduction. As a prelude to the stress test exercises, the second section explains the spirit behind them. The third proposes several statistical models to assess just how sound the Colombian financial system is in terms of dealing with eventual macroeconomic shocks. The last section discusses the weaknesses of the exercises and concludes.

1. What Do Stress Test Exercises Entail?

Because stress test exercises are a fundamental tool for assessing financial stability, their nature is examined in detail in this section, as well is the difficulties that arise when developing exercises of this type, with a particular emphasis on credit risk[5]. The exercises simulate hypothetical crisis scenarios in an attempt to determine if the financial system would be able to resist these events were they to occur. Unlike the so-called early warning models, these exercises do not assign a likelihood of occurrence to such events.

A. What should they involve?

Stress testing exercises have four basis phases. The scope of the exercise should be designed in the first phase and the macroeconomic scenario under stress, in the second. The impact of the stress on the system's stability should be assessed in the third phase and the possible feedback effects, in the fourth. The details of these phases are discussed below.

In an exercise of this type, the first step is to define its scope. Doing so implies defining the set of relevant financial institutions and how they will be approached. For example, some studies focus exclusively on banks, occasionally only the largest. It also is necessary to decide on the type of financial assets to be examined. Many studies concentrate solely on the banking book, particularly consumer, commercial and interbank credit.

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[4] See the July 2005 Financial Stability Report.

[5] This section closely follows Bunn et al. (2005) and Sorge (2004).

The second step is to design the stressed macroeconomic scenario, based on one or more shocks. Considering the type of risk being analyzed (credit risk in this case), it is essential to identify the risk factor(s) to be taken into account, the parameters under stress (e.g. prices, volatility, correlations) and the time horizon. A properly constructed macroeconomic scenario will take into account the correlation existing among the different variables[6]. However, many analyses include sensitivity exercises that examine the effect of an extreme change in a particular variable, without considering its impact on the others.

Selecting the size of the shock is crucial to the design of the scenario. Shocks that are too small or too large can deprive the exercise of its validity. In any case, the exercise should be extreme but plausible. Generally speaking, there are four different methods for deciding on the size of the shock. The first consists of gauging or weighing the shocks against historical observations, so the one selected replicates an event in the past. The second, which involves probability, implies selecting a shock that is found in a high percentage of the variable's distribution (e.g. in 95% of the movement in interest rates on consumer loans). The third is based on hypothetical scenarios, which do not necessarily have a parallel in reality. Another approach is to answer the question: How big a shock is required to generate losses above a particular threshold?

In the third phase, the impact of the shocks on the system's stability should be quantified. This necessitates first taking a look at how the new macroeconomic environment affects debtor's balance sheet. Deteriorating economic conditions for debtors eventually lead to portfolio problems for financial institutions. It also is important to contemplate the impact on the balance sheets of these institutions, solely as a consequence of the change in the macroeconomic scenario. Once these two effects have been quantified, the impact on the health of the financial institutions should be assessed.

Taking into account the feedback effect also can enrich the analysis. For example, it permits a look at contagion on the interbank market or added deterioration in the macroeconomic panorama, as a result of how the agents respond (decline economic activity due to less credit supply). However, this is a particularly complicated aspect, given the difficulties that emerge when trying to model interaction between financial institutions fragility households and corporations.

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[6] 6 For example, the Bank of England uses the ones established in the Inflation Report as base scenarios.

B. Difficulties

Difficulties, both methodological and data-oriented, generally arise when conducting exercises of this type. For one thing, the events under consideration are fairly unlikely; that is, they are extraordinary and difficult to design or predict. The opposite is true in the case of price stability, where the central scenarios and not the tails of the distributions are considered. For example, in terms of traditional econometric modeling, which implies linear relations, this poses a problem. At times of crisis, these models cease to be a good approximation, because the size of the shocks makes nonlinearity an important element to consider. (See Bunn et al. (2005).

On the other hand, the lack of sufficiently time series long makes it difficult to arrive at statistical estimates of relations existing during the crisis periods. Not only is this a problem in the Colombian case; it occurs regularly at the international level. As an example, for the Colombian case, monthly bank balance-sheet information is available only as of 1990. For corporations, it is available on an annual basis only as of 1995. There is no adequate regularly information on households, which means we have only one observation of a crisi: that of the late nineties.

