Temas de Estabilidad Financiera
Septiembre de 2011, no. 60

Measuring Systemic Risk in the Colombian Financial System: A Systemic Contingent Claims Approach*

Laura Capera*
Esteban Gómez†
Mariana Laverde‡
Miguel Ángel Morales§

Abstract

The financial crisis of the late 2000's underscored the importance of identifying systemically significant institutions and developing mechanisms for the latter to internalize the externalities they create on the economy should they fail. Using monthly data for the period comprised between September, 2001 - March, 2011, we calculated bank-specific probabilities of default and expected losses given default. Subsequently, we estimated the joint distribution of such expected losses and found the aggregate cost of the implicit bailout option for the government. Our results suggest that even though systemic risk is currently not a major concern in the Colombian banking system, it is necessary to enhance the supervisory and regulatory framework to include quantitative measures of this risk.

JEL classification: *G120, G130, G180, G210, G280*
***Keywords*:** Contingent Claims, Systemic Risk, Macroprudential Supervision, Black-Scholes-Merton, Copula.

Resumen

La crisis financiera de 2008-2009 resaltó la importancia de identificar a instituciones sistemáticamente importantes y de desarrollar mecanismos para que éstas internalizaran las externalidades que crean en la economía ante una eventual quiebra. Utilizando datos mensuales para el periodo comprendido entre Septiembre de 2001 - Marzo de 2011, calculamos probabilidades de default y pérdidas dado incumplimiento a nivel individual para un grupo de bancos comerciales. Consecuentemente, estimamos la distribución conjunta de dichas pérdidas y encontramos el costo agregado de la opción implícita de rescate de parte del gobierno. Nuestros resultados sugieren que si bien el riesgo sistémico no parece ser una preocupación mayor en este momento en el sistema bancario, es necesario fortalecer el marco de supervisión y regulación para incluir medidas cuantitativas de este riesgo.

Clasificación JEL: *G120, G130, G180, G210, G280*
***Palabras clave*:** Reclamos Contingentes, Riesgo Sistémico, Supervisión Macroprudencial, Black-Scholes-Merton, Copulas.

*The authors would like to thank Dairo Estrada and Nancy Zamudio for useful comments and suggestions. The opinions contained herein are the sole responsibility of the authors and do not reflect those of Banco de la República or its Board of Governors. All errors and omissions remain our own.

*Profesional, Departamento de Estabilidad Financiera. E-mail: lcaperro@banrep.gov.co
†Profesional experto, Departamento de Estabilidad Financiera. E-mail: egomezgo@banrep.gov.co
‡Profesional, Departamento de Estabilidad Financiera. E-mail: mlaverqu@banrep.gov.co
§Profesional especializado, Departamento de Estabilidad Financiera. E-mail: mmoralmo@banrep.gov.co

1. Introduction

The international financial crisis of the late 2000's is arguably the most significant financial crisis since the Great Depression. The costs associated with the crisis include the collapse of large financial institutions (e.g. Northern Rock, Bear Sterns and Lehmann Brothers), sharp declines in asset prices which deteriorated household wealth (therefore altering consumption and investment decisions) and the bailout of private banks by national governments. The latter included not only capital injections, but the purchase of toxic assets, guarantees over debt and liquidity support from central banks. Additionally, the financial crisis also gave investor confidence a huge blow, effectively drying-up commercial debt markets and overnight funding, and increasing the risk of moral hazard in the economy. Though many of these effects are not directly quantifiable, the International Monetary Fund (IMF), in its *Global Economic Outlook* of April 2010, estimates that the cost to financial institutions of the global economic crisis, due to banking system write-downs, to be close to US$2.3 trillion (International Monetary Fund (2010)).

Nonetheless, the initial attention that was focused on finding the trigger of the financial crisis has gradually shifted towards the lessons that can be learned from it. Importantly, there has been renewed attention on the close link between financial risk and the sovereign, one that became readily apparent when governments were forced to channel national funds to absorb private losses and prevent a more pronounced macroeconomic shock. In this way, the recent crisis highlighted the importance of identifying systemically important institutions (i.e. the so-called too-big or too-interconnected to-fail) as well as that of developing mechanisms for these institutions to internalize the externality that their failing generates on the financial system and the economy as a whole.

Hence, a considerable number of academic and policy papers have recently began to explore the statistical tools at hand to enhance our understanding and measurement of systemic risk (see Gray et al. (2008), Gray & Jobst (2010*b*), Gray & Jobst (2010*a*) and Saldías (2010)). The importance of these initiatives is that they constitute a first attempt at developing methodologies to quantify this risk in a consistent and continuous manner, and are therefore a necessary tool if regulators aim at having financial institutions internalize this cost. In addition, these initiatives should have the side-effect of alleviating some of the moral hazard concerns that followed the chain of bailouts from governments around the globe, and could therefore help restore investor confidence.

Most of the literature aimed at analyzing systemic vulnerabilities stems from the theory and practice of modern Contingent Claims Analysis (CCA), which applied to the measurement of credit risk is commonly called the "Merton Model" (Merton (1974), Merton (1977)). Under this framework, we apply the explicit formulae for both the *distance-to-distress* and probability of default to commercial banks in the Colombian financial system, as well as for the expected loss given default, which in effect constitutes the value of the implicit bailout by the government. Henceforth, we use monthly market data for the period comprised between September:2001 - March:2011 to quantify these variables.

More importantly, given that the implicit bailout by the government can be represented as the price of a put option, we suggest that systemic risk can be quantified by calculating the traditional risk measures (i.e. Value-at-Risk and Expected Shortfall) on a portfolio comprised of such puts. In other words, in this paper we understand systemic risk as the expected aggregate cost for the government, under a default scenario, given the implicit bailout option. We use extreme value theory and copulas to account for the marginal distributions and the dependence between them. Our sample consists of 4 of the 23 banks in Colombia's financial system (which will be referred to as Bank A, Bank B, Bank C and Bank D, respectively), since this is the number of intermediaries that have traded continuously in the stock market for more than 5 years. These four banks are relatively big, representing close to 46% of total assets in the banking system, and so concentrating our analysis on these intermediaries, in a systemic risk setting, seems appropriate.

Our results suggest that systemic risk is currently not a major concern in the Colombian banking system, with expected aggregate losses, considering a one-month holding period, being less than 1% of 2011's first quarter GDP. This is, at least partially, due to the fact that we are only considering the 4 largest commercial banks, and Colombia's financial system has many important non-bank institutions. Additionally, Colombia's banking system is not particularly deep, with the ratio of total loans to GDP being slightly above 30%, and most institutions still operate under a model of traditional intermediation, where assets are divided between loans and government debt securities, and liabilities are concentrated in demand deposits. Moreover, the regulatory framework in Colombia is highly conservative, banning credit institutions from engaging in several financial activities, such as the trading of credit default swaps. These factors contribute to relatively low levels of asset volatility, and ultimately, low probabilities of default.

Nonetheless, we strongly believe that monitoring and regulating systemic risk is of crucial importance. As Colombia's financial system continues developing, interactions between institutions will become more complex, potentially increasing the risk of contagion across markets and thus, overall systemic vulnerability. Our approach, by allowing to quantify the individual contribution of each bank to aggregate risk, constitutes an ideal regulatory tool in an attempt to solidify risk-based capital charges in the spirit of Basel III. Finally, as our risk measures are based on forward-looking variables (i.e. the one-month-ahead *expected* loss given default), we believe that it represents a useful instrument for macroprudential as well as financial stability purposes.

This paper is organized as follows. Section 1 presented a brief introduction, while Section 2 provides the framework in which systemic risk is developed. Section 3 outlines the theoretical model and our empirical strategy. The Government implicit bailout option is described and quantified in section 4 whilst results on the calculated risk measures can be found in Section 5. Finally, Section 6 concludes.

2. Systemic Contingent Claims Analysis

Given the recent events in financial markets, significant efforts have been concentrated in developing methodologies to determine the contribution of individual financial institutions to systemic risk, considering the contagion between them. Four of the main systemic risk models proposed are CoVaR (Adrian & Brunnermeier (2010)), Systemic Expected Shortfall (SES) (Acharya et al. (2010)), Distress Insurance Premium (DIP) (Huang et al. (2009)) and the Systemic Contingent Claims Analysis (CCA) (Gray & Jobst (2010*b*)). The latter approach is highlighted because it is a more comprehensive and flexible model in comparison with CoVaR or SES (which can be viewed as a subset of the Systemic CCA approach) in the sense that it provides a direct quantification of the dependence structure between all individual institutions in the network. In addition, Systemic CCA outperforms the dependence structure of DIP models, capturing tail risk and average risk in a more sophisticated way than through simple linear correlation.

Accordingly, a considerable number of empirical research regarding systemic risk has focused on the the study of CCA[1]. As mentioned above, this approach is a generalization of the option pricing theory

[1]The CCA approach is based on the estimation of the default probability by an entity on its obligations, and is widely used by rating agencies to asses creditworthiness in the corporate sector. A contingent claim is any financial asset whose future payoff depends on the value of another asset.

developed by Black & Scholes (1973), Merton (1973) and Merton (1974), which applied to the analysis of credit risk is commonly referred to as the "Merton Model". In general, this framework is used to estimate a firm-specific risk indicator, namely, the expected probability of default, and it can be extended to asses the systemic risk of a financial market. Hence, assuming that the government will bailout the banks in the event of bank distress, the Systemic CCA approach allows to asses total implicit government liabilities and the individual banksŠ contribution to this potential cost (Gray & Jobst (2010*b*)).

The idea behind the basic CCA is to quantify the creditworthiness of a debt issuer, which could be a firm, a government or a bank, as in our case. Under this approach, the debt-holders of the bank effectively own the institution's assets until their liabilities are payed off. In this sense, owners of equity in leveraged banks hold a call option on the bank. In non-default scenarios, they are the residual claimants on the bankŠs asset value once outstanding debts have been settled, and have limited liability in the event of default. Thus, implementing the Black-Scholes-Merton (BSM) formulation it is possible to value the call option held by equity owners and, subsequently, to study the behavior over time of both default probabilities and the *distance-to-distress*, at an individual level.

Recent empirical works developed by the IMF and some central banks have used the CCA methodology in order to quantify the level of risk posed by individual financial institutions in emerging economies. For instance, using both book-value and market data, Gray & Walsh (2008) on Chile, Souto & Abrego (2008) on Colombia, Blavy & Souto (2009) on Mexico and Antunes & Silva (2010) on Portugal, employ the CCA methodology to asses risk indicators to banking sector[2]. Findings show that there is a significant heterogeneity among banks' risk indicators, and that bank soundness is related to macro-financial variables[3]. Moreover, the risk indicators (distance-to-default and probabilities of default) capture episodes of bank stress appropriately.

However, there is a wide heterogeneity between the manner in which CCA is applied in practical work. For instance, some studies like Souto et al. (2009) on Brazil and Souto (2008) on Uruguay, performed the same analysis of the traditional Merton Model using only book-value data into the estimation of credit risk indicators due to the absence of market data. The results of this modified framework appear to be appropriate for countries without developed equity markets, at least in the sense that they reflect episodes of financial stress accordingly. Additionally, other studies like Gray & Jones (2006) on Indonesia and Keller et al. (2007) on Turkey, have used the BSM model not only to analyze banking sector risk, but to analyze sovereign risk using government balance sheet information.

Nonetheless, the risk indicators obtained from the BSM formulation only allow for analysis at a bank-specific level. Consequently, in order to build a measure of systemic risk it is necessary to account for the dependence structure of the individual market valuations of the relevant contingent claims. According to Gray & Jobst (2010*b*), the systemic CCA framework approximates the joint asymptotic tail behavior of expected losses and associated contingent liabilities of financial institutions using a multivariate density estimation. Hence, the most recent analysis, such as that of Gray et al. (2011) on Israel, builds on recent work using the Systemic Contingent Claims (Systemic CCA). The authors apply the CCA framework in order to calculate individual contingent liabilities for banks and insurance companies using information from equity and CDS markets, and finally use extreme value theory (EVT) to generate a multivariate limiting distribution that captures extreme realizations of expected losses of the system.

[2] Some of those studies do not calculated directly the default probabilities, instead they use MoodyŠs-KMV estimates of expected default frequencies (EDFs) as a direct measure risk.

[3] This analysis are performed by building models (as VAR, Panel data models) that estimate the relationship between default probabilities and macro-financial indicators.

In concordance with the fact that Systemic CCA is a more complete measure for systemic risk in comparison with other popular approaches, and that some individual risk indicators have been quantified for the colombian financial system under the CCA perspective (Souto & Abrego (2008)), we consider the Systemic CCA analysis as the ideal tool in measuring systemic risk in the colombian banking sector.

3. The Merton Model

The Merton Model, in its basic formulation, is built upon the simplest capital structure one can assume for a firm (or bank for that matter). Namely, at any point in time t, it consists of common equity (E_t) and a risky zero-coupon bond (D_t) with face value at maturity given by D (i.e. the book value of total liabilities). Hence, the total market value of assets at any point is given by:

$$\begin{aligned} \text{Assets} &= \text{Risky Debt} + \text{Equity} \\ A_t &= D_t + E_t \qquad (1) \end{aligned}$$

Moreover, the model assumes that there can only be default at time T, which is the maturity date for the bond[4]. Assets are stochastic and assumed to follow a lognormal distribution. The diffusion process for the asset's return is of the form:

$$\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dW_t^P \qquad (2)$$

$$= \mu_A dt + \sigma_A \epsilon \sqrt{t} \qquad (3)$$

where μ_A is the drift rate or asset return, σ_A is equal to the standard deviation of the asset return, and $\epsilon \sim N(0,1)$. Formally, this process is known as "Geometric Brownian Motion" and $dW_t^P \equiv \epsilon\sqrt{t}$ is a normally distributed "Wiener Process" under the real-world probability measure P. Note that the latter assumption is what implies that assets follow a lognormal distribution. Using the Ito-Doeblin theorem, one can easily show that such a process for A_t implies:

$$A_t = A_0 e^{\left(\mu_A - \frac{\sigma_A^2}{2}\right)t + \sigma_A \epsilon \sqrt{t}} \qquad (4)$$

In order to determine the probability of default, we first define the default set as $\varepsilon = \{A_T \leq D\}$. Hence, the real-world probability of default at time T, conditional on the information at t, $PD(t,T)$, is given by:

$$\begin{aligned} PD(t,T) &= Prob(A_T \leq D) &(5)\\ &= Prob\left(A_t e^{\left(\mu_A - \frac{\sigma_A^2}{2}\right)(T-t) + \sigma_A \epsilon \sqrt{T-t}} \leq D\right) &(6)\\ &= Prob\left(\epsilon \leq -\frac{ln\left(\frac{A_t}{D}\right) + \left(\mu_A - \frac{\sigma_A^2}{2}\right)(T-t)}{\sigma_A\sqrt{T-t}}\right) &(7)\\ &= Prob\left(\epsilon \leq -DD_t\right) &(8)\\ &= N(-DD_t) &(9) \end{aligned}$$

[4]Black & Cox (1976) relax this assumption and extend the analysis to allow for default to occur when the value of the firm's assets reaches a lower threshold.

where DD_t is the *distance-to-distress* at time t for an asset with drift μ_A, volatility σ_A and distress barrier D, and the function $N(\cdot)$ is the cumulative distribution function for a standardized normal distribution. Note that DD_t is the number of standard deviations that the asset is from the default barrier, D.

