Sovereign Risk and Macroeconomic Fluctuations

Franz Hamann*
North Carolina State University

12th November 2002

Abstract

This paper investigates the impact of sovereign risk on the stochastic rational expectations equilibrium of a pure exchange small open economy. International borrowing and lending arise from the interaction between a risk averse sovereign representative agent in a small open economy trying to self insure against idiosyncratic shocks and risk neutral international lenders. The credit market is imperfect because the sovereign cannot commit to repay its outstanding debt and chooses to default when it is optimal to do so. The possibility of default induces an endogenous sovereign risk premium on foreign debt and endogenous rationing by foreign creditors. The model is parameterized and solved numerically. The experiments conducted here generalize the results of Eaton and Gersovitz (1981) into environments with varying degrees of persistence and volatility in the underlying stochastic income process.

1 Introduction

The financial crises in emerging markets during the 90's has revealed some empirical regularities in their business cycles. Periods of financial distress in emerging economies are characterized by large current account reversals and

*I would like to thank Enrique Mendoza, Paul Fackler and John Seater for their orientation and helpful comments. This paper has benefited from the comments of the participants at the Macroeconomics Seminar at NCSU and the International Economics Seminar at Duke University. The usual disclaimer applies.

sudden stops in capital inflows (Calvo, 1998 [4]), soaring sovereign country risk reflected in hikes in the international interest rates faced by the economy and large output contractions (Oviedo, 2001 [18] and Neumeyer and Perri, 2001 [17]), accompanied by collapses in equity prices (Mendoza and Smith 2001, [15]) and prices of nontradable goods relative to tradable goods (Mendoza, 2000 [14]). In several cases, the magnitude of the crises led countries to default on their outstanding debt (Argentina 2001, Russia 1998, Ecuador 1999 and Indonesia 1998).

In a recent study, Reinhart (2002, [19]) has highlighted the statistical significance of the interaction between default and emerging markets crises. The study finds that 84% of the defaults occurred in emerging markets are associated with currency crises and about 50% of the currency crises are linked to defaults. This result seems to be a particular characteristic of emerging markets.[1]

Despite the renewed interest on sovereign defaults, its occurrence is a characteristic of international lending. During the nineteenth century many nations have at some point defaulted on their foreign debt and have lost access to international credit markets (Cole and English, 1995 [6]). According to Standard & Poor's figures, in the 1830's 31% of the total number of governments that issued sovereign debt had defaulted on their obligations (Beers, 2000 [3]).[2] At the beginning of the twentieth century (1920) a boom in sovereign lending ended with a large number of defaults (21%) during the Great Depression (Cantor and Packer, 1995 [5]).[3] During the postwar period until the 1970's the trend was a decreasing one. In 1975 less than 5% of the governments were in default. In that year the rate started to climb dramatically, in 1982 it reached the 25% to reach another historical peak in 1990 at 31%. Since then the rate of defaults has declined to 13% in 2000.

[1] The study finds that there is no statistical significance of the relationship between default and currency crises for developed economies.

[2] Standard & Poor's survey tracks the number of sovereign debt issuers year by year since 1824, as well aas the number of defaulting countries in foreign currency debt in loans and bonds. Nowadays there are 201 countries in the survey.

[3] These authors have documented that 21 out of 58 nations defaulted on international bonds between 1930 and 1935.

Table 1 presents a summary of some of the sovereign debt defaults on foreign currency (bonds and bank loans) between 1975 and 2001.

These figures as well as the recent episodes of the Argentinean default default (November 2001) and the defaults of Russia (1998), Ecuador (1999) and Indonesia (1998), provide evidence that default is relatively a frequent event in international lending.

In addition to the occurrence of default, the behavior of the sovereign country risk reflected on the interest rate that the economy faces in the international credit markets appears to be closely related to the sharp movements in the current account, the collapse in private consumption and the currency crises. Another interesting finding has been that sovereign credit ratings (used as an indicator of the likelihood of default) not only have significant impact on sovereign bond yield spreads, but also serve as good predictors of the occurrence of defaults (Larraín et al., 1997 [12]). In consequence, it is not surprising that during periods of financial distress lower ratings are observed and countries face more difficulties borrowing from international credit markets as they have to pay higher interest rates for limited amount of funds.

The nature of the dynamics of the crises in emerging markets and the empirical findings described above challenges the study of the business cycles in small open economies. The smooth movements in the current account and the level of foreign debt, as well as the neutrality of the business cycle to the external interest rates shocks predicted by conventional models of business cycles in a small open economy[4] are inconsistent with the dynamics of the emerging markets crises and the sudden stops of capital inflows. One important reason of this inconsistency with the facts is the role assigned to international creditors. Since international credit markets are assumed to be perfect, a small open economy can borrow funds at a fixed risk-free rate up to a point limited only by the extent of their wealth.

In a recent survey, Arellano and Mendoza (2002, [2]) point out that the common starting point of much of the literature on emerging markets crises has been to introduce some type of financial-market imperfection that

[4]See Mendoza (1991, [16]) and Correia et al. (1995, [7]).

Table 1: Year of Sovereign Debt Defaults Episodes 1975-2001

Defaulting Country	Bonds	Loans
Argentina	1989,2001	1982
Bolivia	1989	1980,86
Brazil		1983
Chile		1983
Cook Islands		1995
Costa Rica	1984	1981
Croatia		1992
Dominican Rep.		1982
Ecuador	1999	1982
Egypt		1984
Indonesia		1988
Jamaica		1978,81,87
Jordan		1989
Mexico		1982
Morocco		1983,86
Pakistan	1999	1998
Panama	1987	
Paraguay		1986
Peru		1976,78,80,83
Phillipines		1983
Poland		1981
Romania		1981,86
Russia	1989	1991,98
Slovenia		1992
South Africa		1985,89,93
Trinidad &Tobago		1988
Turkey		1978,82
Uruguay		1983,88,90
Venezuela	1995	1983,90

Source: Standard & Poor's.

distinguishes emerging economies from industrial countries. However, the majority of studies focuses on partial equilibrium models that qualitatively predict results consistent with the dynamics of the crises. Little is known about the quantitative predictions, not only of these type of models but also of the equilibrium models of business cycles of small open economies.

In this paper, the imperfection arises from the inability of sovereign debtors to commit to repay their their outstanding debt. The small open economy sovereign representative agent can borrow funds at an increasing interest rate up to a point limited by the perception of lenders of the sovereign's willingness to repay the debt. The interest rate that the small open economy faces and the availability of funds are the result of its interaction with foreign lenders. The aim is to explore quantitatively the determinants of sovereign country risk and its role on the dynamics of emerging markets crises in the context of an equilibrium model of business cycles of a small open economy. As the results in Reinhart (2002, [19]) suggest sovereign risk may play an important role on the occurrence of emerging markets crises.

This paper borrows the main elements from the earlier work of Eaton and Gersovitz (1981, [8]) in the context of an equilibrium model of a small open economy to explore two aspects: first, to quantify how different properties of the economy (degree of risk aversion of the agents, the income volatility, the persistence of the shocks) affect the sovereign country risk and the access to international credit. Second, the consistency of the model with the business cycle dynamics of a financial crisis. The model is extended to allow for a more general class of stochastic processes. The rest of the paper proceeds as follows. Section 2 presents the structure of the model and characterizes the equilibrium. Section 3 describes briefly the algorithm for the solution of the model. Section 4 reports the results of different experiments designed to study the determinants of the equilibrium probability of default and the endogenous ratiuoning limit. Section 5 concludes and outlines the direction for future research.