In addition, this analysis is made even more complex by the rapid and significant changes witnessed recently on the financial markets. The development of new instruments and new practices, coupled with the advent of new participants in this market, have made it more difficult to examine.

II. Exercises

The exercises were done to assess the ability of commercial and mortgage banks to withstand macroeconomic shocks that heighten credit risk[7]. For this purpose, models to explain portfolio quality, which is understood as the post due loans/total loans portfolio ratio, were estimated for consumer, mortgage and commercial credit. The nature of the macroeconomic shocks in question is historical. We assume they are independent, even when all the variables in certain statistical models are endogenous (i.e. the multivariate error-correction models). Impairment in financial activity

[7] We examined these institutions because they are the most important intermediaries in the Colombian financial system. The sample includes 21.

because of changes in the macroeconomic situation is not taken into account, nor do the exercises have feedback effects.

In terms of consumer and mortgage credit, the household analysis is problematic because balance sheets do not exist. Hence, it difficult to trace the effect shocks to macroeconomic variables have on household finances. As a result, the impact changes in the variables have on the post due loans/ total loans portfolio ratio is analyzed directly. The contrary occurs with corporations, where balance-sheet information is available and, therefore, it is possible to analyze how changes in macroeconomic variables affect corporate balance sheets.

A. Statistical Models

1. The Household Portfolio

Due to the lack of household account balances, one must look directly at the relationship between macroeconomic variables and the portfolio quality indicator. However, in most cases, the use of macroeconomic data is characterized by probability distributions that change over time. This is known as non-stationarity, As a result, some studies, such as one by Delgado and Saurina (2004), use time series techniques to analyze these relationships. This particular work uses cointegration techniques along the lines of Johansen (1988) and Johansen and Joselius (1990). It is important to point out, as Hoggart, Soransen and Zicchino (2005), that these models imply the stress tests are conditional to the historic correlation of the variables. This approach also allows for identifying possible feedback effects, such as those mentioned in the previous section.

2. Consumer Credit

It is to be expected that a decline in economic activity and higher interest rates will bring on increase post due loans, insofar as households' income decline and assume more of a financial burden. Taking this factor into account, a VEC model was estimated for the credit quality indicator, the GDP and the reference interest rate (DTF)[8] . The exercise covers the period from January 1994 to March 2005[9] .

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[8] 8The estimated model also contains centered seasonal dummies and dummies for different episodes. In the long-term relationship, it has a constant and a tendency; there is only a constant (cdrift) in the short-term relationship. The GDP in logarithms is used.

[9] GDP was adjusted monthly, pursuant to the method proposed by Feibes and Lisman (1967) and Denton (1971). See the attachment.

TABLE 1

ADDITIONAL PAST DUE CONSUMER LOAN
(PERCENTAGE)

Period	Shock 1 1/	Shock 2 2/	Shock 3 3/
6 months	3.4	4.4	7.8
12 months	6.4	5.9	12.2

1/ 450 bp increase in the DTF.
2/ 6.8% drop in GDP.
3/ 1 and 2.
Source: The author's calculations.

In the estimated model, all the variables are part of the cointegration variable; they are not stationary and none is weak exogenous. The residues are normal and show no autocorrelation. The trace test, corrected by a small sample as suggested by to Cheung and Lai (1993), produced only one cointegration vector to corroborate the previous intuition (i.e. correct signs).

Because all the variables in the cointegration analysis are endogenous, the system's response to changes in some of them has to be examined by looking at the impulse response functions (See Lutkepohl, 1993)[10].

The following scenarios were considered to determine how the portfolio quality indicator is affected by different shocks to macroeconomic variables: 1) an increase of 450 basis points (bp) in the DTF, similar to what occurred between May and June 1998; 2) a drop of 6.8% in GDP, as observed in the second half of 1999; and 3) a combination of both these scenarios. The results are shown in Table 1.