3.1. Calculating the Probability of Default

From equation (7) above, it is clear that in order to determine the probability of default we need the following vector of inputs $(A_t, D, \mu_A, \sigma_A, T-t)$. As mentioned before, D is given by the book value of total liabilities (and is hence readily available from the bank's balance sheet), while $T-t$ is, following the standard in the literature, set to one year (i.e. $T-t=1$). The market value and volatility of assets, on the other hand, are not observable. However, the market value of equity and the volatility of equity returns are quoted in the market, and under the framework of the Merton model, this allows us to derive explicit relationships between the observable and unobservable variables such that we can back-out the implied market value of the latter.

Given the simple capital structure of the model, note that equity is defined as a residual claim on assets once debt-holders are paid-out in their entirety. In other words, equity is a call option, with asset value as the underlying, the distress barrier (D) as the strike and $T-t$ as the time to maturity. Hence, the value of equity at any moment in time t, is given by the conditional expectation (under the risk-neutral probability measure Q) of the terminal payoff:

$$E_t = \mathbf{E}_t^Q \left[e^{-r(T-t)} max(A_T - D, 0) \right] \tag{10}$$

$$= C^{BSM}(A_t, D, T-t) \tag{11}$$

where C^{BSM} is the price for a European call option using the Black-Scholes-Merton (BSM) pricing formula (see Black & Scholes (1973) and Merton (1973)). The closed-form solution for the price of equity follows from the fact that assets (i.e. the underlying) are assumed to follow a lognormal distribution and both the risk-free rate and asset volatility are assumed to be time-invariant. The latter thus implies that we can calculate the market value of equity at time t as:

$$E_t = A_t N(d_1) - De^{-r(T-t)} N(d_2) \tag{12}$$

$$\text{where} \quad d_1 = \frac{ln(\frac{A_t}{D}) + (r + \frac{\sigma_A^2}{2})(T-t)}{\sigma_A (T-t)} \tag{13}$$

$$= DD_t^Q \tag{14}$$

$$\text{and} \quad d_2 = d_1 - \sigma_A \sqrt{T-t} \tag{15}$$

Moreover, note that since equity is a function of assets, we can use the Ito-Doeblin theorem to derive an expression for the diffusion process followed by the former. Explicitly, we have that:

$$dE_t = \frac{\partial E}{\partial A} dA_t + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} A_t^2 \sigma_A^2 dt \tag{16}$$

$$= \frac{\partial E}{\partial A} (A_t \, r \, dt + A_t \sigma_A dW_t^Q) + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} A_t^2 \sigma_A^2 dt \tag{17}$$

$$= \left(\frac{\partial E}{\partial A} A_t \, r + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} A_t^2 \sigma_A^2 \right) dt + \frac{\partial E}{\partial A} A_t \sigma_A dW_t^Q \tag{18}$$

where we use the fact that under the risk-neutral probability measure Q, assets follow a geometric brownian motion process with a drift given by the risk-free interest rate r (i.e. $dA_t = A_t\, r\, dt + A_t \sigma_A dW_t^Q$). Assuming that equity also follows a lognormal process implies that:

$$dE_t = E_t\, r\, dt + E_t \sigma_E dW_t^Q \tag{19}$$

Matching terms in equation (18) with those in equation (19) implies that:

$$E_t\, r = \left(\frac{\partial E}{\partial A} A_t\, r + \frac{1}{2} \frac{\partial^2 E}{\partial A^2} A_t^2 \sigma_A^2 \right) \tag{20}$$

and

$$E_t \sigma_E = \frac{\partial E}{\partial A} A_t \sigma_A \tag{21}$$

$$\sigma_E = N(d_1) \frac{A_t}{E_t} \sigma_A \tag{22}$$

Recall that our limitation to calculate the probability of default lay in our inability to observe the market value of assets and asset return volatility. Nonetheless, since the market value of equity and the volatility of equity returns are both quoted by the market, we can use equations (12) and (22) to obtain the market value of assets and asset return volatility implicit in them. Our empirical strategy consists simply of solving this system of equations simultaneously.

3.2. Empirical Strategy

In order to calculate the *distance-to-distress*, we use monthly data for the banks that quoted in the stock market for the entire period comprised between September:2001 - March:2011. This criteria implies that our entire sample is comprised of four banks. However, we believe in the representativeness of these 4 banks from a systemic perspective, given that they accounted for nearly 46% of total bank assets as of April 2011[5]. The variables used are as follows[6]:

- risk-free rate (r): 3-month average Certificate of Deposit (CD) rate[7]
- distress barrier (D): book value of total liabilities
- market value of equity (E_t): current market capitalization at time t
- equity returns volatility (σ_E): annual volatility of monthly stock price returns[8]
- time to maturity ($T - t$): one year
- market value of assets (A_t): obtained from solving equations (12) and (22) simultaneously

[5] This proportion has increased importantly during the period of analysis, going from 28.4% in September, 2001 to 46.4% in April, 2011.

[6] The recent behavior of these series is found in Appendix A.

[7] We also conducted the exercises described below using the 3-month interbank rate as our risk-free rate. Results are, in essence, identical.

[8] In order to effectively calculate equity returns volatility, we use daily stock prices for the 4 banks for the period comprised between July:2000 - March:2011. To calculate the volatility of equity, we first define monthly returns as:

$$\nu_i = ln \frac{P_{last}}{P_{first}} \tag{23}$$

where P_{last} is the closing price on the last trading day of a given month and P_{first} is the closing price on the first day of the same month. Subsequently, we use these monthly returns to calculate the annual volatility of equity returns (at a monthly frequency) as:

- asset return volatility (σ_A): obtained from solving equations (12) and (22) simultaneously

Once we have all our inputs, we calculate the *distance-to-distress* and probability of default under the risk-neutral probability measure as:

$$DD_t^Q = \frac{ln\left(\frac{A_t}{D}\right) + \left(r - \frac{\sigma_A^2}{2}\right)(T-t)}{\sigma_A\sqrt{T-t}} \tag{25}$$

$$PD^Q(t,T) = N(-DD_t^Q) \tag{26}$$

On a final note, we also calculate an additional measure of the *distance-to-distress*. The latter is based on an intuitive definition of the *distance-to-default* proposed by Moody's KMV model (see Crosbie & Bohn (2001)), and we will refer to it as the *theoretical distance-to-distress* (TDD). Given the widespread use of the KMV methodology, we believe this extra measure provides a natural benchmark with which to compare the *distance-to-default* of the "pure" BSM model.

The TDD is simply defined as the ratio between the bank's net market worth and the size of a one standard deviation move in asset value; In other words, the number of standard deviations that asset value is away from default. Hence, we can compute the theoretical probability of default (TPD) as:

$$TDD_t = \frac{\text{Market value of Assets} - \text{Default Point}}{\text{Market Value of Assets} \cdot \text{Asset Volatility}} \tag{27}$$

$$= \frac{A_t - D}{A_t \cdot \sigma_A} \tag{28}$$

$$TPD_t = N(-TDD_t) \tag{29}$$

At this point, it is worth mentioning that the risk measures calculated with the theoretical probability of default are expected to be more stringent than those of the BSM model. This follows from the fact that one can easily show that $TDD_t \leq DD_t$ for any value of the relevant vector of parameters (A_t, D, r, σ_A), and hence the corresponding default probabilities and expected losses will, *ceteris paribus*, be higher[9].

$$\sigma_E = \frac{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\nu_i^2 - \frac{1}{(n-1)n}\left(\sum_{i=1}^{n}\nu_i\right)^2}}{\sqrt{\frac{1}{n}}} \tag{24}$$

where n is 12.

[9]Explicitly, we want to show:

$$TDD_t \leq DD_t,$$

which is true if and only if

$$1 + ln\left(\frac{D}{A_t}\right) - \frac{D}{A_t} \leq \underbrace{\left(r + \frac{\sigma_A^2}{2}\right)}_{\geq 0}$$

Therefore, it is sufficient to show that:

$$1 + ln\left(\frac{D}{A_t}\right) - \frac{D}{A_t} \leq 0.$$

For simplicity, we rewrite this condition as:

$$1 + ln(x) - x \leq 0$$

Results of the *distance-to-distress* and default probabilities using both methods for the 4 banks can be viewed in Figure 1 below.

and check to see if this function has a global maximum or minimum:

$$\frac{\partial}{\partial x} \longrightarrow \frac{1}{x} - 1 = 0$$

$$x^* = 1$$

$$\frac{\partial^2}{\partial x^2} \longrightarrow -\frac{1}{x^2} < 0 \qquad \forall x$$

Thus, we see that $A_t = D$ is a global **maximum** of this function. Moreover, we have that:

$$1 + ln\left(\frac{D}{A_t}\right) - \frac{D}{A_t}_{|_{A_t=D}} = 0$$

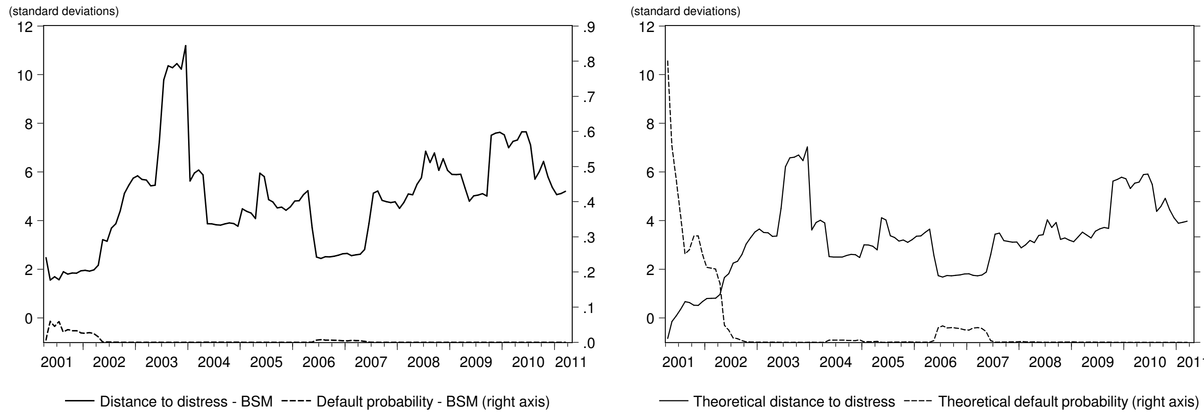
Hence, for any value of $\frac{D}{A_t} \neq 1$, we have that:

$$1 + ln\left(\frac{D}{A_t}\right) - \frac{D}{A_t} < 0$$

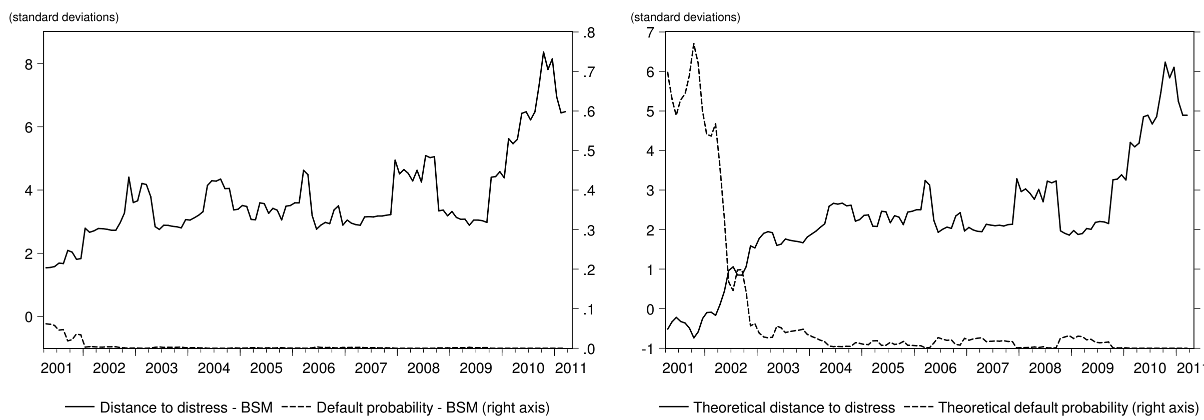
Q.E.D.

Figure 1: Distance to Distress and Default Probability, by bank

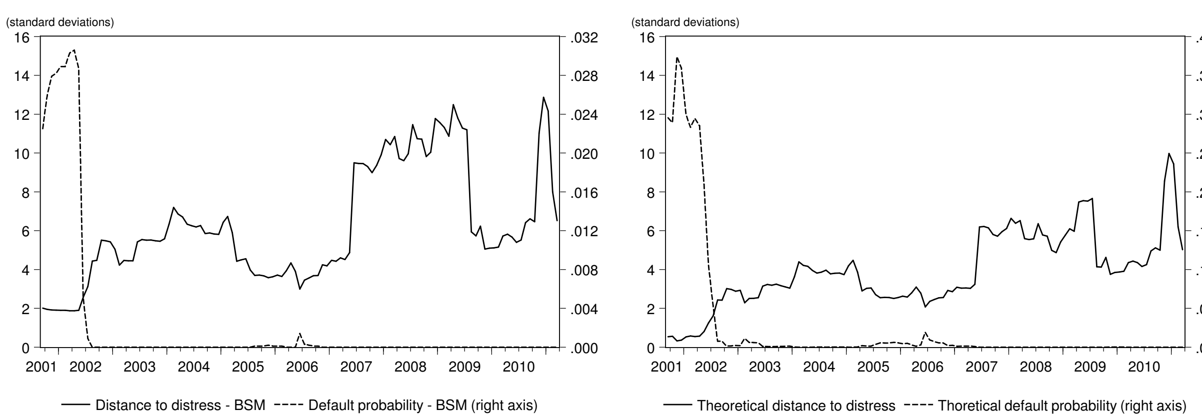
Bank A



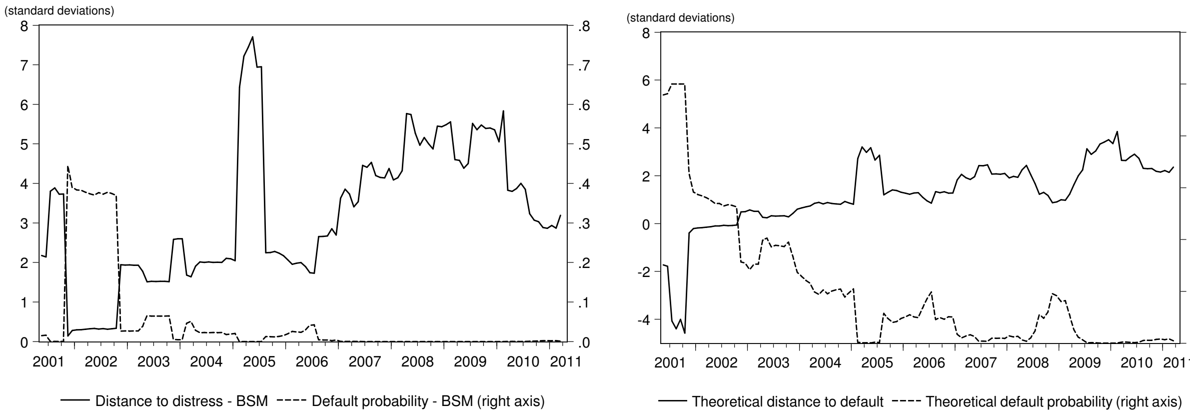
Bank B



Bank C



Bank D



From the observed results, we would like to highlight five facts. First, clearly, a smaller *distance-to-distress* implies a greater default probability. Second, as expected the TDD is always smaller than the one computed using the BSM framework. This implies that the theoretical default probability is always greater than the one calculated with BSM. Third, default probabilities are at their lowest levels of the last ten years, and where highest during 2001-2002, just a few years after the mortgage crisis of 1998-1999. Fourth, results for the four banks indicate that, in general, the periods of higher bank distress (i.e. higher probabilities of default), correspond to 2001-2002, 2006 and 2008-2009, which are effectively periods of significant imbalances in Colombia's financial market. Finally, Bank D exhibits the highest default probabilities (and hence the shortest *distance-to-distress*) in the sample. This is due to the fact that this bank has both the highest equity (and asset) volatility and the largest gross balance sheet leverage ratio of the banks under study (see Appendix A)[10].