2 The Model

There are two types of agents in the model. A small open economy representative agent (country) borrows funds from foreign risk-neutral competitive lenders. The country can default on its debt but at the cost of permanent exclusion from future borrowing. Therefore the borrower's default decision is the result of optimally balancing the cost of exclusion, given by the forgone benefits of consumption smoothing, against the direct costs of repayment, given by the short run disutility of repaying the loan. Risk-neutral lenders are able to assess the probability of default of the country and will restrict the amount of funds available for lending to limit optimally their degree of exposure to default risk. In this fashion, an endogenous borrowing constraint arises.

2.1 The Sovereign

Time is discrete, and each period the sovereign receives a stochastic endowment of a perishable consumption good that asumes discrete values derfined over the set $Y = \{y_1, ..., y_{n_1}\}$. The country's endowment follows a Markov process with stationary transition probability $\pi(y'|y) = \Pr(y_{t+1} = y'|y_t = y) > 0$ for y, $y' \in Y$. The preferences defined over consumption are given by:

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]$$

where $\beta \in (0,1)$ and $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. In this setting, the sovereign can smooth consumption by accessing the international credit market. The country can borrow funds in the form of one-period consumption loans available in the form of a set of discrete values, $A = a_1, ..., a_{n_2}$. This set is determined by international lenders. The loan level must remain above an endogenous borrowing limit $\underline{a} \in A$. If $a > 0$ the country is a lending (at the risk free rate), but if $a < 0$ the country is borrowing. The country can choose to repay or default on its outstanding debt. If the country decides to repay,

then the resources constraint is given by:

$$c + a' \leq y + (1 + r)a \tag{1}$$

where $c \geq 0$ and $a \geq \underline{a}$, and r is the interest rate *function* that the country faces in the international credit market. This function depends on the endowment level and the level of debt, (y, a), and the probability of default perceived by lenders. The interest rate is determined endogenously by the interaction of the sovereign and the international lenders. If the country decides to default, it is not allowed to borrow new loans and has to consume the stochastic endowment thereafter. So, the constraint in the case of default is $c \leq y$. Notice that default is an absorbent state.

The country's state position at a point in time is described by a state vector $s \in S$. The state $s = (y, a, d)$ indicates a country's net foreign assets a, the level of output y and the default state d. The state space is $S = Y \times A \times D$, where $Y = \{y_1, ..., y_{n_1}\}$, $A = a_1, ..., a_{n_2}$ and $D = \{1, 0\}$. If the country decides to repay $d = 1$, otherwise $d = 0$.

The problem of the sovereign at time t can be described as exercising an option to default. At any point in time, the expected discounted value of the utility of the sovereign country must satisfy the following functional equation:

$$v(s; r) = \max \left\{ \max_{(c,a') \in \Gamma(s)} \left\{ u(c) + \beta \sum_{y'} \pi(y'|y) v(s') \right\}, w(y) \right\} \tag{2}$$

where

$$\Gamma(s) = \{(c, a') : c \leq y + (1 + r(s))a - a'; c \geq 0; a' \geq \underline{a}; \}$$

and

$$w(y) = E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau - t} u(y_\tau) \right] \tag{3}$$

d' indicates the decision to repay or default in the next period:

$$d' = \begin{cases} 1 \text{ if } \max_{(c,a')\in\Gamma(s;r)} \left\{u(c) + \beta \sum_{y'} \pi(y'|y) v(s';r)\right\} \geq w(y) \\ 0 \text{ otherwise} \end{cases} \tag{4}$$

so if the value of the program of repaying the loans exceeds the value of the program to return to autarky, then the country will repay the loans. Otherwise, it will default. To obtain a solution the sovereign's problem, it is assumed that a bounded measurable solution v to functional equation (2) exists and that it is optimal. If v is the optimal value function, then the policy functions $c : S \to R_+$, $a : S \to A$ and $d : S \to D$ are the optimal decision rules provided $c(s)$, $a(s)$ and $d(s)$ are measurable and satisfy (2). These sovereign's policy rules map the current state of the economy, represented in the level of loans and output, into the consumption decision the next period's loans level and the next period's default decision. In the next subsection the international lenders behavior is described.

2.2 International Lenders

The international lenders are assumed risk neutral. They borrow funds from an independent market at the risk free rate, $\widetilde{r}$. They know that the sovereign defaults at t if and only if $w(y) > v(s)$ therefore the probability λ of default at t anticipated at $t-1$ is given by $\lambda(s) = \Pr[w(y) > v(s)]$. The higher the level of debt the higher the probability of default.Since banks are risk neutral they will lend as long as they are paid a return that is at least as high as the risk free rate:

$$[1 - \lambda(s)](1 + r(s)) \geq (1 + \widetilde{r}) \tag{5}$$

To determine the borrowing limit that lenders impose on the sovereign, it is assumed that they will allow the sovereign to borrow up to the annualized value of the lowest possible realization of the stochastic endowment (in every future period) discounted at the interest rate associated with that income stream. Aiyagari (1994,[1]) has shown that as consumption goes to zero its marginal utility goes to infinity, then an agent will never hold an amount of

assets that may induce non-positive consumption. Thus a borrowing limit is implied by this fact. In his model the interest rate is a constant risk free rate, so the value of the borrowing constraint is the annuity of the worst possible income realization discounted at the risk free rate.

Here the interest rate is stochastic and depends on the probability of default, which in turn depends on the preferences and the stochastic properties of the small open economy. As Zhang (1994,[21]) has pointed out, a natural generalization of Aiyagari's setting is to allow the agent to borrow up to the present value of the worst possible income stream, discounted at the interest rate associated with this income stream, conditional on the constraint binding. More formally, a given probability of default, λ, implies an interest rate r, so the maximum amount of funds available for borrowing is the value of $\underline{a}$ such that

$$\underline{a} = \frac{\underline{y}}{\max(r(\underline{y}, \underline{a}))} \tag{6}$$

where $\underline{y}$ is the lowest possible realization of the endowment and the maximum is taken conditioned on the borrowing constraint binding.

Intuitively, the lenders think about the worst case scenario for the repayment of their loans: every period the agent borrow up to the limit and receives the lowest possible endowment. If lenders allow the agent to borrow more than this amount, there will be a positive probability that the present value of the worst income stream is less than the amount of debt, violating the non-negativity of consumption.

The computation of this borrowing limit is as follows. Start with an arbitrary borrowing limit and find the optimal interest rate function, r. The new borrowing limit is the right hand side of (6). Update the set A and find a new optimal interest rate function, r. The computation of the optimal interest rate function is described later. In the next section the equilibrium of the model is characterized.