3. Mortgage Credit

Recent changes in the mortgage market make it difficult to design econometric models for the latest trend in the mortgage portfolio. First of all, the market has shrunk considerably. At December 1998, the mortgage portfolio accounted for 10.9% of GDP, as opposed to only 2.6% at December 2004. Secondly, in the wake of the crisis, the credit indexing system no longer depends on market rates (e.g. DTF) and is now a function of inflation. Finally, mortgage-portfolio securitization processes are now regulated by law and have had a major impact on the financing system.

In spite of these problems, a VEC model was estimated for the period from January 1994 to December 2004. The following variables were used: GDP, the DNP housing price index and the quality index for the mortgage loans. Decline in economic activity and lower housing prices, which increase loan to value, should undermine the quality of the mortgage portfolio[11]. Centered seasonal dummies were included, as were control dummies for portfolio securitization.

[10] In an exercise of this type, although the shock occurs in just one time period, it lasts (with I (1) as the variable) and, because of the endogenous nature of the system, affects all the other variables in the model. This is important to bear in mind.

[11] See Amaya and Martínez (2005).

The trace test, corrected by Cheung and Lai (1993), produced two cointegration vectors; only one of which showed adequate signs. Impulse response functions were calculated to quantify growth in the post due loans as a result of the slowdown in economic activities and lower housing price. We assumed a drop equal to the decline in consumer credit use for GDP and an 8% drop for housing prices, which is equivalent to the average decline for both consumer credit and housing prices during 1996-2000. Table 2 contains the results.

TABLE 2

ADDITIONAL OVERDUE MORTGAGE PORTFOLIO
(PERCENTAGE)

Period	Shock 1 1/	Shock 2 2/	Shock 3 3/
6 months	3.27	4.27	7.53
12 months	8.58	7.26	15.84

1/ Drop in the housing prices index (8%).
2/ 6.8% drop in GDP.
3/ 1 and 2.
Source: The author's calculations.

4. *Corporate Portfolio*

The Securities, Corporate and Banking Superintendencies provided the data used in this exercise. The sample includes 14 major economic sectors during the period from 1998 to 2004. A two-stage procedure was conducted to gauge the increase in the post due loans as a result of changes in macroeconomic conditions. The effect of macroeconomic shocks on the income statement of the corporations in the sample was analyzed in the first stage to determine profits after the shocks[12]. For example, a drop in sales immediately means less operating income. This spells a change in profits, which affects corporate profitability. A panel data model was estimated at the same time, with the indicator of portfolio quality as the dependent variable and asset profitability return on asset as the independent variable. Deterioration in the commercial portfolio was calculated with the new profitability and the estimates from the model.

The statistical tests produced a random-effects model where the intercept and slopes vary by individual[13]. The following was the resulting model:

$$(1) \qquad Y_{i,t} = \sum_{k=1}^{K} b_{k,i} X_{k,i,t} + e_{k,i,t}$$

As shown in Table 3, the quality of the commercial portfolio is inversely related to corporate profitability. Agriculture, livestock, hunting/forestry and construction were the sectors most sensitive to changes in profitability (hence, their portfolio quality also would be affected the most). The least sensitive sectors were education and real estate, business and rental activities.

[12] The selected shocks include a 450 bp increase in interest rates and a 9% drop in sales, as occurred during the crisis, plus a combination of the two.

[13] The Hausman test identified the model as one of random effects. The Swamy test then determined that the slopes varied among individuals.

An additional exercise was done to consider tradables and nontradables corporations separately. In the case of the tradables producers, the statistical tests determined that a random-effects model was required, in which the intecept varied among individuals, but not the slopes. A random-effects model with an intercept and slopes that varied among individuals was estimated for the nontradables producers. Table 4 shows the 2004 values of the variables included in the panel and the coefficients estimated by group.