Although these results suggest that a bank failure seems implausible at present (and as such we could expect systemic risk to be relatively low), they hold a latent warning. Specifically, that default probabilities can change dramatically in relatively short periods of time. The latter implies that although probabilities of default can be at low levels today, a constant monitoring of these variables is warranted.

4. Government's Implicit Bailout

Obtaining the probability of default of the 4 major banks is a relevant result for financial stability purposes in itself, as it provides supervisory authorities with an explicit and tractable early-warning indicator at a firm-specific level. Nonetheless, it can also be further exploited to account for a more system-wide measure of risk. The recent financial crisis highlighted the importance of large and interconnected institutions and the imperative need to measure and regulate the systemic risk they pose.

In order to do so, we first specify the implicit bailout from the government. We assume that the sovereign will effectively be liable to debt-holders and will therefore repay all outstanding debt net of the market

[10]Gross balance sheet leverage, calculated as the ratio of total assets to total book equity, reached 5.1, 6.7, 7.1 and 10.8 in April 2011 for Bank A, Bank B, Bank C and Bank D, respectively. Average leverage for the entire financial system stood at 7.1, implying that only Bank D was above the system average in our sample.

value of assets. The latter implies that the payoff that the government expects to undertake at time T is thus given by:

$$\text{Payoff}_T = Max(D - A_T, 0) \tag{30}$$

Note that equation (30) effectively defines the payoff of a put option, and hence we can express the discounted expected value of the implicit bail-out (under the risk neutral probability), as the BSM price of such an option. The expected loss given default (or expected implicit bailout) is therefore:

$$P_{E,t} = De^{-r(T-t)}N(-d_2) - A_t N(-d_1) \tag{31}$$

Where d_1 and d_2 are given in equation (13) and (15) above. Recall from the previous section that we computed two versions of the *distance-to-distress*, and therefore we will have two distinct calculations of the expected loss given default. We can rewrite equation (31) in terms of the risk-neutral probability of default (RNPD) and the loss given default (LGD) as:

$$P_{E,t} = \underbrace{N(-d_2)}_{RNDP} \underbrace{\left(1 - \frac{N(-d_1)A_t}{N(-d_2)De^{-r(T-t)}}\right) De^{-r(T-t)}}_{LGD} \tag{32}$$

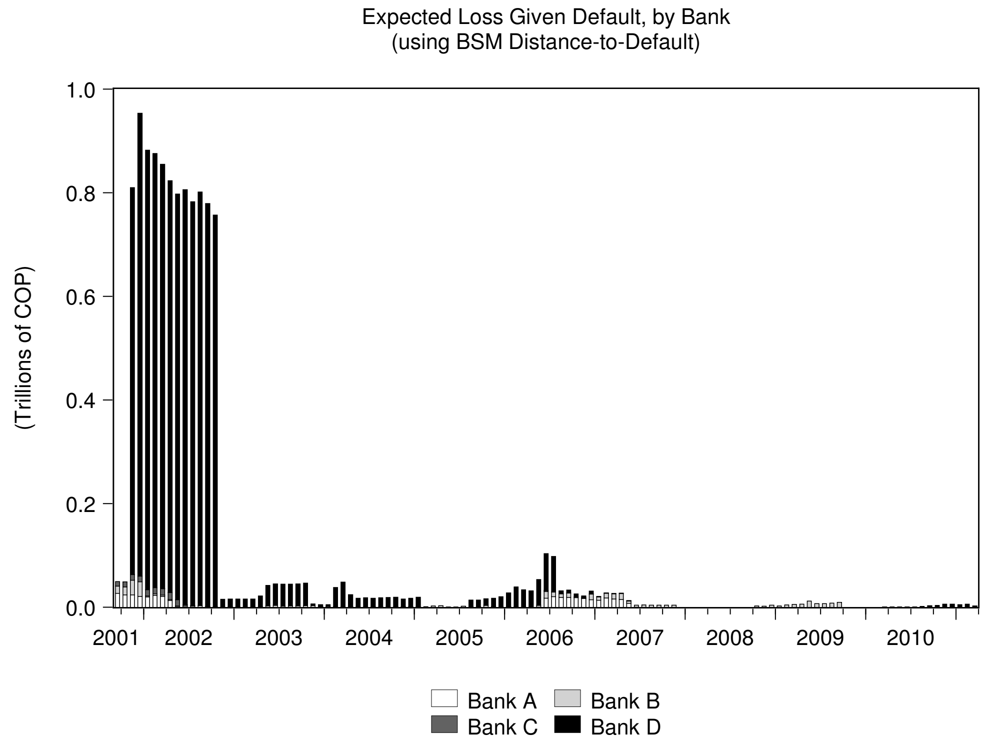
Figure 2 shows the historical contribution of each bank to the system-wide expected loss given default, computed both with the BSM and the theoretical *distance-to-distress*. As anticipated, the expected LGD computed with the TDD is greater than the one computed under the BSM framework. Under both approaches, Bank D is the bank that exhibits the greatest contribution to potential systemic sovereign risk, while Bank C has the least.

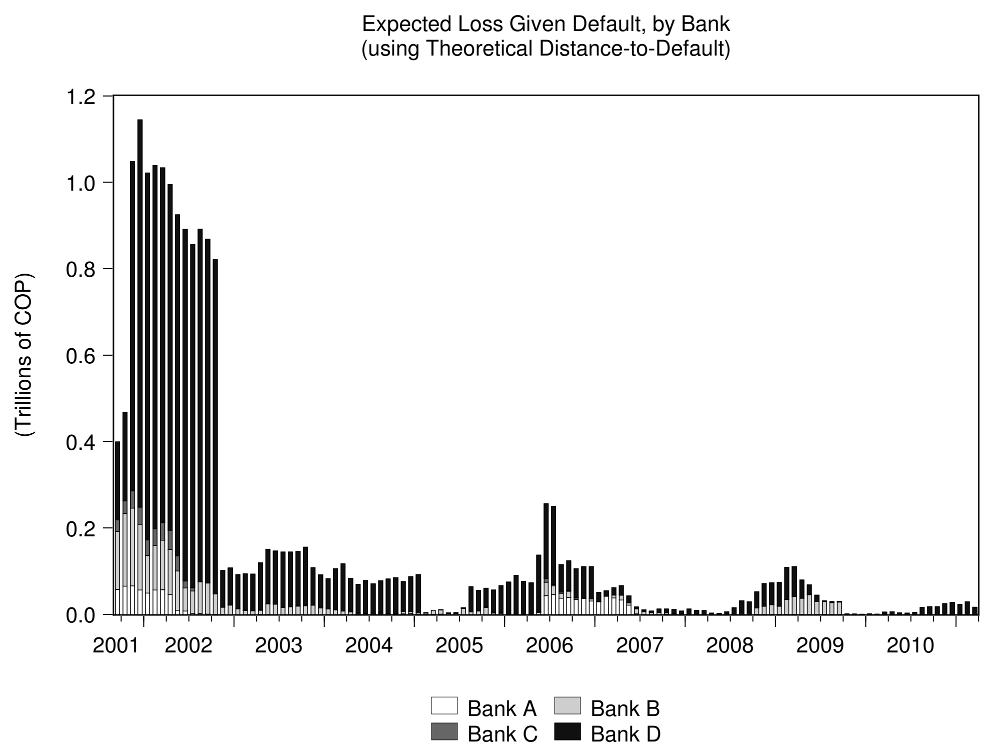
Interestingly, historical results show that periods where the implicit government bailout increased correspond with periods of financial distress: 2001-2002, 2006 and 2008-2009. However, the main causes of each of these periods of stress are fundamentally distinct. While the 2008 crises seems intimately linked to a materialization of credit risk, and the 2006 crisis period can be better explained by an increased exposure to market risk, the 2001-2002 period seems to have elements of both. What is important is that since the default probability used to compute the expected LGD is based on the market value of equity returns, it should encompass, assuming the Efficient Market Hypothesis holds, all the relevant information needed to evaluate the performance of a bank in all of its fronts.

In the first of these three periods, the maximum loss (corresponding to the theoretical framework) that would have faced the government in the event of default would be greater than COP$1.1 trillion[11] (t) (around US$496 million), the biggest in the period studied but still no larger than 2% of 2001's fourth quarter GDP. Not surprisingly, recall from Section 3.2 that the early 00's also showed default probabilities at their highest levels for all banks, possibly as a lagged symptom of the 98-99 mortgage crisis, one of the deepest scenarios of financial distress in Colombia.

[11] We use the short-scale convention, so that 1 trillion is equal to $1 \cdot 10^{12}$.

FIGURE 2: Implicit Government Bailout





The roots of the 98-99 crisis period go back to 1991, when Colombia's Congress issued the Financial Reform Law, which eased controls on capital flows to facilitate foreign investment. The reform was successful, and given the intimate relation between capital flow and loan cycles in Colombia (see Villar et al. (2005)), their increase led to significant credit growth, especially mortgages. As consumption increased, housing prices grew to their highest historical levels. This surge in prices was, nonetheless, followed by a sharp increment in interest rates in response to the sudden stop in capital flows. As loan-to-value ratios elevated, loan quality began to quickly deteriorate, which ultimately led to a general reduction in the availability of credit (i.e. a credit crunch). With credit markets partially frozen, financial institutions began to increase their holdings of Government securities. Higher demand from market investors, coupled with lower interest rates since the beginning of 2000 as the Central Bank sought to restore market liquidity, created ideal conditions for exceptional growth in this market. However, the increase of interest rates between July and September of 2002, given the change in country risk, adversely affected the price and volatility of these assets, causing additional losses in financial institutions balance sheets (*Financial Stability Report* (2002)).

However, financial turmoil linked to the sovereign debt market was not exclusive of the 2002 episode described above. In effect, the 2006 period was also characterized by imbalances stemming from the investment portfolio of banks' balance sheets. In the first semester of this year, the interest rate of the secondary market for government bonds increased rapidly due to the massive liquidation of positions by investors who expected increments in foreign rates. This phenomena caused an increase in market volatility and consequently, financial institutions holding long positions in sovereign debt faced significant valuation losses (*Financial Stability Report* (2006)). The maximum total expected cost for the government was close to COP$0.26 t (around US$101 m), a little over 0.25% of 2006's second quarter GDP.

Finally, the financial crisis of 2008-2009, which had considerable effects on international markets, had only minor repercussions in the Colombian economy. Contagion of this crisis to the colombian banking system occurred mainly through its effect on the real sector and increased risk aversion. The reduction in international demand, coupled with significant drops in commodity prices, weakened the real sector's balance sheet, thus affecting their repayment capacity and ultimately increasing non-performing loans indicators. Moreover, increased risk aversion, especially from banks, implied tighter credit conditions and a reduced supply of financial funds. Still, the maximum aggregate expected loss for the government was COP$0.11 t (around US$45 million), less than 0.1% of 2009's first quarter GDP.

Still, the practical importance of being able to express the expected loss given default as a simple put option goes beyond providing us a useful tool to analyze systemic risk in hindsight. Specifically, it implies that we can quantify a forward-looking systemic risk measure by constructing a portfolio of put options (i.e. one option per bank). Once we have defined such a portfolio, and taken into account the dependence structure between the different put options, we can calculate any risk measure desired on the derivatives portfolio. Of course, the crucial element in this analysis is how to best capture the dependence structure between our random variables (i.e. the put options). In order to do this we turn to our powerful statistical allies; copulas.

4.1. Copulas

In general terms, copulas are functions that approximate the joint behavior among random variables with pre-specified marginal distributions. Although copulas are abstract in nature, they are one of the most used tools for modeling dependence between random variables, specially when modeling the extremes of a distribution. Formally, if $U_1, \cdots, U_n$ are uniform random variables, the function $\mathbf{C} : [0,1]^n \to [0,1]$ that satisfies

$$\mathbf{C}(u_1, \cdots, u_n) = \mathcal{P}\{U_1 \leq u_1, \cdots U_n \leq u_n\} \tag{33}$$

is the copula of $U_1, \cdots, U_n$.
In the case where the marginals are not uniform variables, say, $X_1, \cdots, X_n$ with cdf's $F_{X_1}, \cdots, F_{X_n}$, respectively, the copula of the uniform random variables

$$U_1 = F_{X_1}(x_1), \cdots, U_n = F_{X_n}(x_n) \tag{34}$$

is called the copula of $(X_1, \cdots, X_n)$. The dimension of the copula is the dimension of the domain of $\mathbf{C}$. In this case n.

As mentioned above, a copula enables to reconstruct the joint distribution of random variables taking the marginal distributions as inputs. Sklar's theorem[12] provides the theoretical foundation for the application of copulas. In particular, the theorem states that for a multivariate cumulative distribution function $F(x_1, ..., x_n)$ with marginal cdf's given by $F_i(x_i)$ for $i = 1, \cdots, n$, there exists a unique copula $\mathbf{C}$ such that

$$F(x_1, ..., x_n) = \mathbf{C}(F_1(x_1), ..., F_n(x_n)) \tag{35}$$

In this way, this formulation enables to decompose $F(x_1, ..., x_n)$ into marginal distributions and a copula, where the latter summarizes the dependence structure of the random variables.

Using this result, it is clear that in order to obtain the joint distribution of any set of random variables it is sufficient to approximate the unique copula $\mathbf{C}$ that satisfies equation (35). In practice, it is necessary to estimate, using a semi-parametric approach, the dependence parameter(s) of the copula model that best fits the empirical data. The theory on copulas has defined different families which suit distinct types of data. Among these, the Archimedean Family is popular because the copulas in this family allow to model dependence in high dimensions with only one parameter; which is convenient in practical applications. The Gumbel copula, which is an Archimedean copula, appears naturally in the analysis of dependence between financial series as it is usually used to model variables with heavy-tailed distributions (Carmona, 2004). Given that our financial variables are indeed heavy-tailed, we will estimate the dependence between them using a Gumbel copula.

Therefore, to estimate the risk faced by the sovereign it is crucial to understand the dependence structure of the put options computed before, and as we are studying extreme events, the copula methodology allows us to model the dependence of the tails of these distributions explicitly. In this way, we estimate what would be an extreme but possible loss in the financial system.