2.3 Equilibrium

The equilibrium concept used is that one of a stochastic, stationary rational expectations equilibrium represented by a time invariant probability of de-

fault function, λ, a time invariant interest rate function r, a time invariant optimal set of loans and a time invariant optimal probability distribution over the state space of the small open economy, ψ. The stationary equilibrium is defined recursively. Given the transition rule $a(s)$, and the transition probabilities $\pi(y'|y)$, the optimal transition probability $P(s, B)$ can be computed. $P(s, B)$ is the probability that a country in state s will reach a state vector that lies in B.[5] More formally, the stationary stochastic rational expectations equilibrium of the model is characterized by:

1. an optimal interest rate function r and optimal default function λ
2. an optimal set of loans A
3. an optimal borrowing policy $a'(s)$
4. a stationary probability measure $\psi(B) = \int_S P(s, B) d\psi$

such that:

1. The sovereign solves (2) subject to (1).
2. Expected profits for international lenders are zero, i.e. (5) holds with equality.
3. The sovereign demand for loans is in the available set of loans, $a'(s) \in A$

The first condition states that the sovereign optimize to find a set of decision rules for consumption, debt allocation and default that depend on the current state of the economy. The second condition states that the profits for international lenders are zero, because the credit market is competitive. The third condition is a market clearing condition.

[5] The optimal transition probability $P(s, B)$ is very useful to compute the probability of default. Since there are transient and absorbent states, the probability of default is given by the probability of reaching the set of absorbent states.

3 Algorithm

The characterization of the equilibrium of the model presented above does not yield a closed form solution. However, the recursive nature of the model allows the computation of the equilibrium and the study of its long-run characteristics using numerical methods. The computational method used for the solution of the model is similar to that of Huggett (1993, [9]) and Aiyagari (1994, [1]). The main differences are that here there is an additional discrete state and and action (default-repay) and the market clearing condition. To simplify notation, let $x = g(s)$ denote the policy functions $(a', d') = (a(s), d(s))$. The method is based on the following steps:

1. Start with an arbitrary probability of default function $\lambda^{(i)}$ and borrowing limit $\underline{a}^{(i)}$

2. Given $\lambda^{(i)}$ compute $r^{(i)}$ using equation (5) and compute the optimal policy function $g(s)$ and the optimal transition probability matrix $P(s, B)$ using the contraction equation (2).

3. Update $\lambda^{(i)}$ to $\lambda^{(i+1)}$ using the optimal transition probability matrix $P(s, B)$ and update the borrowing limit to $\underline{a}^{(i+1)}$ using equation (6).

4. Check whether the zero profit condition is approximately satisfied. If not, use $\lambda^{(i+1)}$ and $\underline{a}^{(i+1)}$ and repeat steps 2 to 4.

5. After convergence of the algorithm, given the optimal policy $g(s)$ and the optimal transition probability matrix $P(s, B)$ iterate on $\psi^{(i+1)}(B) = \int_S P(s, B) d\psi^{(i)}$ from an arbitrary initial distribution $\psi^{(0)}$ to obtain the stationary probability distribution.

The first step of the algorithm is solved by policy function iteration on equation (2), for a given $r^{(i)}$ function.

The second step requires to obtain $P(s, B)$. Notice that the optimal policy $g(s)$ and the Markov chain π on y induce a Markov chain on s, $P(s, B)$ via the formula:

$$\Pr[s_{t+1} = s' | s_t = s] = \Pr[d_{t+1} = d' | s_t = s] \Pr[a_{t+1} = a' | s_t = s] \Pr[y_{t+1} = y' | y_t = y]$$

or:

$$\Pr[s_{t+1} = s' | s_t = s] = \iota(d', a', s)\pi(y, y') \tag{7}$$

where $\iota(d', a', s) = 1$ if $a' = a(s)$ and $d' = d(s)$ and 0 otherwise. This indicator function identifies the time t states s that are sent into (a', d') at time $t+1$. Equation (7) defines an $n \times n$ matrix P where n is the number of total possible states. The matrix P is used to compute the ergodic distribution, ψ and the probability of default.

The computation of the probability of default in the third step deserves some comment. Typically, when there is no default option, all the states of the Markov Chain associated with P will be recurrent and its stationary distribution, ψ, can be interpreted as the fraction of time that the country spends in each state. When the possibility of default exists there are two types of states: repayment states, which are transient and defaulting states which are absorbent (once one of the states of this set of states has been reached, the state of the system moves only among them). The numerical results show that the matrix P has the following structure:

$$P = \begin{bmatrix} Q & R \\ 0 & U \end{bmatrix}$$

where Q is a matrix that yield the probability of moving within a set of transient states (repayment) in the next period, R is a matrix whose elements express the probability of moving from a repayment state to a defaulting state in the next period, and U is a matrix whose elements yield the probability of moving within the defaulting states. Note that the matrix 0 indicates that once the decision of default has been taken there is no possibility to reach a repayment state in the next period. The optimal probability of default in period $t+1$ at a given time t can be computed as[6]:

$$\lambda_{t+1} = \sum_{\tau=0}^{t} Q^{t-\tau} R U^{\tau} \tag{8}$$

[6] For a proof of this result, see Medhi (1994, [13]) pages 116-117.

where R is the probability of default at $t = 1$.

Step 4 updates r , using the zero profit condition. If $\left\| \lambda^{(i+1)} - \lambda^{(i)} \right\| < \sqrt{\varepsilon}$ then stop. Otherwise continue iterating on the previous steps. The last step is the standard computation of the ergodic probability distribution.

4 Results

This section presents the functional forms and the parameters used in the solution of the model. The robustness of the solution is tested by varying the values of the parameters. At the same time this exercise permits the exploration of the determinants of the probability of default and the borrowing limit. The functional form used to represent the preferences is a standard CES utility function, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$where γ is the coefficient of relative risk aversion, assumed to be between 1 and 2. The discount factor, β, is set between 0.94 and 0.96. These relatively large small discount factors reflect the high degree of impatience that characterizes emerging market economies. The output process is assumed to be a Gaussian AR(1) $y_t = \ln(Y_t)$ with law of motion:

$$y_t = (1 - \rho)\mu + \rho y_{t-1} + \epsilon_t$$

where ϵ_t is i.i.d. $N(0, \sigma_\epsilon^2)$. The log of output volatility, σ_y^2, is assumed to be between 0.27 and 0.4, and its mean, μ, is assumed to be 1. Different degrees of persistence include values between $\rho = 0$ and $\rho = 0.8$. These numbers illustrate the impact of the properties of the stochastic endowment (persistence and volatility) on the probability of default.

The solution of the model is performed by state-space discretization. The state space S is partitioned in a grid of 2400 points. Output is discretized following Tauchen and Hussey (1991, [20]) and the asset levels are discretized by an equally spaced grid between $[-\frac{\underline{y}}{r}, 10]$. The lower bound on A is the default risk free natural debt limit.

4.1 Properties of the Solution

Figure 1 shows the optimal value functions under autarky and the optimal value function of the sovereign representative agent of the small open economy when $\gamma = 1.5, \beta = 0.96, \tilde{r} = 1.5\%, \sigma_y = 0.3$ and $\rho = 0.5$. This is the benchmark parameterization. The value function is strictly concave in both directions, increasing in output and decreasing in the debt level (i.e. negative values of a). The value of continued access to the international credit market is lower when the economy exhibits higher levels of debt and lower levels of output. Default is more tempting during recessions and high levels of indebtedness.