B. Financial Institutions Soundness

The foregoing models determine the additional percentage of the post due loans that would result from the macroeconomic shocks. However, it also is important

TABLE 3

COMMERCIAL PORTFOLIO BY ECONOMIC SECTORS

Sector	Percentage of		Profitability	Constr.	$\hat{b}$
	Portfolio	Overdue Portfolio			
Agriculture	3.90	12.80	1.30	0.41	(9.29)
Fishing	0.10	2.80	(2.30)	0.16	0.79
Mines	1.70	3.90	31.40	0.36	(1.27)
Industry	37.30	7.20	4.80	0.27	(1.58)
Electricity	6.70	8.40	6.30	0.08	0.29
Construction	5.90	16.70	4.20	0.44	(3.25)
Commerce	19.10	5.60	6.10	0.22	(2.51)
Hotels and Restaurants	0.50	36.00	1.60	0.38	(3.19)
Transport and Communications	5.90	10.50	1.10	0.19	(1.12)
Financial Intermediation	6.20	6.80	3.70	0.14	(1.10)
Real Estate Agencies	6.40	2.10	0.70	0.21	(0.40)
Education	1.10	3.10	1.50	0.12	(0.65)
Health	1.50	12.10	8.00	0.35	(1.19)
Others	3.70	5.90	2.20	0.25	(3.80)
Total	100.00	7.70	5.10	0.25	(1.59)

Source: Orozco y Zamudio.

TABLE 4

COMMERCIAL PORTFOLIO: TRADABLES AND NONTRADABLES SECTORS

Sector	Percentage of		Rentabilidad	Const,	$\hat{b}$
	Portfolio	Overdue Portfolio			
Tradables	42.5	7.61	7.56	0.27	-0.978
Nontradables	57.5	7.72	3.16	0.25	1.593

Source: Orozco y Zamudio.

to see how these shocks affect the soundness of the financial institutions in question. To do so, the losses generated by these shocks as a portion of profits and the impact this would have on the capital adequacy ratio (CAR) were quantified in the exercises.

It is assumed the post due loans affects the balance sheets of banks in two ways. First, banks must register an additional expense for provisions on their income statement, due to deterioration in the portfolio. As done by IMF (2005), we assume 45% of the losses. Secondly, the post due loans do not generate any interest income. To determine this cost, the implicit lending rates, constructed as income from interest on the performing loan, are calculated for each type of loan, as is the income that is no longer perceived. It is assumed the loan stops earning interest (i.e. becomes non-performing) the moment it falls due. Both these effects influence profits, as well as the solvency ratio, through technical equity.

Table 5 shows how the selected macroeconomic shocks impact the financial institutions as a whole, per type of loan and shock. In this case, the cost of the shock was calculated as a portion of the profits. Both the loan portfolio and the profits of these financial institutions at October 2005, for a 12-month period, are used in the exercise. However, this leads us to underestimate the outcome, since adverse economic shocks should reduce the extent of profits.

At an aggregate level, the results suggest the financial system would be able to resist heavy price or output shocks. However, if a situation similar to the crisis in the late nineties were to occur with simultaneous shocks, the financial system as a whole could not cover these obligations, at least with profits as they now stand.

Tables 6 and 7 offer a breakdown of the results. The number of banks where the cost of the shock would exceed profits is shown in Table 6. Table 7 lists the number of banks whose CAR would fall below the minimum (9%) and the total impact of the shock on the CAR of the banks as a whole. It should be noted that the sum of the banks, per type of portfolio, is not equal to the total. This is because the cost of the shock might be moderate for an

TABLE 5

COST AS A PERCENTAGE OF PROFITS
(12 MONTHS)

	Shock 1	Shock 2	Shock 3
Commercial	4	53	55
Consumer	18	24	36
Mortgage	12	11	19
Total	34	88	111

Source: Author's calculations.

TABLE 6

NÚMERO DE BANCOS CUYOS COSTOS SERÍAN MAYORES A LAS UTILIDADES
(12 MESES)

	Shock 1	Shock 2	Shock 3
Commercial	1	7	7
Consumer	1	1	3
Mortgage	0	0	1
Total	2	10	15

Source: Author's calculations.

TABLE 7

NÚMERO DE BANCOS CUYA RELACIÓN DE SOLVENCIA CAERÍA POR DEBAJO DEL MÍNIMO
(12 MESES)

	Shock 1	Shock 2	Shock 3
Commercial	0	4	5
Consumer	4	4	6
Mortgage	0	0	1
Total	5	8	12
Relación de solvencia (%)	11.8	9.8	8.9

Source: Author's calculations.

institution, depending on the type of portfolio. However, the sum of the costs for each type of portfolio could exceed the reported profit level or be enough to lower the CAR considerably.