4.2. Empirical Strategy Revisited

In order to estimate the risk measures of our portfolio we use Monte-Carlo simulations, sampling from the joint distribution of the put options. As explained in Section 4.1, the joint distribution is defined by the marginal of each put and the copula between them. Thus, in estimating the risk measures for our portfolio we require a two-step process: First, we estimate the marginal distribution of the expected LGD

[12] See Sklar (1959).

of each bank and then we fit a copula to describe the dependence between the random variables (i.e. the puts).

4.2.1. Fitting Extreme-Value Distributions

In estimating the marginal distribution of the expected LGD, we closely follow the approach suggested by Carmona (2004). The author underscores the fact that traditional density estimators (such as kernel density estimators or histograms) cannot estimate the tails of the distribution precisely, because there are not enough data points there. Consequently, he suggests the use of a semi-parametric approach involving Extreme-Value Theory (EVT). Under this method, one uses standard nonparametric techniques to estimate the center of the distribution whilst parametric techniques are used to estimate the polynomial decay of the density in the tails (see Zivot & Wang (2006) and Becerra & Melo (2008)).

In applying this semi-parametric approach, we use monthly data on put prices (i.e. expected loss given default) for the period comprised between September:2001 - March:2011[13]. An important consideration here is which theoretical distribution to use in the fit of the tails. In overcoming this minor impasse, we turn to Figure 3, which depicts the Quantile-Quantile plot of the the empirical distribution of the option prices against that of the standard normal. The presence of leptokurtosis on the right tail of the distribution is readily apparent[14]. Hence, in trying to fit a theoretical distribution to the tail of our data, we use a class of extreme-value distributions known as Generalized Pareto Distributions (GPD). These distributions allow for a large number of unusually large extremes in each sample, a characteristic that is evident in our put prices.

Thus, the question is how to characterize such a distribution for our data. Said class of distributions has a density that decays polynomially, and the task of the researcher is then to estimate the degree of the polynomial decay in the tails (i.e. to estimate the shape parameter, ξ). In doing so, we use a semi-parametric approach based on the Peaks over Threshold (POT) method. In a nutshell, this method consists of defining an overall threshold and extracting all points above that threshold to then develop a distributional model for these points. The GPD discussed above provides a useful distributional model for univariate extreme value data since it indicates what type of extreme value model is appropriate[15]. Schematically, we abide by the following steps in fitting a marginal extreme-value distribution to the tails[16]:

I. Using a Q-Q plot we determine which cut-off value (i.e. the quantile) should separate the tail from the bulk of the distribution.

II. Once we have determined such a quantile, we can calculate the probability mass beyond this point, that is, the percentage of data points that will comprise the tail.

[13] We also replicated the exercise using the following sub-samples: September:2001 - December:2008, September:2001 - June:2009, September:2001 - December:2009 and September:2001 - June:2010. The empirical strategy is analogous to the one described below.

[14] Note that the distribution of the expected loss given default has no lower/left tail, since losses are bounded at 0.

[15] The type of model will depend on the sign of ξ. Namely, $\xi = 0$ is equivalent to an extreme value Gumbel distribution; $\xi > 0$ is equivalent to an extreme value Fréchet distribution; and $\xi < 0$ is equivalent to a reverse Weibull distribution.

[16] In the particular case of the put options here used, we need only fit the GPD to the right tail of the distribution (recall that the expected LGD has no left tail). However, the procedure is described for the general two-tailed case for completeness.

III. We then use a POT Plot, which graphs the maximum likelihood estimates of the shape parameter ξ as a function of the threshold value for each tail of the distribution of the monthly put prices (see Figure 4).

IV. Using this visual aid we define a threshold (and ultimately a value for ξ) by weighting two considerations:

- *i.* That the threshold is such that the percent datapoints above (below) the threshold for the right (left) tail are approximately equal to those suggested by examining the Q-Q plot.

- *ii.* That the estimated value for ξ is robust to small changes in the threshold value (i.e. that the line depicted in Figure 4 is "flat" in the neighborhood of the chosen threshold).

V. Having chosen a value for ξ we fully characterize the GPD fitted to our data, and all that is left is to examine the goodness-of-fit of such a distribution.

Figure 3: Q-Q Plot of Put Option Prices, by Bank

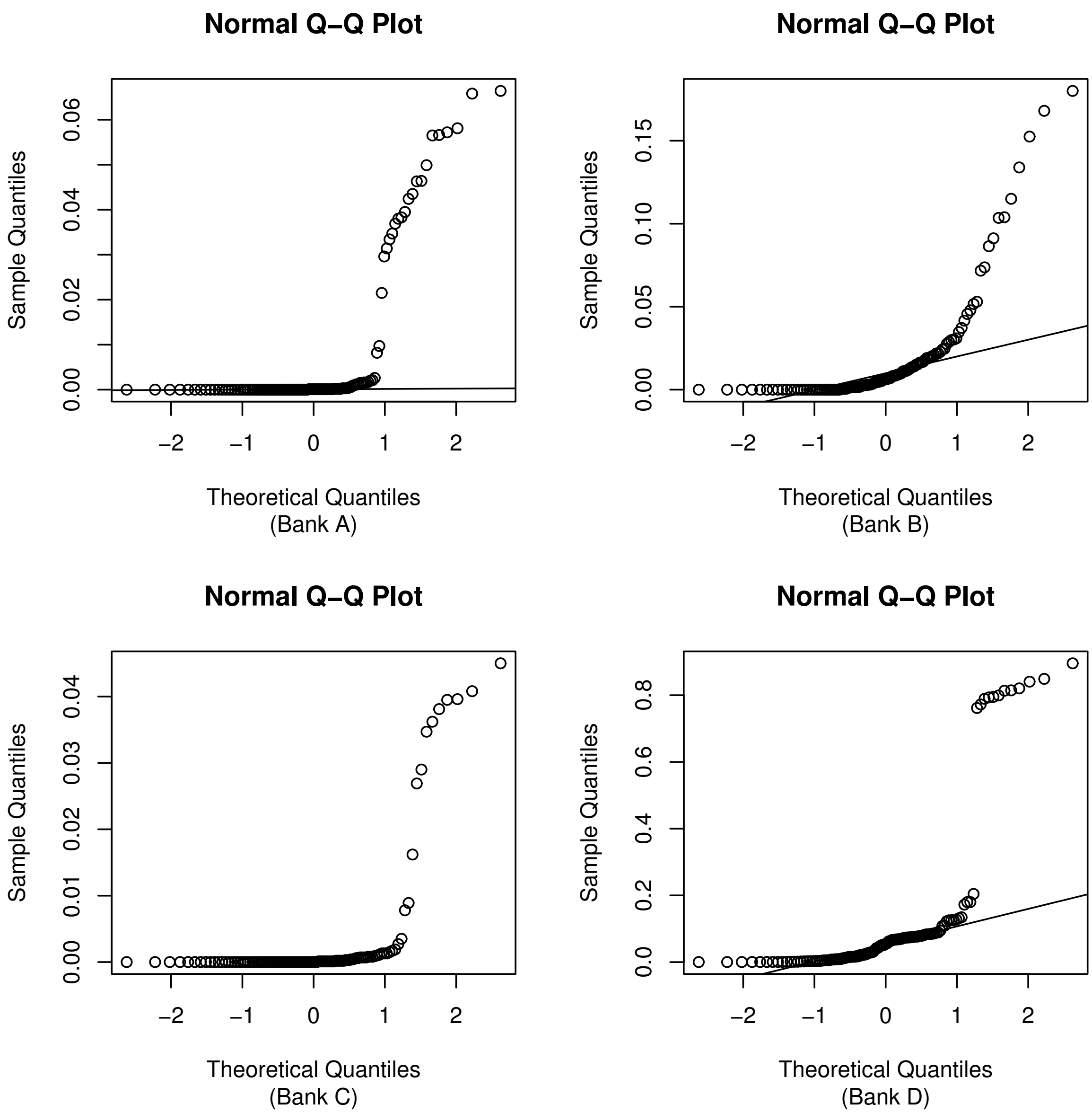
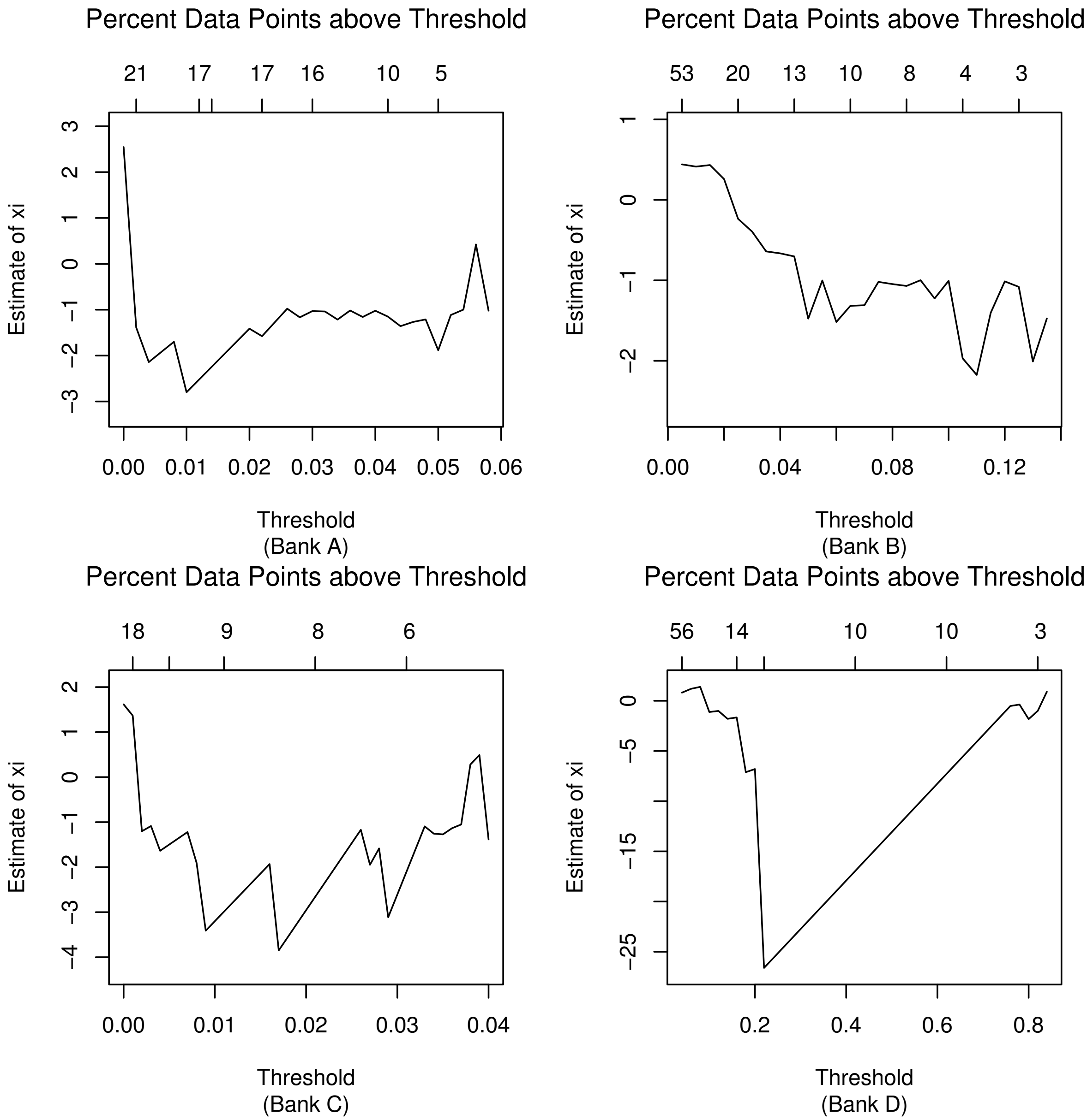
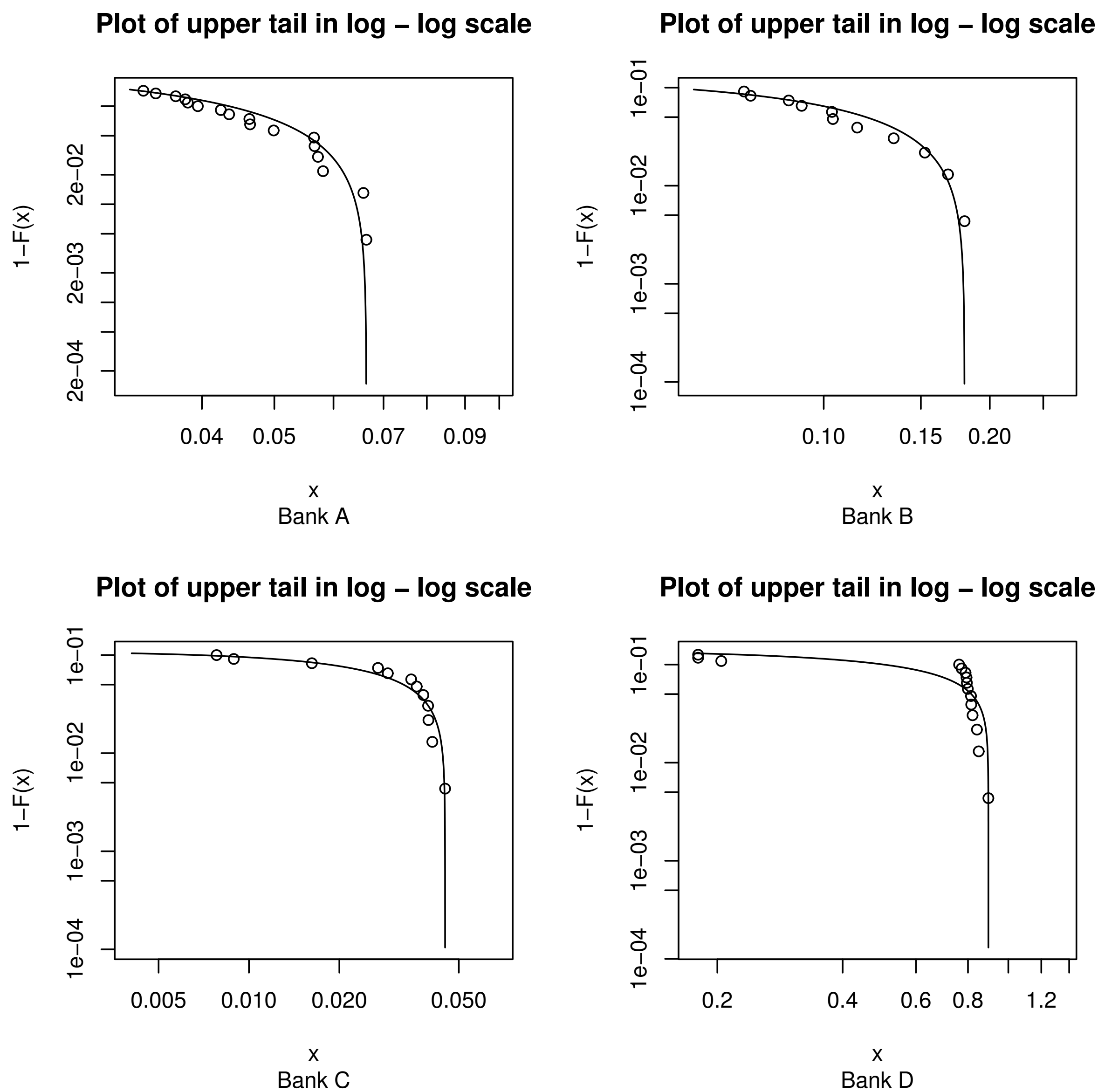


Figure 4: Maximum Likelihood Estimates of the Shape Parameters for the Right Tail, by Bank



In the particular case of the put prices here considered, an examination of the Q-Q plots reveals that significant deviations from the bulk of the standard normal distribution generally occur 1 deviation away from the mean. On a one-tailed distribution, this accounts to roughly 16% of the probability mass. Hence, in choosing our thresholds we try to account for roughly this percentage, while at the same time guaranteeing robustness in the estimated ξ. The fit of our GPDs can be visually assessed in Figure 5. It is clear that the GPD does a good job in accounting for the extreme losses present in our empirical data.