The optimal policy $a(s)$ is shown in Figure 2. Optimal borrowing is decreasing in the available level of resources. When the difference between output and debt is low (low resources) the country wants to borrow to smooth consumption. Eventually the country will hit the borrowing limit at $\underline{a} = -5.4409$ and won't be able to accumulate a higher debt. When the level of income is low and the debt level is high enough the sovereign decides to default and the economy reverts to autarky, so $a(s) = 0$. In contrast, when the level of resources is high the country borrows less and may be eventually not borrow at all and may lend funds.[7] The properties of these functions are the consequence of the asset market incompleteness and precautionary savings (Aiyagari 1994, [1]) in conjunction with the existence of sovereign country risk.

The optimal probability of default function, λ, is shown in Figure 3. The sovereign is more likely to default when the debt level is high and income is low. The probability of default approaches 1 as income levels approach 0 and debt levels are considerably high. For higher levels of income, the probability of default approaches zero as the debt level goes to zero. However, there is a positive probability of default even for low debt levels since income can go to zero. The shape of the probability depends on the interactions of all the parameters of the model. In the next section, the sensitivity of the

[7] The apparent "ceiling" on the policy function for high positive values of a is induced by the size of the grid. Setting a higher value for $\overline{a}$ would imply a more separated grid and will reduce the accuracy of the algorithm.

probability of default to changes in these parameters is studied.

The ergodic probability distribution of income level and foreign asset holdings is shown in Figure 4. The distribution is well defined because the interest rate is endogenous. There is a cutoff at the level of the endogenous rationing limit. Funds above the limit will not be available for consumption smoothing. This is due to the presence of endogenous limits to borrowing additional funds and the existence of default. As the small open economy increases the level of debt it will approach the borrowing limit. While this happens the interest rate raises and reduces the incentives to increase the debt level. It is possible that at the limit, the sovereign is willing to pay a higher interest rate for additional funds. However, lenders may not find it profitable because the possibility of default is high. In consequence the small open economy may be rationed.

4.2 Determinants of the Probability of Default and the Borrowing Limit

In this section the response of the endogenous probability of default and borrowing limit to changes in the parameters of the model is evaluated. Several experiments are performed by changing the values of the degree of relative risk aversion, the discount factor, the risk free rate and the stochastic properties of the income process and evaluating the impact on the equilibrium probability of default function and the borrowing limit.

The first exercise evaluates the impact of a higher degree of relative risk aversion of the representative agent of the small open economy. Figure 5 shows the this impact in a two dimensional plot.[8] A lower degree of relative risk aversion increases the probability of default. A sovereign with a low degree of risk aversion doesn't care much about smoothing consumption in the future. Therefore not paying back the debt becomes relatively attractive and so the probability of default is higher, even if the sovereign holds a small amount of debt. On the contrary a highly risk averse sovereign values more

[8]The horizontal axis indicates the position of the state in the discrete grid. Values to the left indicate higher levels of debt. Next to each debt level, output is in decreasing order.

the possibility of consumption smoothing and will not default unless he holds a large amount of debt.

Before discussing the impact on the borrowing limit it is convenient to recognize the factors that determine it. Equation (6), shows that this limit has two basic components: the interest rate function and the lowest possible realization of income. The first one depends on the probability of default, while the second depends on the stochastic nature of the income process. For a given variance and persistence of the stochastic income process, the higher the maximum interest rate conditional on the constraint binding (i.e the maximum probability of default, conditional on the constraint binding) the lower the borrowing limit. So, parameters that impact the equilibrium probability of default but not the income properties (these are $\gamma, \beta, \widetilde{r}$ and ρ) will affect the borrowing limit directly: a higher probability of default will tighten the borrowing limit. In contrast, the effect of the income volatility will impact both the equilibrium probability of default and the lowest income value. Therefore the effect of changes in σ_y depends on whether the impact on $\underline{y}$ is larger or smaller than that one on the probability of default.

In the case of the first experiment, for a given income volatility and persistence, a higher degree of risk aversion increases the probability of default and so relaxes the borrowing limit from -5.44 to -4.15.

The second experiment examines the impact of a higher discount factor. Figure 6 illustrates the impact. A higher time preference parameter, β, reduces the probability of default. The intuition is similar as the previous case. As the value of the discount factor increases the sovereign places a higher value on future consumption smoothing and risk sharing. The value of having access to the international credit markets is increased and so the probability of default is lower. In consequence, for the given volatility and persistence of income shocks the borrowing limit relaxes from -5.44 to -4.15.[9]

The third and fourth exercises analyze the effects of increased volatility

[9]The value of the borrowing limit after the parameter change is similar to that one of the previous experiment because the grid is not fine enough to generate a numerical difference. To achieve a higher degree of accuracy the grid should be refined. Of course, this comes at a higher computational cost.

and persistence of the stochastic endowment. Figures 7 and 8 shows these two impacts respectively. Higher uncertainty on the income stream reduces the probability of default.[10] Increased uncertainty implies higher benefits from consumption smoothing, but also the possibility of lower output realizations. The disutility of making repayments during a recession is not enough to offset the long term benefits of consumption smoothing, and so the probability of default increases.[11] The reduction in the probability of default induces the foreign lenders to reduce the set of loans available to the country.

Contrary to the case of increased volatility, a higher degree of (positive) persistence of income shocks tends to increase the equilibrium probability of default. In the event of a negative shock, if the income process is positively correlated, the income level with tend to stay a higher period of time in lower level. A lower output not only increases the likelihood of a default but also induces the sovereign to increase its debt holdings, which rises further the probability of default.

Table 2 shows how the value of the borrowing limit changes as the degree of volatility and persistence changes.[12]

Table 2: Effect of Income Properties on the Borrowing Limit

$\sigma \vert \rho$	0.1	0.2	0.3	0.4	0.5
0.27	-6.9267	-6.9264	-6.9259	-6.9247	-6.9236
0.30	-5.4515	-5.4512	-5.4508	-5.4498	-5.4490
0.33	-4.2928	-4.2926	-4.2922	-4.2914	-4.2909
0.36	-3.3820	-3.3818	-3.3815	-3.3808	-3.3805
0.39	-3.1238	-3.1236	-3.1233	-3.1227	-3.1225

[10] This effect depends on the value of β. To generate a positive relationship between σ_y and the probability of default requires implausible values of β lower than 0.3.

[11] This result is also found in the literature of endogenous incomplete markets. An increase in the variance increases the availability of assets for insurance, so the agents can obtain more assets to insure against idiosyncratic risk. See Kehoe and Perri (1999, [10]) and Krueger and Perri (2000, [11]).

[12] As a percentage of income the values of the borrowing limits are between 91.4% and 19.4% of the maximum level of income.

As mentioned earlier, the impact of increased volatility on the borrowing limit depends on the balance between two effects: the reduction in the lowest possible realization of income and the reduction of the equilibrium probability of default. A higher variance implies that $\underline{y}$ can be lower. This effect tends to tighten the borrowing limit. In contrast, the reduction in the probability of default tends to relax the borrowing limit. The results show that as the volatility of the income shocks increase, the first effect dominates the second, and the borrowing limit tightens. Conditional on the borrowing limit binding, the effect of a lower probability of default is not enough to outweigh the negative impact of the possibility of occurrence of the lowest income realization.