As the tables indicate, if the country were to face another situation similar to the crisis in the late nineties, most banks would be in trouble. Shocks to economic activity are particularly relevant, as they are for the entire system. In terms of profits, it would be enough to cause losses for 10 intermediaries and to place the CAR at 9.77%. A combination of shocks to prices (i.e. interest rates and housing prices) and economic activity would spell losses for 15 institutions and place the capital adequacy ratio (CAR) below the mandatory minimum.

It is important to remember that the exercises presented in this article examine only credit risk. None of the other risks (e.g. market and liquidity risks) are considered. These surely would increase in unstable environment as the assured above. Generally speaking, the results denote fragility among the financial institutions when it comes to dealing with historically large macroeconomic shocks. Despite record-high profit levels, a CAR well above the minimum and extremely low credit-risk levels, drastic changes in macroeconomic conditions could significantly affect the financial system's stability.

III. Constraints And Conclusions

Although these exercises are far from a complete analysis of the financial system's vulnerability to credit risk, they do offer an initial frame of reference. Despite very high profits and a comfortable capital adequacy ratio (CAR) in the financial system, these exercises show that macroeconomic shocks similar to the ones observed during the last crisis, particularly in economic activity, would create serious stability problems for the system. These findings underscore the importance of moving forward with regulations on credit risk and anti-cyclical provisions.

In terms of stress-testing techniques to help assess the soundness of financial institutions, much remains to be done, even internationally. The exercises conducted for this article have certain weaknesses that are important to point out for the sake of progress towards more sophisticated and comprehensive procedures. To begin with, the relationship between exogenous shocks to the economy and the balance for the agents in the sample needs to be improved (mapping). This implies adequately designing the channels through which movement in the macroeconomic variables affects the soundness of debtors and credit institutions alike, and the relationship between them. Part of the problem in this respect is the lack of data, particularly on households. Having this data would enable us to broaden the analysis to include other types of risk. Secondly, it is important to

understand how shocks correlate to one another. For example, we need to understand how interest rate hikes by the central bank can affect output, the interest on government bonds, the exchange rate, etc. This would help to create consistent macroeconomic scenarios. Because they assume independence among the variables, the present exercises may be somewhat biased.

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Econometric Appendix

The monthly GDP derivation based on the quarterly GDP follows the method proposed by Boot et al. (1967) and Denton (1971). The problem, as Denton illustrates, is a specific case where an attempt is made to adapt a high frequency series, from a particular source, to another that is less frequent and originates with a different source.

The problem in adjusting an original low-frequency series to one with a high-frequency can be resolved with a method that somehow minimizes the distortion of the original series and satisfies the condition which stipulates that the sum of the monthly values must be equal to the quarterly total. For example, in the case of GDP, the sum of the monthly GDP estimated during a quarter should be equal to the quarterly figure reported by DANE.

One possible consideration would be to minimize the sum of the differences between the successive months, with the restriction that, for each quarter, the sum of the months should be equal to that of the quarter.

Formally speaking, if we have n years, we want to choose the x_i that minimizes the following expression:

$$(2) \quad \min \sum_{i=2}^{12n} (x_i - x_{i-1})^2 \quad \text{s.a.}$$

$$(3) \quad \sum_{i=12k-11}^{12k} x_i = t_k \; (k = 1, 2, \ldots, n)$$

Here, x_i represents the monthly observation and tk, the total for quarter k.

As is traditional, the problem is resolved by using the lagrangian function and derivation with respect to x_i y and λ_k. The x_i are the monthly values that solve the problem and satisfy the following first-order condition:

$$(4) \quad \begin{pmatrix} B & -J' \\ J & 0 \end{pmatrix} \begin{pmatrix} x \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 \\ t \end{pmatrix}$$

Here, x is the vector of $12n$ elements, λ is the vector of n lagrangians, t is the vector of n total quarters and 0 is a zero matrix of dimension $n \times n$.

The J matrix, of dimension $n \times 12n$, for $n = 2$, is provided by:

$$(5) \quad J = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$