FIGURE 5: Verifying the Fit of the GPD on the Put Price Data, by Bank



4.2.2. Estimating a Copula Model

The second and final step in calculating the risk measures for our puts portfolio is to estimate the degree of dependence between the random variables via a copula model. As mentioned in Section 4.1, a copula is the joint distribution of uniformly distributed random variables, and so we must first *transform* our put price data in order to implement this model. The required procedure is simple, and requires us to make use of the inverse transformation method (or Smirnov transform), which states that if X is a continuous random variable with cumulative distribution function F_X, then the random variable $Y = F_X(x)$ has a uniform distribution on [0,1].

Here, an important caveat applies: since the put price data is comprised primarily by zeros, the transformation does not generate a uniformly distributed random variable and therefore thwarts the use of the copula as the appropriate model to capture the dependence between the variables. However, rather than giving up on the method altogether, we make an additional assumption on the joint behavior of the expected LGD of the banks under study. In particular, we assume that the dependence structure between these variables is identical to that between the equity returns of the banks. Hence, we can estimate the dependence parameter between the equity returns and use **this same parameter** to characterize the copula model of the put prices. We believe the strength of this assumption lays in the fact that, if equity returns appropriately reflect the market's valuation of the bank's net worth, then joint movements in these variables should reflect the dependence structure between the expected LGD of the banks, as these are a direct function of the bank's equity value.

Thus, in estimating the copula model, we use monthly data on the continuously compounded (monthly) return on equity prices, for the period comprised between October:2001 - April:2011[17]. However, one of the assumptions in the estimation of the copula parameter is that the data used comes from an *i.i.d.* sample. In practice, this assumption implies the data should not exhibit any type of pattern or temporal regularity, a requirement almost implausible for financial variables. Therefore, utilizing the usual tools in time series analysis, we identify a VAR(p) - GARCH(p,q) model on the four put option price series and work on the residuals, as suggested in Grégoire et al. (2008). The approach followed consisted of the following steps:

I. We fit a VAR(4) model[18] and use the resulting residuals from this model as our "new" returns

II. We model the conditional volatility of the "new" returns using a GARCH(1,1) model, and obtain the "weakly stationary returns" (WS returns) as the residuals of this model

III. We check that these WS returns effectively behave like weakly stationary variables

IV. We fit a marginal distribution to the data following the steps described in Section 4.2.1

V. Using the inverse transformation method, we obtain uniformly distributed random variables

VI. Finally, we estimate the parameter of a copula of the Gumbel (or logistic) family. The estimation is based on the inversion of Kendall's tau method[19].

Residual diagnostic tests to verify the suitability of the VAR-GARCH model used can be found on Table 3 in Appendix B. In general, the tests show that no significant autocorrelation or ARCH effects persist. Moreover, the appropriateness of the estimated GPD on our data can be verified by looking at the fit of such a distribution on the tails, which is shown in Figure 6. Again, the GPD seems to get the job

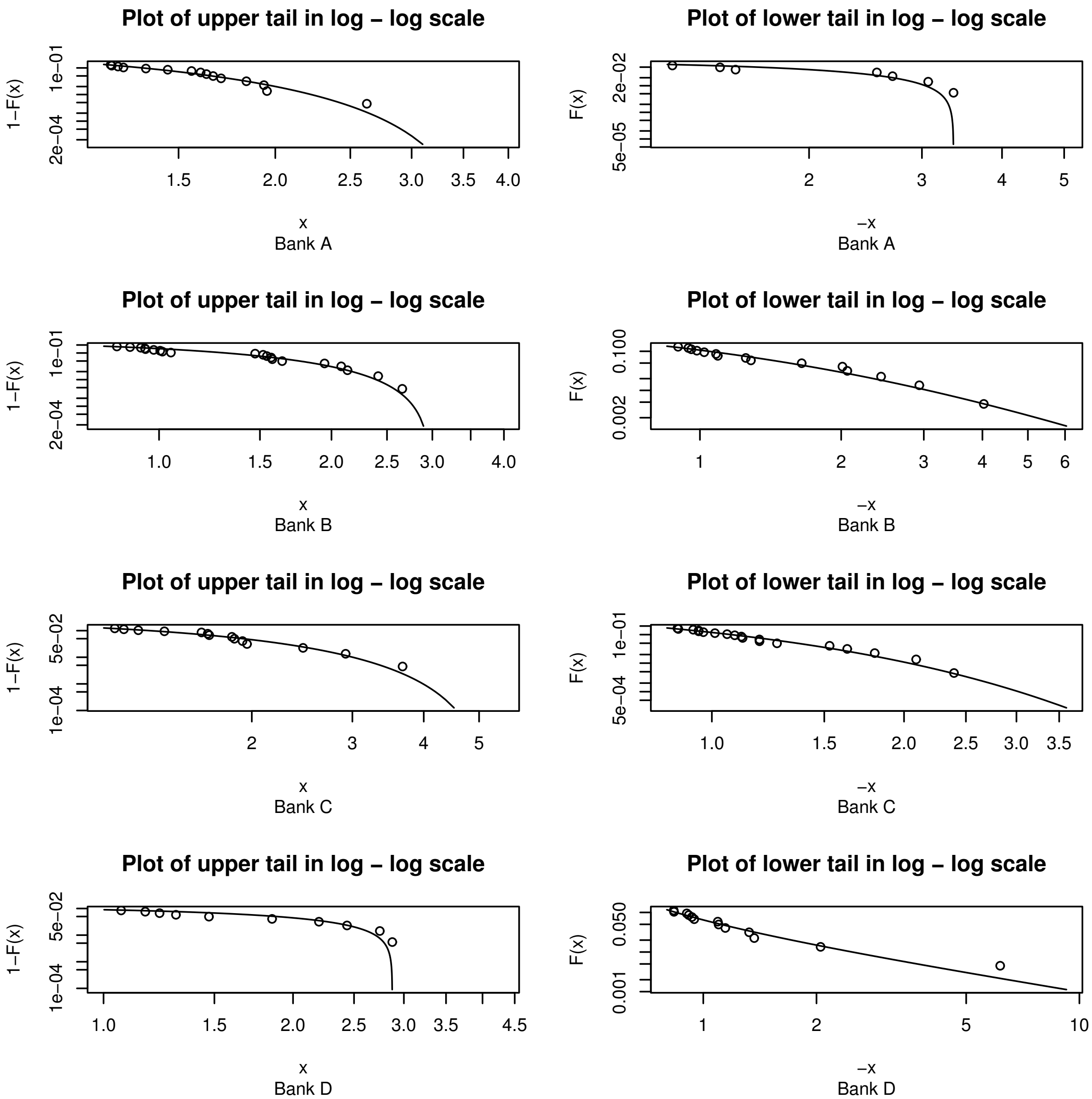
[17] We also replicated the exercise using the following sub-samples: October:2001 - December:2008, October:2001 - June:2009, October:2001 - December:2009 and October:2001 - June:2010. The empirical strategy is analogous to the one described below.

[18] The order of the estimated VAR was chosen by minimizing the AIC and HQ information criteria.

[19] In calculating the dependence parameter of our copula model, we use several estimation methods. Namely, maximum pseudo-likelihood, inversion of Kendall's tau and inversion of Spearman's rho. In deciding which estimated parameter to use to characterize the dependence between the variables, we first checked the statistical significance of the coefficient, and then chose the method which yielded the lowest standard error.

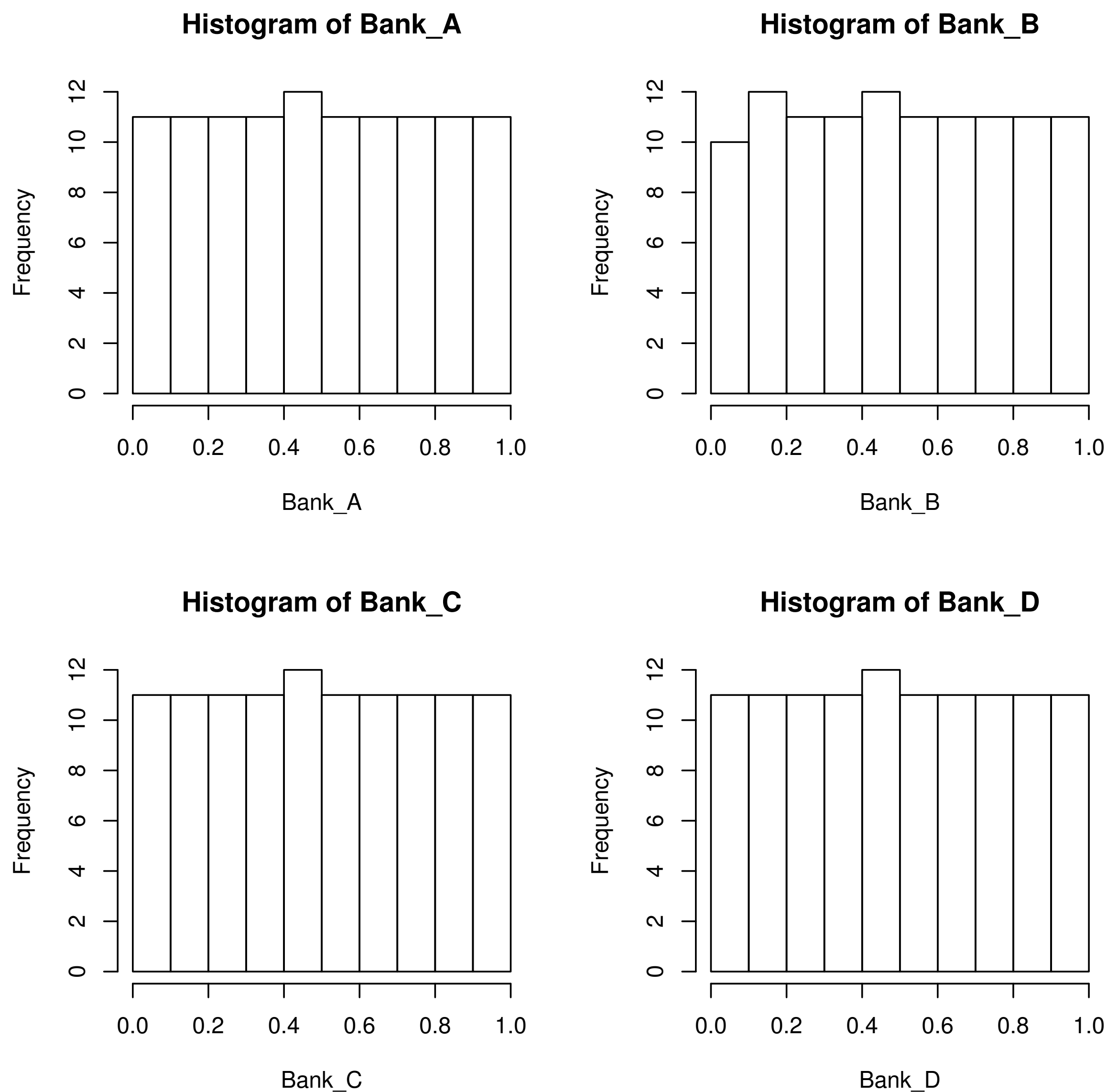
done when it comes to capturing distributions with extreme events. The uniformity of the transformed variables was checked using the Kolmogorov-Smirnov test and can be visually assessed in Figure 7[20].

Figure 6: Verifying the Fit of the GPD on WS Equity Returns, by Bank



[20]Results of the Kolmogorov-Smirnov test can be found in Appendix C. Additionally, the POT plots used to estimate the shape parameter of the GPD for each bank and the Q-Q plots of the uniform distribution against the sample quantiles of our transformed variables are also included in this Appendix.

Figure 7: Histogram of Transformed WS Equity Returns, by Bank



5. Simulations and Risk Measures

Once we have estimated the marginal distributions of the put option prices, as well as the dependence between the variables, we can generate multivariate random samples from the joint distribution of the expected LGD. This is because the joint distribution can be fully characterized by the marginal distributions and the copula, as explained in Section 4.1. Indeed, the first component of a random sample generated from our estimated copula forms a univariate sample uniformly distributed on [0,1]. Thus, applying the quantile function of the first estimated marginal GPD effectively generates a sample from such a distribution. The same can be done with the second, third and fourth component in the random sample from the copula model. By definition, these multivariate random samples not only have the right marginals, but also the right copula (Carmona (2004)). Thus, in order to calculate our risk measures we:

I. Generate a random sample of size $N \times 4$, where N is the number of simulations, from the Gumbel copula with the estimated dependence parameter.

II. We *transform* each uniform simulated sample by applying the inverse cdf (i.e. the quantile function) of the estimated probability distribution, so that the resulting sample has the estimated GPD.

III. We define the Government's implicit bailout as the sum of the simulated expected LGD of each bank.

IV. Since we have N possible one-month-ahead loss scenarios, we can effectively characterize the distribution of the implicit bailout from the government, and use this distribution to calculate our risk measures (i.e. VaR and ES).

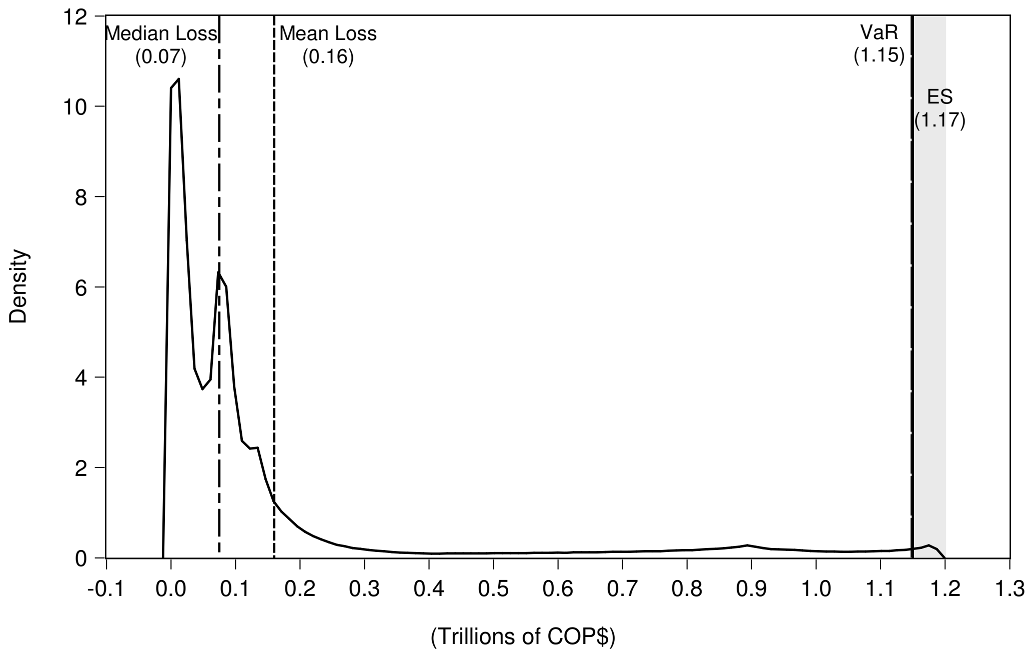
Calculating both the VaR and ES using Monte Carlo simulations is fairly straightforward. Since we have a simulation of possible one-period-ahead losses obtained from the joint distribution of the put option prices, the former is nothing more than an empirical quantile, whilst the latter is the arithmetic mean of the losses above and including the VaR.