Now, for a given variance a higher persistence of income does not affect the lowest value that income can take. Only the effect of the increased probability of default operates and so the borrowing limit is tighter.

Finally, the last experiment evaluates the impact of changes in the default risk-free international interest rate. Figure 9 shows that increasing the default risk free rate from 1.5% to 2% raises the equilibrium probability of default and the borrowing limit is tightened from -5.44 to -4.09. A higher interest rate affects both, international lenders and the sovereign. Foreign lenders will lend funds as long as expected rate of return on those loans exceed the default risk-free international interest rate. So, a higher default risk free international interest rate means that the opportunity cost of lending to sovereign countries increases. Less funds will be available for borrowing. On the other side, the sovereign will face a higher cost of borrowing, for a given level of debt and output. Therefore the probability of default will be higher.

5 Conclusions

This paper analyzes the determinants of sovereign risk on the stochastic rational expectations equilibrium of a pure exchange small open economy. International borrowing and lending arise from the interaction between a risk averse sovereign representative agent in a small open economy trying to self insure against idiosyncratic shocks and risk neutral international lenders.

The sovereign representative agent cannot commit to repay its outstanding debt and chooses to default when it is optimal to do so. The possibility of default induces an endogenous sovereign risk premium on foreign debt and an endogenous rationing limit that depend on the preferences of the sovereign, the default risk free interest rate and the stochastic properties of the income process.

The numerical analysis undertaken here generalize the results of Eaton and Gersovitz (1981, [8]). The results show that, in general, factors that increase the value of having access to the international credit markets reduce the likelihood of default and relax the access to international credit. For a given stochastic income process of a small open economy, a higher degree of risk aversion and a higher discount factor of the sovereign representative agent reduce the probability of default and ease the conditions for access to the international credit market. The same effect has a lower default risk free interest rate at which lenders obtain resources in the international credit market. The nature of the income process plays an important role in determining not only the shape of the equilibrium probability of default but also the borrowing limit. If income shocks tend to persist over time, the likelihood of a default increases and the borrowing limit is tighter.

The exception to this symmetry of effects on the probability of default and the borrowing limit occurs in the case of increased volatility. A higher uncertainty of income shocks increases the value of consumption smoothing through the access to the international credit market and reducing the probability of default, but also increase the possibility of having stronger recessions. This latter effect dominates the former and in response international lenders tighten the borrowing limit.

The model has several drawbacks. The main limitation is that the environment in which agents interact is a pure exchange economy. The agents have no other alternative than borrowing and lending in the international credit market to insure against stochastic shocks. A deeper understanding of the dynamics of the emerging market crises requires a model that allows domestic capital accumulation as an additional savings channel. This natural extension permits the study of the effect of sovereign country risk on

the interaction of savings and investment. The endogeneity of the default risk premium would break the type of Fisherian separation in savings and investment decisions that characterizes conventional models of business cycles in small open economies. The international interest rate may play a more important role as a determinant of the macroeconomic fluctuations. In addition, the existence of an endogenous borrowing limit may improve the ability of these type of models to explain the dynamics of recent crises in emerging markets and shed some lights on the evaluation of alternative macroeconomic policies.

Another limitation is that the type of debt contract studied here rules out the possibility of re-entering the international credit market. There is no possibility of re-contracting. If recontracting is allowed, one can conjecture that the cost of exclusion will be lower and so the probability of default may be higher, all other things equal. An extension in that direction would bring the model closer to reality.

References

[1] S. Rao Aiyagari. Uninsured idiosyncratic risk and aggregate savings. *Quarterly Journal of Economics*, pages 659–684, August 1994.

[2] Cristina Arellano and Enrique Mendoza. Credit frictions and sudden stops in small open economies: An equilibrium business cycles framework for emerging markets crises. Working Paper 8880, National Bureau of Economic Research, 2002.

[3] David T. Beers. Sovereign defaults: Hiatus in 2000? Standard and Poor's, 2000.

[4] Guillermo Calvo. Capital flows and capital market crises: The simple economics of sudden stops. *Journal of Applied Economics*, 1:35–54, 1998.

[5] R. Cantor and F. Packer. Sovereign credit ratings. *Current Issues in Economics and Finance*, 1(3), 1995. Federal Reserve Bank of New York.

[6] Harold Cole and William English. Default, settlement and signalling: Lending resumption in a reputational model of sovereign debt. *International Economic Review*, 36:365–385, 1995.

[7] Isabel Horta Correia, Jo .ao César das Neves, and Sérgio Rebelo. Business cycles in a small open economy. *European Economic Review*, 39(6):1089–1113, June 1995.

[8] Jonathan Eaton and Mark Gersovitz. Debt with potential repudiation: Theoretical and empirical analysis. *Review of Economic Studies*, 48:289–309, 1981.

[9] Mark Huggett. The risk-free rate in heterogeneous-agent economies. *Journal of Economic Dynamics and Control*, 17:953–969, 1993.

[10] P.J. Kehoe and F. Perri. International business cycleswith endogenous incomplete markets. Working Paper 7870, National Bureau of Economic Research, 2000.

[11] Dirk Krueger and Fabrizio Perri. Does income inequality leads to consumption inequality? empirical findings and a theoretical explanation. Department of Economics, Stanford University, 2000.

[12] Guillermo Larrain, Helmut Reisen, and Julia von Maltzan. Emerging market risk and sovereign credit ratings. Working Paper 124, OECD Development Centre, April 1997.

[13] Jyotiprasad Medhi. *Stochastic Processes*. John Wiley and Sons, 1994.

[14] Enrique Mendoza. Credit, prices and crashes: Business cycles with a sudden stop. Department of Economics, Duke University, 2000.

[15] Enrique Mendoza and Katherine A. Smith. Margin calls, trading costs and asset prices in emerging markets: The financial mechanics of the sudden stops phenomenon. Department of Economics, Duke University, 2001.

[16] Enrique G. Mendoza. Real business cycles in a small open economy. *American Economic Review*, 81(4):797–818, September 1991.

[17] Pablo A. Neumeyer and Fabrizio Perri. Business cycles in emerging economies: The role of interest rates. Department of Economics, New York University, 2001.

[18] Pedro M. Oviedo. Business cycles in a small open economy with a banking sector. North Carolina State University, 2001.

[19] Carmen M. Reinhart. Default currency crises and sovereign credit ratings. Working Paper 8738, National Bureau of Economic Research, 2002.

[20] George Tauchen and Robert Hussey. Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models. *Econometrica*, 59:371–396, 1991.

[21] Harold Zang. Endogenous borrowing constraints with incomplete markets. *The Journal of Finance*, 52(5):2187–2209, 1997.

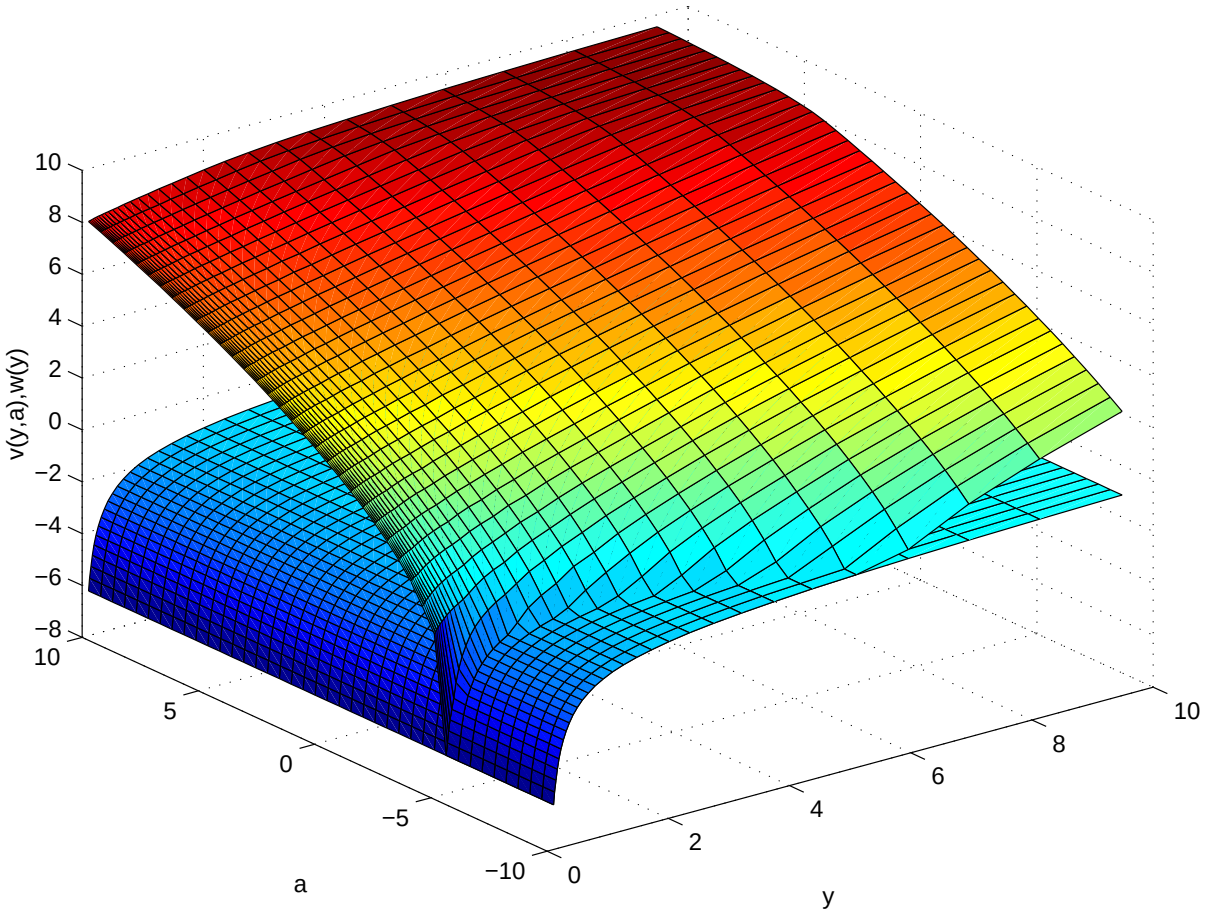
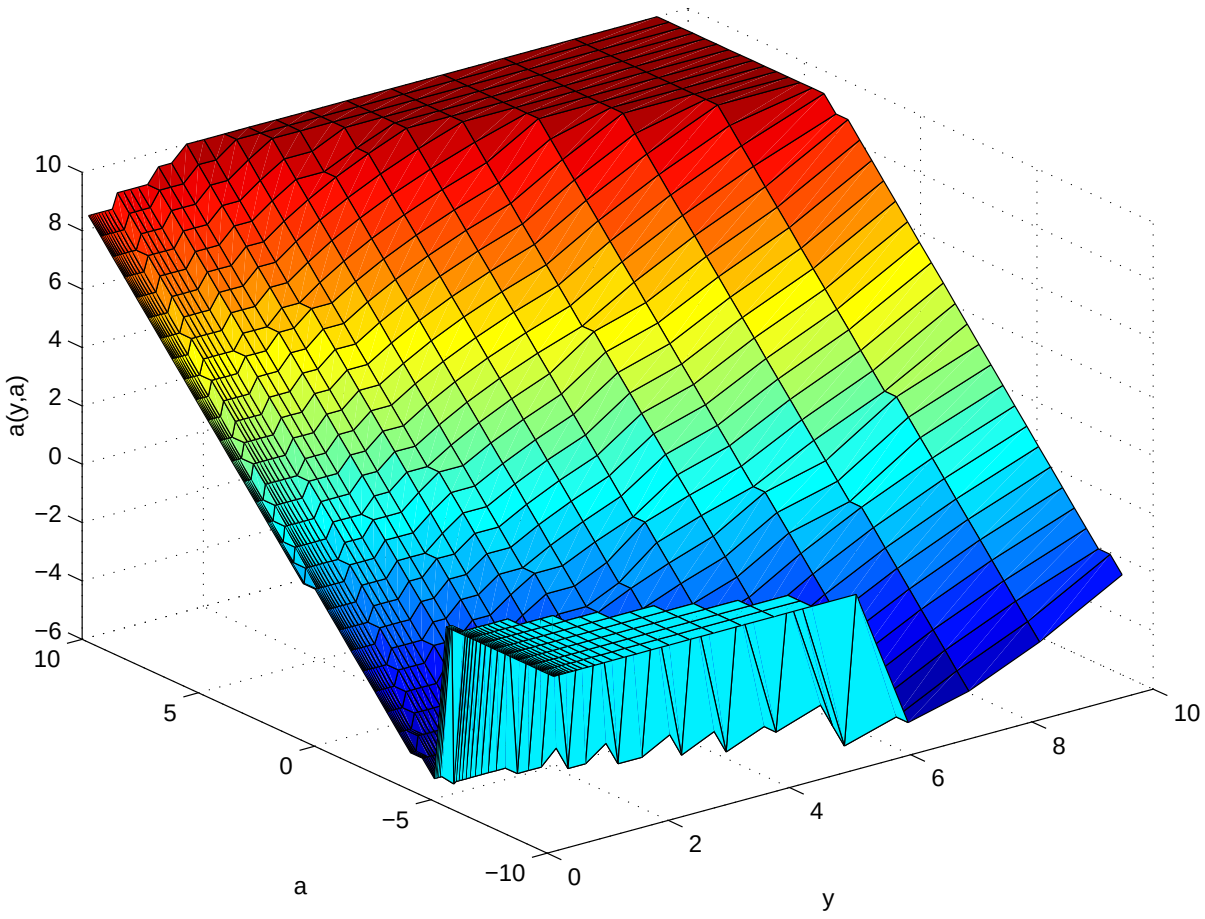


Figure 1: Autarky and Repayment Value Functions



(a)