In our particular exercises, we use 1 million simulations and a confidence level of 99% to calculate our risk measures. The latter are represented graphically in Figure 8, where we also included the mean and median expected LGD for completeness, employing the theoretical framework. Under this assumptions, we find the worst expected loss that will only be exceeded with a probability of 1% within a one-month horizon is COP$1.15 trillion (t) (around US$635 million), while the average loss should that 1% situation materialize, amounts to COP$1.17 t (around US$645 million). These losses are not significant and represent less than 1% of 2011's first quarter GDP, so that they do not pose an immediate threat to financial or overall macro stability. The latter is due to at least two important factors. On the one hand, we are only considering the 4 largest commercial banks, and Colombia's financial system has many important non-bank institutions, so analyzing only a specific fraction of the whole system is bound to bias the impact on the downside. Additionally, Colombia's banking system is fairly traditional (i.e. banks are almost exclusively dedicated to traditional loan operations and their investments are almost entirely devoted to buying safe government securities) and regulation is highly conservative. These factors make asset volatility relatively low, which coupled with low leverage levels (compared to those seen in the U.S, in the pre-crisis period)[21] implies low probabilities of default and hence, a small expected LGD.

The next step in the analysis is to consider the behavior of the traditional risk measures here considered (VaR and ES) during different moments of our period of analysis. In order to do this, we defined four sub-samples: i) September:2001 - June:2009, ii) September:2001 - December:2009, iii) September:2001 - June:2010 and iv) September:2001 - December:2010. For each one of these sub-samples we calculated both the VaR and the ES as well as the bank-specific contribution to the system-wide loss.

[21] Average gross balance sheet leverage, calculated as the ratio of total assets to total book equity, for the four banks here considered stood at 7.4 in April, 2011. This proportion is very similar to that of the entire financial system (7.1 times). Meanwhile, between 2002Q1 and 2007Q2, average leverage for the 17 largest commercial banks in the U.S. reached 12.5, with a sizeable maximum of 17.6 (Wolff & Papanikolaou (2010)).

FIGURE 8: Distribution of the Expected LGD



Results of the expected loss due to default using both the BSM and theoretical framework can be checked in Tables 1 and 2. Not surprisingly, when using the theoretical default probability and *distance-to-distress*, the VaR and the Expected Shortfall for all the banks is greater than when using the BSM calculations. Again, this is a direct implication of the lower probability of default that results from the BSM approach. More importantly, it is relevant to mention that the risk measures calculated are relatively stable through time, independent of the framework used (i.e. BSM or theoretical). This is not only desirable from a policymaking perspective, as one would not want regulatory charges to be volatile, but also seems to be an indication of the robustness of the methodology here considered.

As mentioned above, the methodology implemented here allows us to approximate the multivariate density of these losses, by means of their marginal distributions and a measure of dependence (i.e. a copula). The simulations obtained from this function can be used to determine the contribution of each institution to the total expected LGD, which can be helpful in multiple ways. On the one hand, it allows us to identify the most systemic and vulnerable agents of the sample. Moreover, it can help us to determine the impact that exogenous shocks or changes in the regulatory framework can have over the size and allocation of systemic risk. Finally, it provides an useful guideline in determining the individual capital charges that the regulator should impose given the contribution of each bank to the externality.

Using the BSM model's results, we see that the bank that contributes the greatest percentage to the total expected loss is Bank D, which accounts for roughly 94% of the VaR and 92.4% of the ES, while the bank with the lowest expected LGD is Bank C, which represents close to 1.4% of the risk measures considered. The percentages for Bank D fall under the theoretical framework to around 76%, though it still remains as the highest contributor to aggregate losses in both of the risk measures considered.

However, it is noteworthy that under this framework Bank B represents an important share of both the VaR and ES, raising the corresponding share from 2% and 3%, in the BSM exercise to around 15% and 14,6%, respectively. It is important to note that the individual contributions have not increased significantly in the last two years, which implies that the systemic importance of each bank has not changed in the last couple of years.

Furthermore, we consider the theoretical model results more suitable for regulatory purposes, because they provide a more acid scenario for all the banks considered. In addition, using the theoretical framework implies a less concentrated bank-specific contribution to aggregate risk. Even when Bank D accounts for most of the expected losses in all the periods here evaluated, other institutions have a greater importance in the aggregate cost or have at least increased their relevance during this period. Hence, if one where to impose capital charges, or to force the constitution of a resolution fund with the sole purpose of providing the necessary funds for the orderly unraveling of systemically important institutions, the theoretical framework would entail a higher degree of risk-sharing between institutions.

On a final note, both the mean and median expected LGD deserve an added remark. Though we do not consider these statistics as the most appropriate in terms of calculating risk-based regulatory charges, we consider their usefulness in terms of macroprudential supervision fundamental. Our risk measures are, by definition, reflecting a worst-case scenario, and so they are likely to react to periods of build-up of risk in a sluggish manner. In contrast, the mean and median measures should readily reflect a higher joint default probability, and in this sense, provide a natural complement to the individual probabilities of default calculated in Section 3.2. Hence, given their potential to detect period of risk build-up, we strongly believe in their relevance as ideal early-warning indicators.

TABLE 1: Individual Contribution to System-Wide Risk Measures
(using BSM *Distance-to-Default*)

Value at Risk (1%)									
	(in trillions of COP$)					(% of total expected loss)			
Period	Bank A	Bank B	Bank C	Bank D	**TOTAL**	Bank A	Bank B	Bank C	Bank D
Jun-09	0.026	0.022	0.013	0.870	**0.931**	2.83%	2.32%	1.44%	93.42%
Dic-09	0.026	0.018	0.013	0.845	**0.903**	2.91%	1.99%	1.48%	93.62%
Jun-10	0.025	0.017	0.013	0.873	**0.928**	2.66%	1.81%	1.43%	94.09%
Dec-10	0.024	0.017	0.013	0.873	**0.928**	2.63%	1.84%	1.42%	94.11%
Mar-11	0.022	0.025	0.010	0.871	**0.928**	2.34%	2.64%	1.12%	93.90%

Expected Shortfall									
	(in trillions of COP$)					(% of total expected loss)			
Period	Bank A	Bank B	Bank C	Bank D	**TOTAL**	Bank A	Bank B	Bank C	Bank D
Jun-09	0.026	0.029	0.013	0.873	**0.941**	2.74%	3.11%	1.39%	92.76%
Dec-09	0.026	0.029	0.013	0.845	**0.912**	2.80%	3.19%	1.42%	92.59%
Jun-10	0.025	0.037	0.013	0.873	**0.948**	2.64%	3.89%	1.37%	92.10%
Dec-10	0.025	0.028	0.013	0.873	**0.939**	2.71%	2.98%	1.38%	92.94%
Mar-11	0.025	0.028	0.013	0.873	**0.940**	2.71%	3.01%	1.38%	92.90%

TABLE 2: Individual Contribution to System-Wide Risk Measures (using Theoretical *Distance-to-Default*)

Value at Risk (1%)									
	(in trillions of COP$)					(% of total expected loss)			
Period	Bank A	Bank B	Bank C	Bank D	**TOTAL**	Bank A	Bank B	Bank C	Bank D
Jun-09	0.056	0.17	0.0432	0.888	**1.161**	5.48%	14.33%	3.71%	76.46%
Dec-09	0.065	0.156	0.044	0.880	**1.145**	5.67%	13.64%	3.85%	76.84%
Jun-10	0.065	0.147	0.043	0.883	**1.138**	5.70%	12.95%	3.81%	77.54%
Dec-10	0.065	0.148	0.041	0.886	**1.139**	5.66%	13.01%	3.58%	77.74%
Mar-11	0.063	0.171	0.039	0.874	**1.147**	5.48%	14.92%	3.37%	76.23%

Expected Shortfall									
	(in trillions of COP$)					(% of total expected loss)			
Period	Bank A	Bank B	Bank C	Bank D	**TOTAL**	Bank A	Bank B	Bank C	Bank D
Jun-09	0.0637	0.166	0.043	0.887	**1.161**	5.48%	14.33%	3.71%	76.46%
Dec-09	0.064	0.169	0.044	0.893	**1.169**	5.48%	14.44%	3.72%	76.35%
Jun-10	0.064	0.167	0.043	0.892	**1.166**	5.51%	14.31%	3.72%	76.46%
Dec-10	0.064	0.168	0.043	0.892	**1.166**	5.45%	14.39%	3.70%	76.46%
Mar-11	0.064	0.171	0.043	0.892	**1.170**	5.45%	14.58%	3.70%	76.28%

6. Concluding Remarks

The financial crisis of the late 2000's sparked a renewed interest on the close link between financial and sovereign risk, and more notably, underscored the significance of identifying systemically important institutions and developing mechanisms for the latter to internalize the externalities they create on the economy, given a default scenario.

Using monthly market data for the period comprised between September:2001 - March:2011, we calculated bank-specific probabilities of default and expected losses given default. Consequently, we estimated the joint distribution of such expected losses and found the system-wide expected cost for the government given the implicit bailout option. Our results suggest that even though systemic risk is currently not a major concern in the Colombian banking system, it is necessary to enhance the supervisory and regulatory framework to include quantitative measures of this risk. As Colombia's financial sector continues deepening, interactions between institutions and across different markets will undoubtedly become more complex, potentially increasing the likelihood of a systemic event materializing. Moreover, given the seemingly good-health exhibited by Colombian banks in the last years and the fact that we are currently on the upside of the business cycle, the timing is optimal to impose additional capital charges.

In practical applications, we suggest working with the theoretical probability of default and hence with the risk measures that arise from using the puts portfolio derived from it. From a macroprudential and regulatory view, conservatism is desirable, and hence we consider the worst-case scenario (i.e. the methodology that yields the highest expected losses). Moreover, the possibility of visualizing the contribution of each bank on system-wide losses is also fundamental from a regulatory perspective. The final report of the *Financial Regulatory Reform* presented by the Department of the Treasury in the U.S., proposed that capital requirements for large, interconnected firms should reflect the large negative externalities associated with the financial distress or disorderly failure of each firm and should, therefore, be above prudential minimum capital requirements during stressed economic and financial times (Department of The Treasury. Financial Regulatory Reform. (2009)). Indeed, the Dodd-Frank Act, in its original draft, suggested that too-big-to-fail institutions should create a US$50 billion fund, whose purpose would be to provide for the orderly unraveling of systemically important institutions without forcing taxpayers to cover the losses (Senate Committee on Banking, Housing, and Urban Affairs (2010)). In this sense, our risk measure allows for an initial estimate of the size of the fund, as well as to the individual contributions of each bank.

Additionally, we recommend the use of bank-specific probabilities of default, coupled with the mean and median expected loss given default that arises from the simulated one-month-ahead joint distribution of put prices. These variables are ideal candidates as forward-looking measures of risk accumulation, and can therefore provide early-warning signals to policymakers with respect to the possible build-up of financial imbalances and the specific institutions behind them. This information could accelerate preemptive supervisory action, reducing the likelihood of risk effectively materializing and thus, meliorating costs for the government and the system as a whole.

Nonetheless, our understanding of systemic-risk is still quite limited. Future research should ideally integrate the non-banking system into the quantification of this risk, as we believe that future vulnerabilities could very well arise from this sector of the financial system. However, even with the self-proclaimed constraints that our analysis has, we believe that continually monitoring probabilities of default and the joint expected loss given default, should shed light in anticipating future stress scenarios. As such, our approach offers an useful instrument both from a macroprudential as well as a financial stability perspective.

References

Acharya, V., Pedersen, L., Philippon, T. & Richardson, M. (2010), 'Measuring systemic risk.', *FRB of Cleveland Working Paper* (No. 10-02).

Adrian, T. & Brunnermeier, M. (2010), 'Covar', *Federal Reserve Bank of New York* (no. 348).

Antunes, A. & Silva, N. (2010), 'An application of contingent claim analysis to the portuguese banking system. financial stability report', *Banco de Portugal* .

Becerra, O. & Melo, L. (2008), 'Medidas de riesgo financiero usando cópulas: Teoría y aplicaciones', *Borradores de Economía.Banco de la República de Colombia* (No 489).

Black, F. & Cox, J. (1976), 'Valuing corporate securities: Some effects of bond indenture provisions', *The Journal of Finance* (31), pp. 351–367.

Black, F. & Scholes, M. (1973), 'The pricing of options and corporate liabilities', *Journal of Political Economy* (81 (3)), pp. 637–654.

Blavy, R. & Souto, M. (2009), 'Estimating default frecuencies and macrofinancial linkages in the mexican banking system', *IMF Country Report* (No. 09/54).

Carmona, R. (2004), *Statistical Analysis of Financial Data in S-Plus*, Springer-Verlag.

Crosbie, P. & Bohn, J. (2001), 'Modeling default risk', *Moody's KMV* .

Department of The Treasury. Financial Regulatory Reform. (2009), 'A new foundation: Rebuilding financial supervision and regulation'.

Financial Stability Report (2002), *Banco de la República* .

Financial Stability Report (2006), *Banco de la República* .

Gray, D. & Jobst, A. (2010*a*), 'New directions in financial sector and sovereign risk management', *Journal of Investment Management* **Vol 8**(1), pp. 23–38.

Gray, D. & Jobst, A. (2010*b*), 'Systemic cca - a model approach to systemic risk. presented in the conference', *Beyond the Financial Crisis: Systemic Risk, Spillovers and Regulation* pp. 28–29.

Gray, D., Jobst, A., Roger, S. & Doyle, P. (2011), 'Financial stability in israel -a systemic contingent claims perspective', *IMF Country Report* (No. 11/23 Chapter V).

Gray, D. & Jones, M. (2006), 'Measuring sovereign and banking risk in indonesia: An application of the contingent claims approach', *IMF Country Report* (No. 06/318 Chapter III).

Gray, D., Merton, R. & Bodie, Z. (2008), 'A contingent claims analysis of the subprime credit crisis of 2007-2008', *Credit 2008 Conference on Liquidity and Credit Risk* .

Gray, D. & Walsh, J. (2008), 'Factor model for stress-testing with a contingent claims model of the chilean banking system', *IMF Working Paper* **WP/08/89**.