Figure 2: Optimal Policy Function

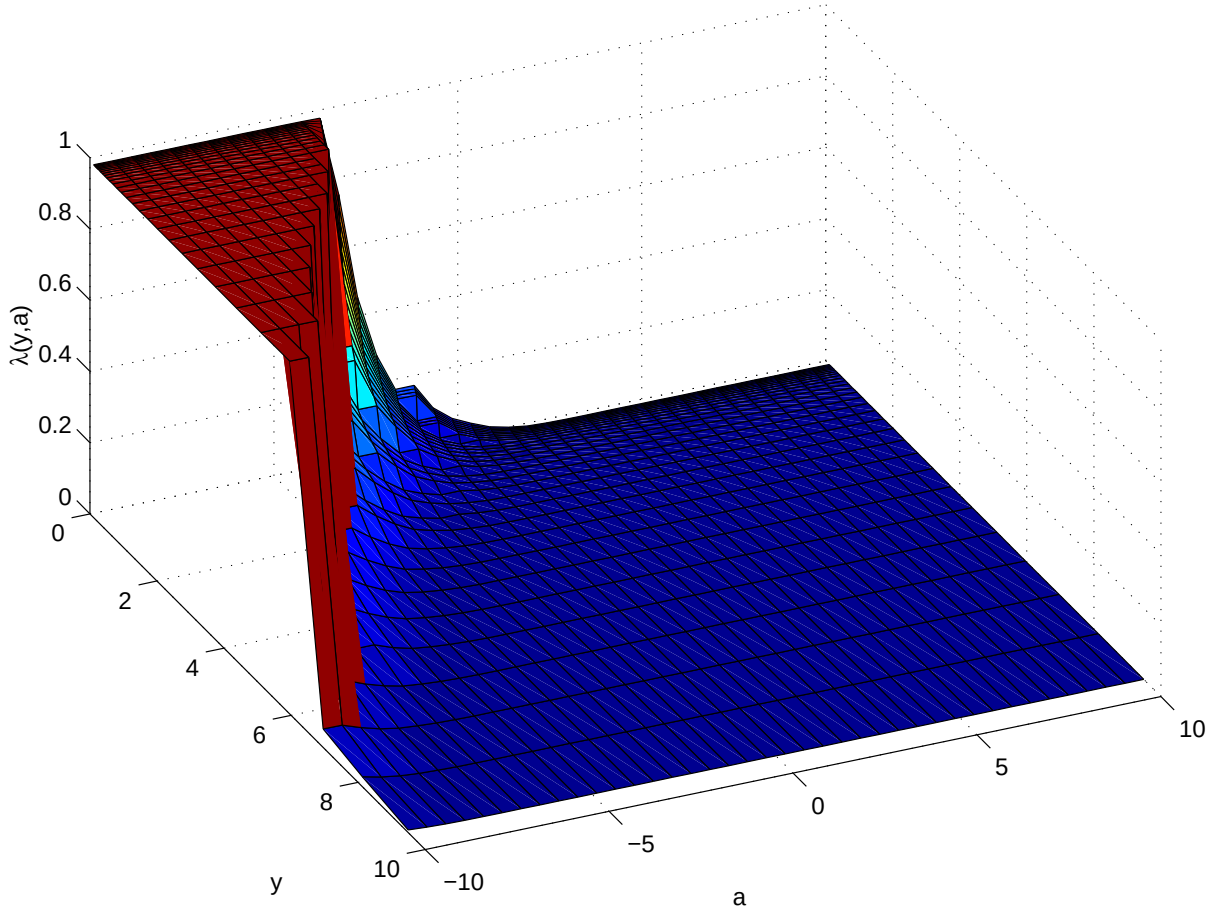


Figure 3: Optimal Probability of Default

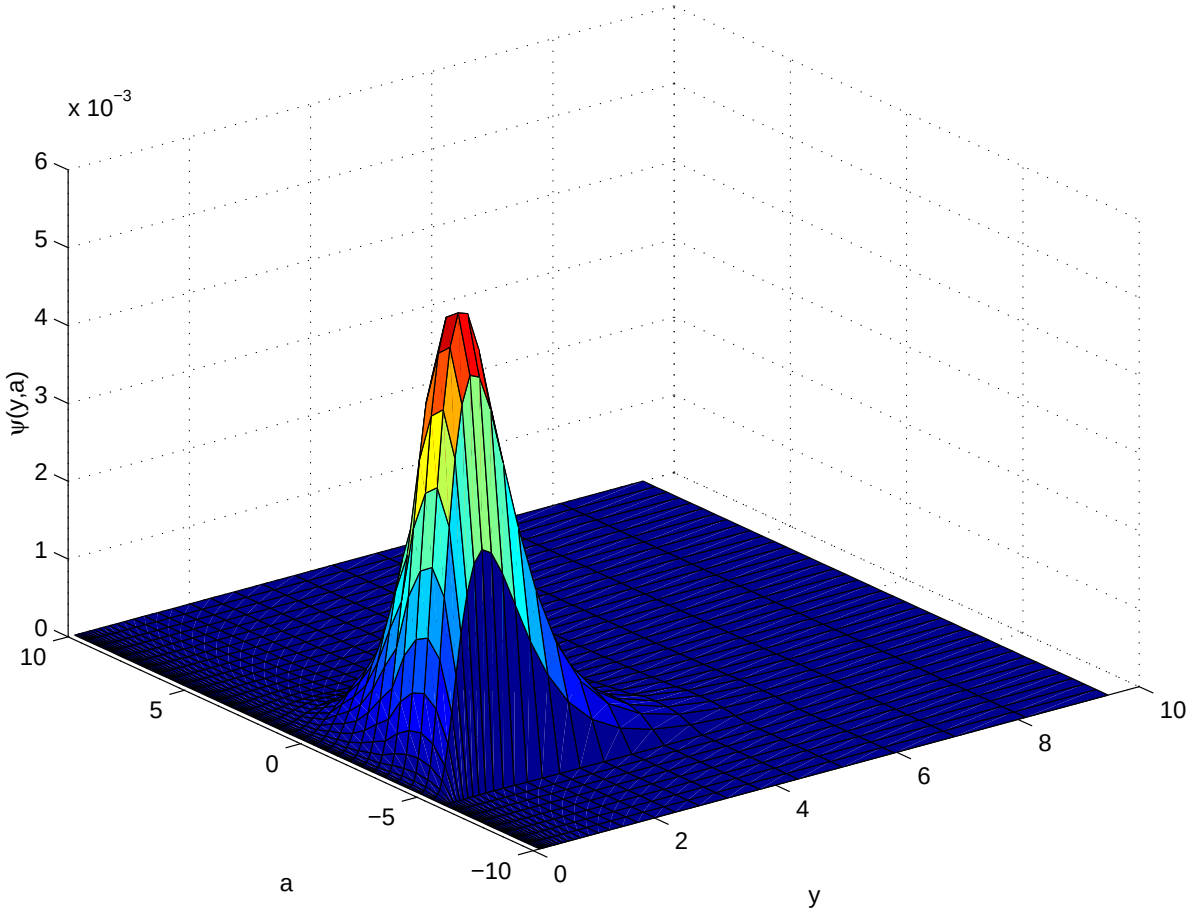


Figure 4: Ergodic Probability Distribution