Grégoire, V., Genest, C. & Gendron, M. (2008), 'Using copulas to model price dependence in energy markets', *Energy Risk* (5(5)), 58–64.

Huang, X., Zhou, H. & Zhu, H. (2009), 'Assessing the systemic risk of a heterogeneous portfolio of banks during the recent financial crisis. finlawmetrics 2010 conference paper', *Revised February 2010* .

International Monetary Fund (2010), 'World economic outlook', *Rebalancing Growth* .

Keller, C., P, K. & Souto, M. (2007), 'Measuring sovereing risk in turkey: An aplication of the contingent claims approach', *IMF Working Paper* **WP/07/233**.

Merton, R. (1973), 'Theory of rational option pricing', *Bell Journal of Economics and Management Science* **Vol. 4**(1), 141 – 183.

Merton, R. (1974), 'On the pricing of corporate debt: The risk structure of interest rates', *Journal of Finance* **29**, pp. 449–70.

Merton, R. (1977), 'An analytic derivation of the cost of loan guarantees and deposit insurance: An application of modern option pricing theory', *Journal of Banking and Finance* (1), pp. 3–11.

Saldías, M. (2010), 'Systemic risk analysis using forward-looking distance-to-default series', *Federal Reserve Bank of Cleveland* (Working Paper No. 2010-05).

Senate Committee on Banking, Housing, and Urban Affairs (2010), 'Summary: Restoring american financial stability'.

Sklar, A. (1959), 'Fonctions de répartition à n dimensions et leurs marges', *Publ. Inst. Statist. Univ. Paris* **8:**, 229 – 231.

Souto, M. (2008), 'Has the uruguayan financial system become more resilient to shocks? an analysis adapting the merton framework to a country without equity market data', *IMF Country Report* (No. 08/46 Chapter VI).

Souto, M. & Abrego, L. (2008), 'The colombian banking sector- a contingent claims analysis', *IMF Country Report* (No. 09/24 Chapter I).

Souto, M., Bejamin, M. & Vasquez, F. (2009), 'Linking financial and macroeconomics factors to credit risk indicators of brazilian banks', *Working Paper Series* **No. 189**.

Villar, L., Salamanca, D. & Murcia, A. (2005), 'Crédito, represión financiera y flujos de capitales en colombia: 1974-2003', *Revista Desarrollo y Sociedad* .

Wolff, C. & Papanikolaou, N. (2010), 'Leverage and risk in us commercial banking in the light of the current financial crisis'.

Zivot, E. & Wang, J. (2006), *Modeling Financial Time Series with S-Plus*, Springer.

Appendix A

The recent evolution of the variables in our model which can be directly quoted in the market can be viewed in Figure 9 below. From here it is worth noting that all 4 banks have shown a significant increase in their liabilities, thus augmenting the size of their balance sheets. Moreover, this increase in funding has been coupled with three important factors. Firstly, an increment in the value of equity, especially for Banks A and B, which also happen to be the banks experiencing the largest growth in liabilities. The importance of this parallel growth is that it implies that there should be no capital restrictions on increasing the asset-side of the balance sheet as well. Secondly, equity has grown in an atmosphere of relatively low levels of volatility; the lowest of the last 10 years for all 4 institutions. The latter should imply, *ceteris paribus*, lower asset volatility. Finally, other than the hike observed in interest rates between 2006-2008, the last few years have seen a pronounced decrease in the cost of money, effectively placing interest rates in the lowest levels of the past decade.

FIGURE 9: Evolution of Observable Variables in the Model

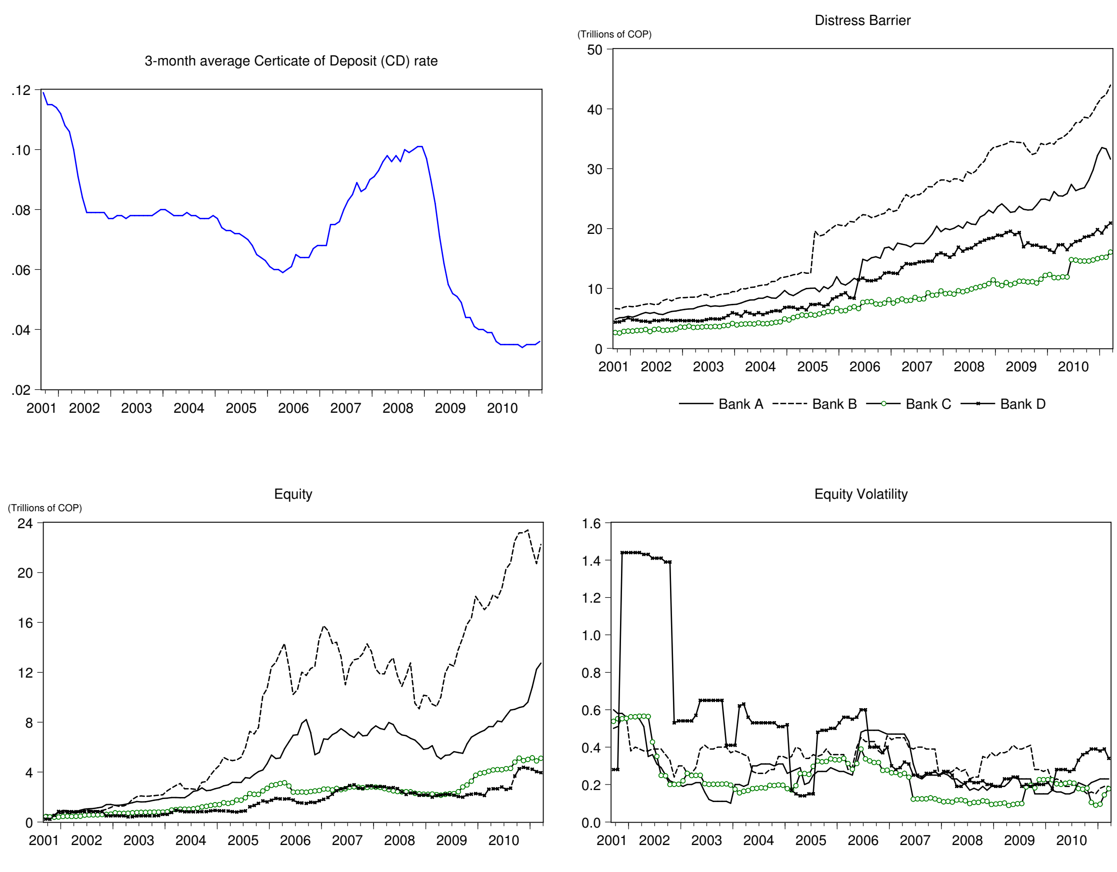
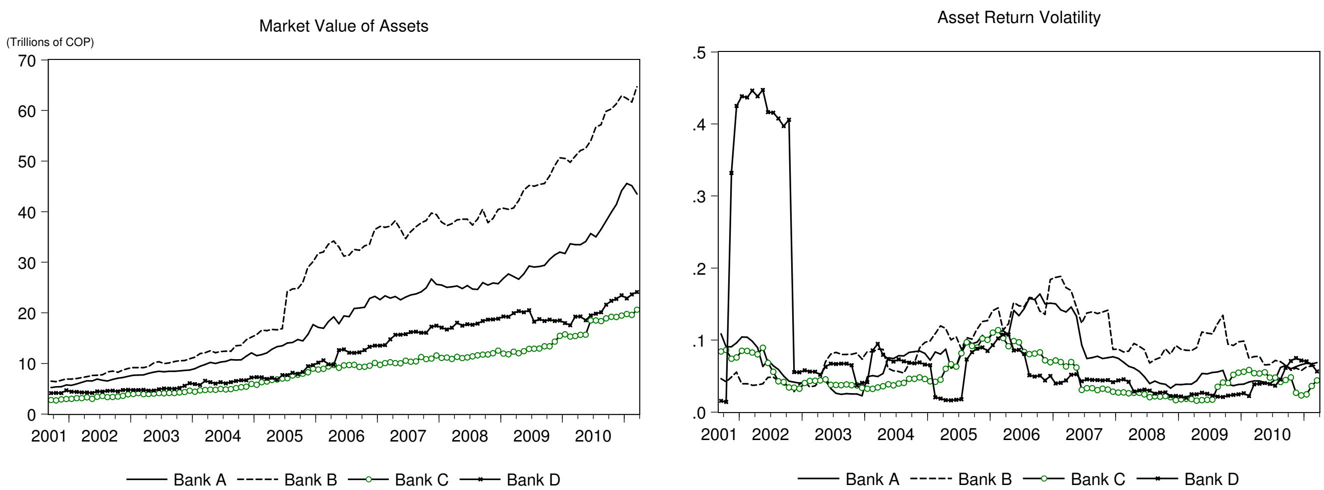


Figure 10 confirms this initial intuition. The market value of assets implied by the model has effectively presented a steady growth path, not unlike that of the liabilities. Additionally, its volatility resembles the trend observed in that of equity, also constituting the lowest levels of the past decade.

FIGURE 10: Evolution of Non-Observable Variables in the Model



Appendix B

TABLE 3: Residual Diagnostics on the VAR(4)-GARCH(1,1) Model Residuals

Univariate Tests

Test	**Null Hypothesis**	**Lags**	**Test Statistic**	**P-Value**
Residual Diagnostics on the residuals of the model for Bank A				
Ljung-Box	No serial correlation	24	15.023	0.920
Box-Pierce	No serial correlation	24	13.150	0.964
Engle	No ARCH effect	6	2.734	0.841
Residual Diagnostics on the residuals of the model for Bank B				
Ljung-Box	No serial correlation	24	24.224	0.449
Box-Pierce	No serial correlation	24	21.044	0.636
Engle	No ARCH effect	6	1.962	0.923
Residual Diagnostics on the residuals of the model for Bank C				
Ljung-Box	No serial correlation	24	9.584	0.996
Box-Pierce	No serial correlation	24	8.110	0.999
Engle	No ARCH effect	6	3.963	0.682
Residual Diagnostics on the residuals of the model for Bank D				
Ljung-Box	No serial correlation	24	25.865	0.360
Box-Pierce	No serial correlation	24	21.986	0.580
Engle	No ARCH effect	6	0.708	0.994

Multivariate Tests

Test	**Null Hypothesis**	**Lags**	**Test Statistic**	**P-Value**
Breusch-Godfrey	No serial correlation	6	19.059	0.266
Portmanteau	No serial correlation	12	125.421	0.548
Portmanteau (Adjusted)	No serial correlation	12	135.225	0.314

Appendix C

Figure 11: Histogram of Put Option Prices, by Bank

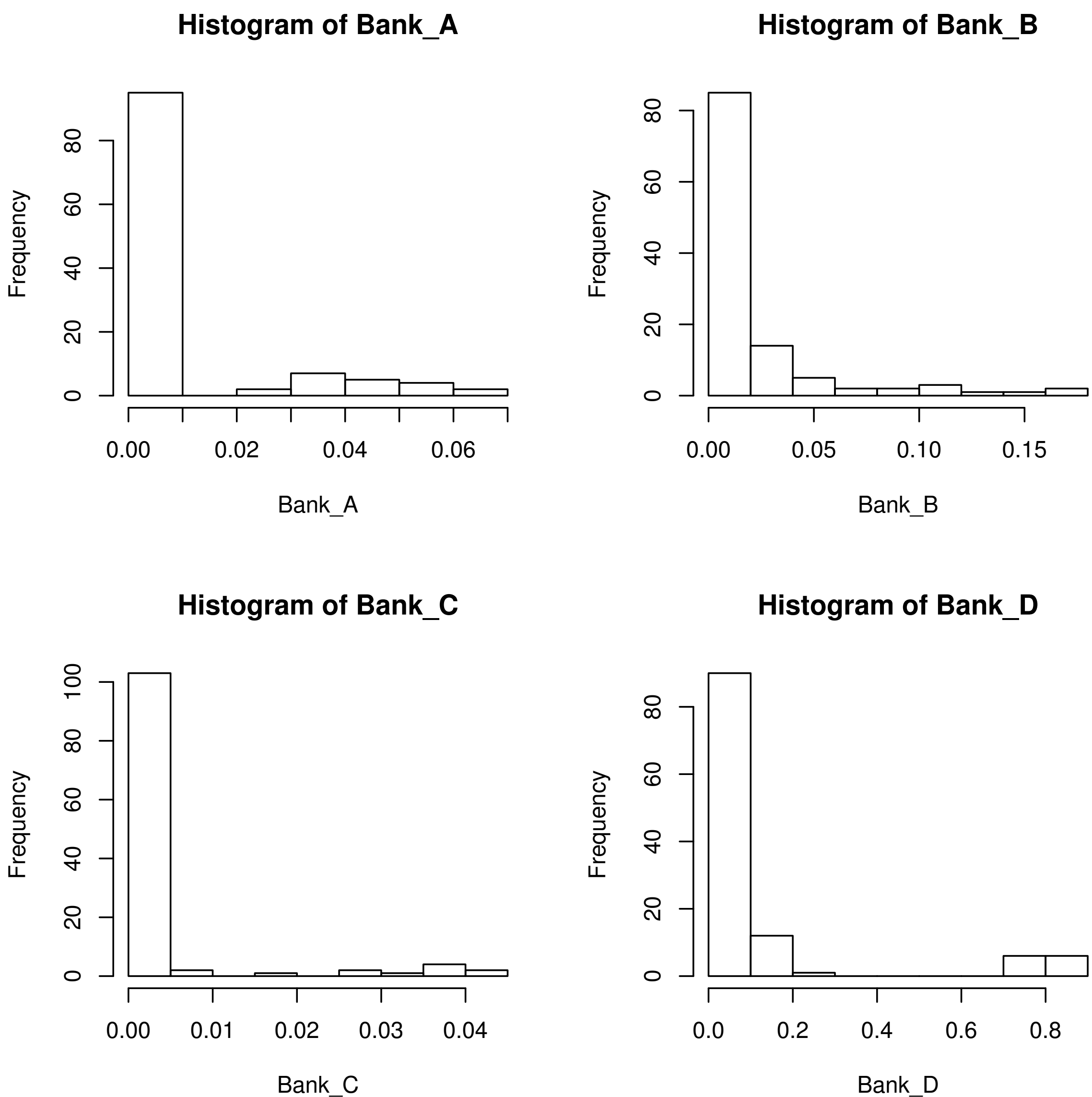


Table 4: Two-sample Kolmogorov-Smirnov Test

Null Hypothesis	**Test Statistic**	**P-Value**
Both series come	0.0375	0.9996
from the same	0.0379	0.9995
continuous distribution	0.0375	0.9996
(in this case a uniform)	0.0375	0.9996

FIGURE 12: Maximum Likelihood Estimates of the Shape Parameters - Bank A's Equity Returns

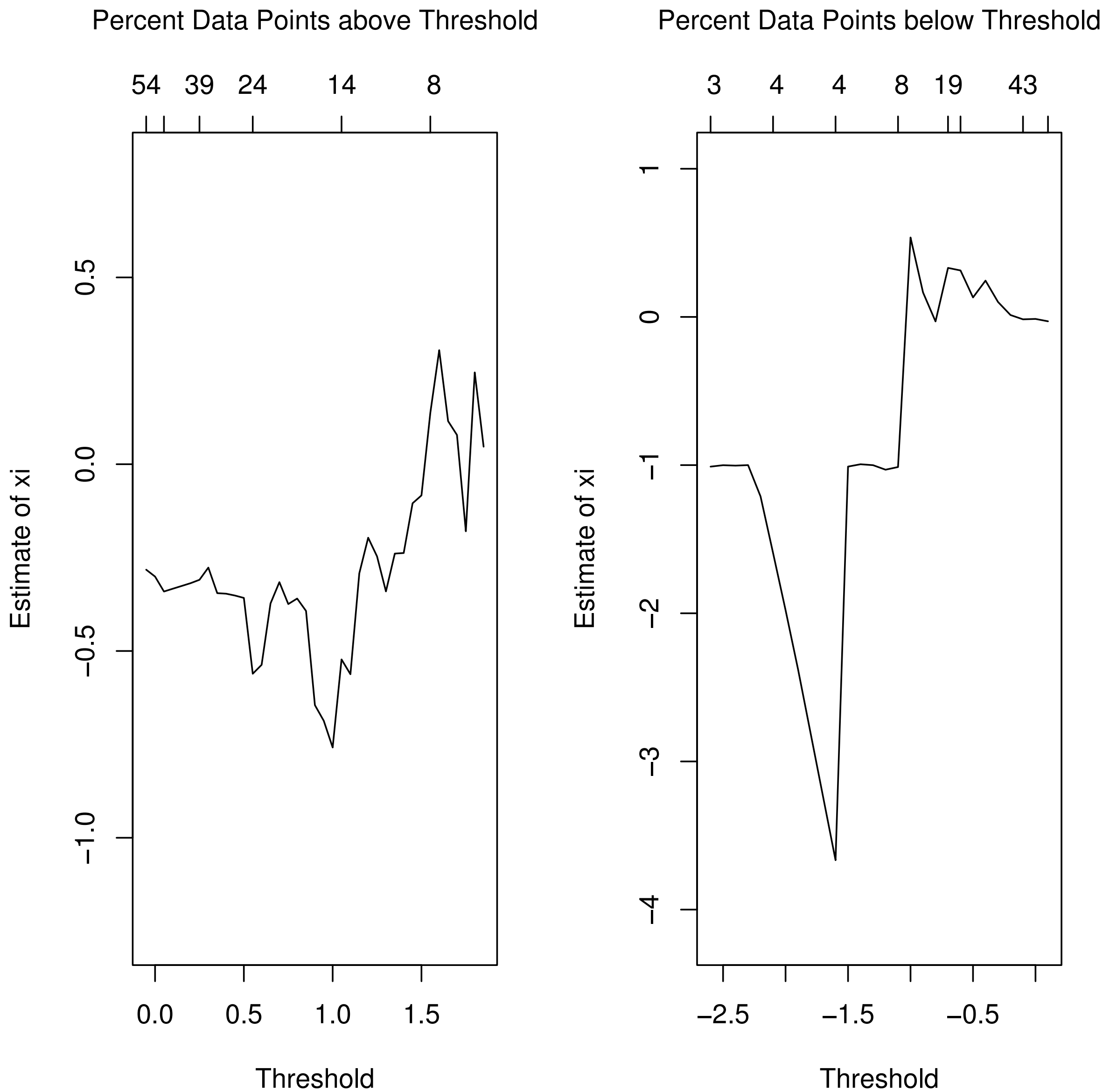


FIGURE 13: Maximum Likelihood Estimates of the Shape Parameters - Bank B's Equity Returns

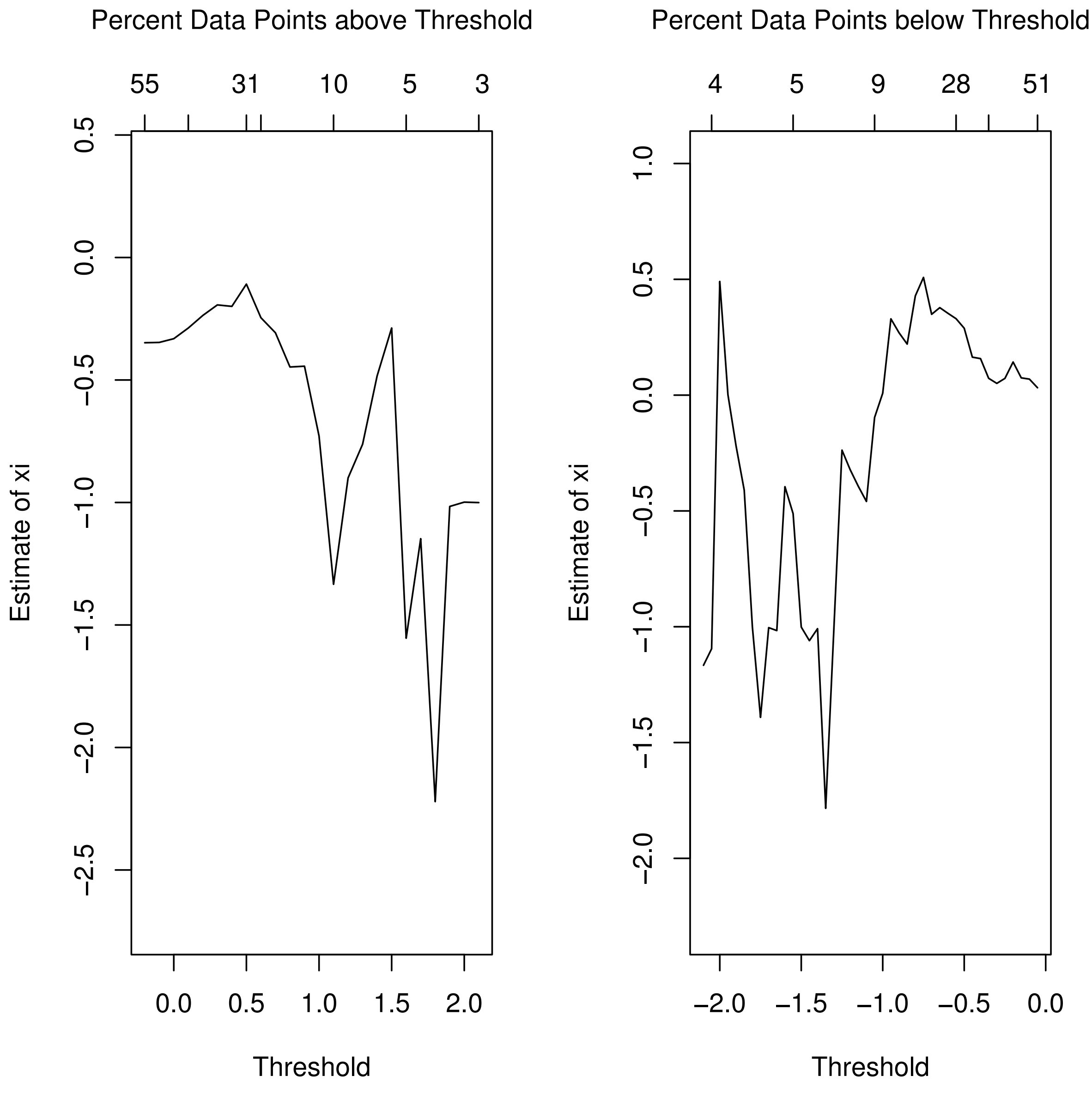


Figure 14: Maximum Likelihood Estimates of the Shape Parameters - Bank C's Estimate Returns

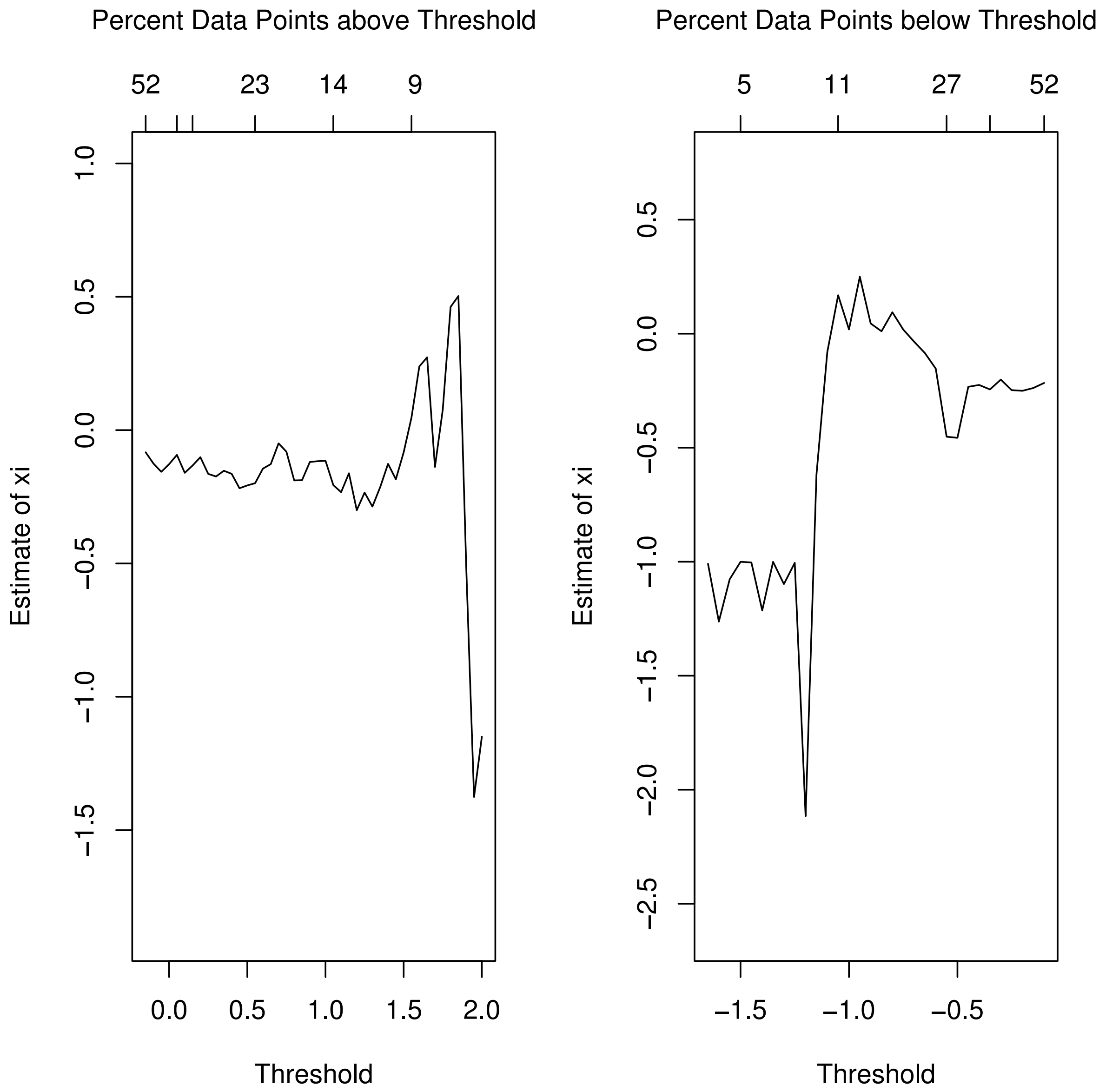


FIGURE 15: Maximum Likelihood Estimates of the Shape Parameters - Bank D's Equity Returns

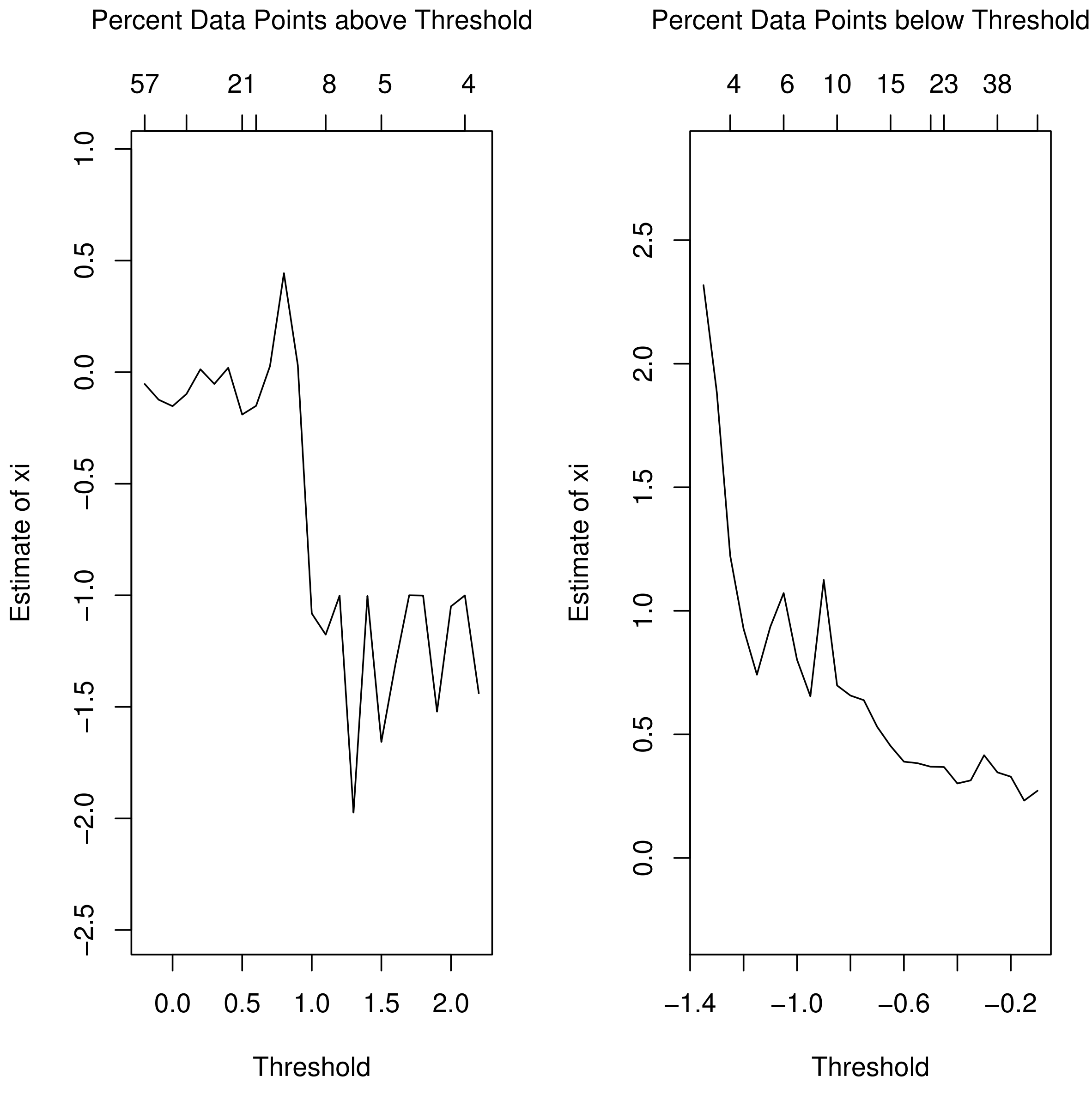


Figure 16: Q-Q Plots of Uniform Distribution Against Sample Quantiles, by Bank