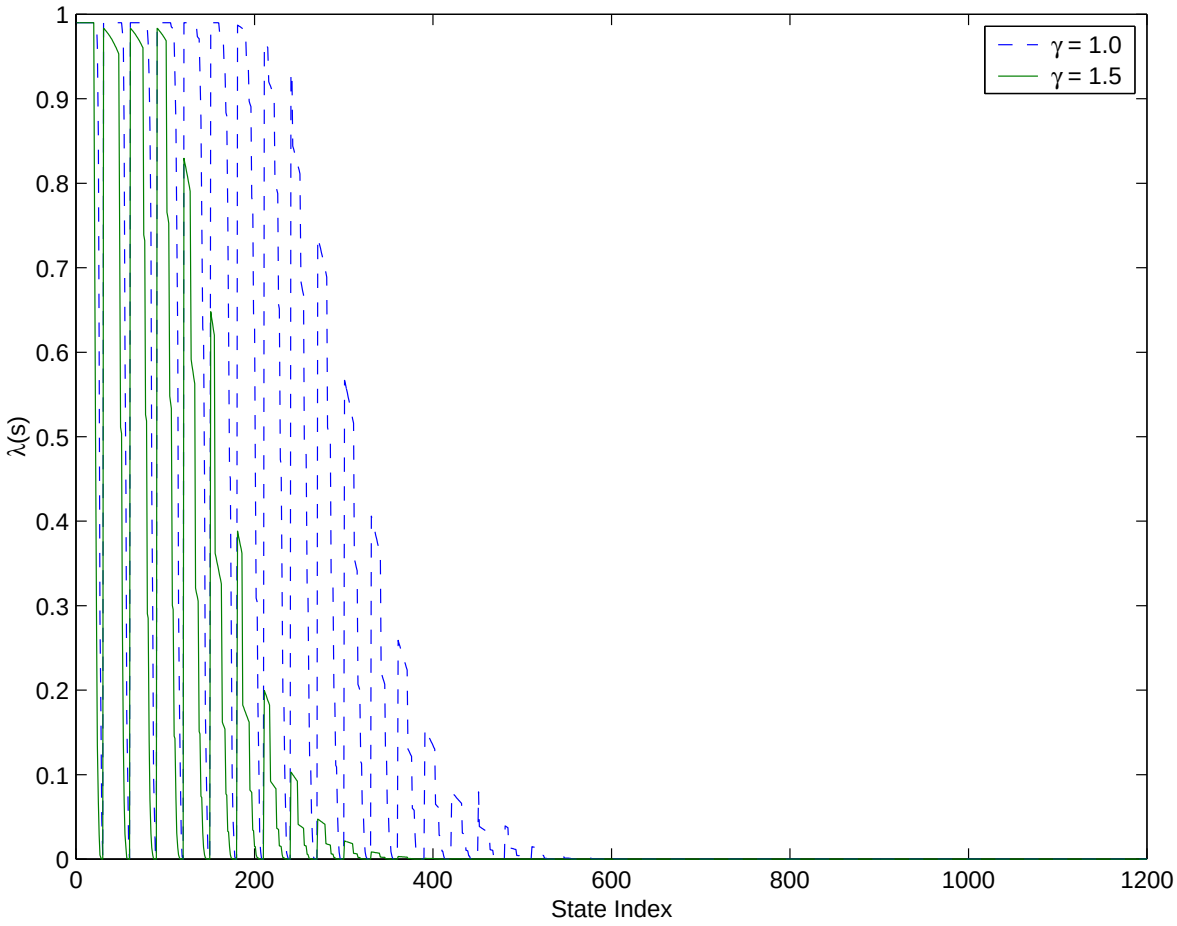


Figure 5: Effect of Higher Relative Risk Aversion, γ

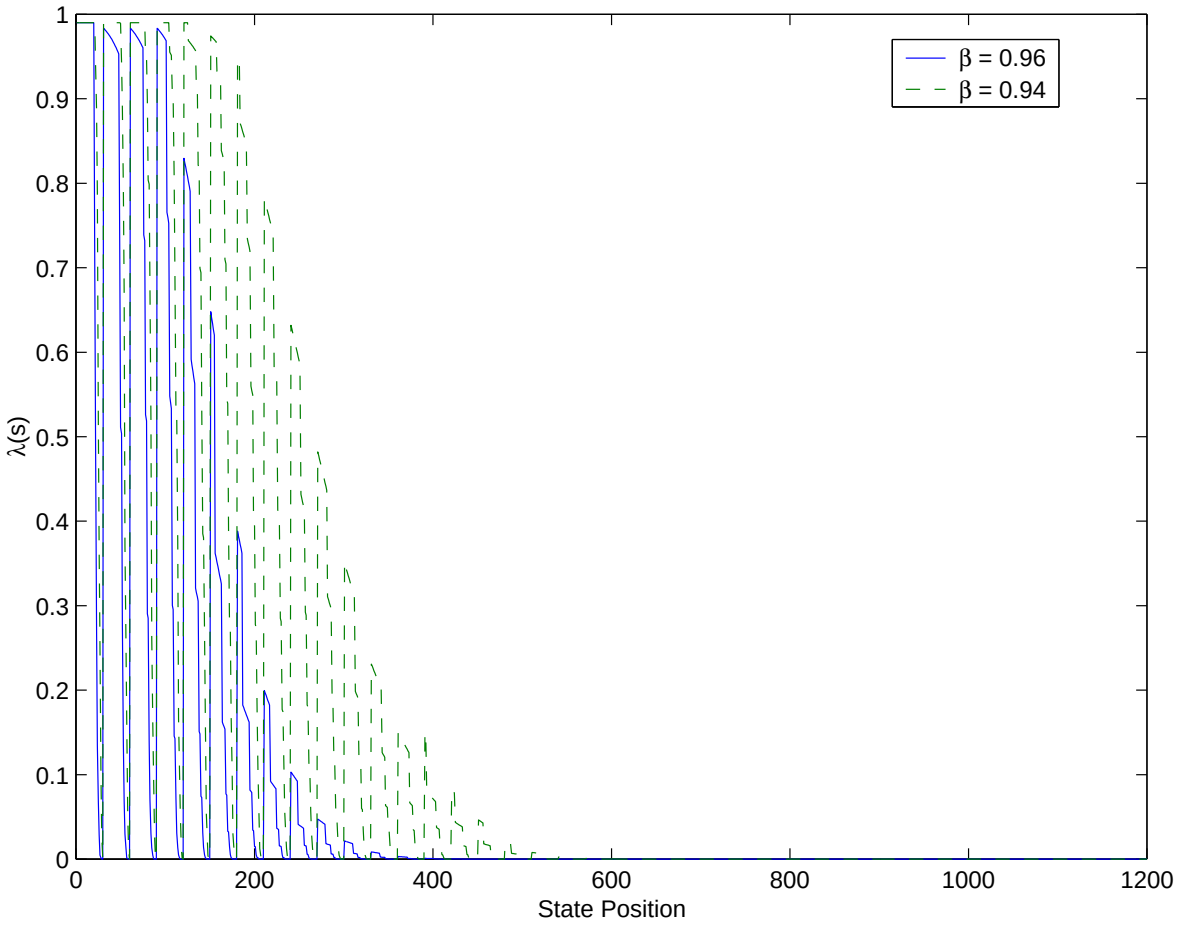


Figure 6: Effect of Higher Discount Factor, β

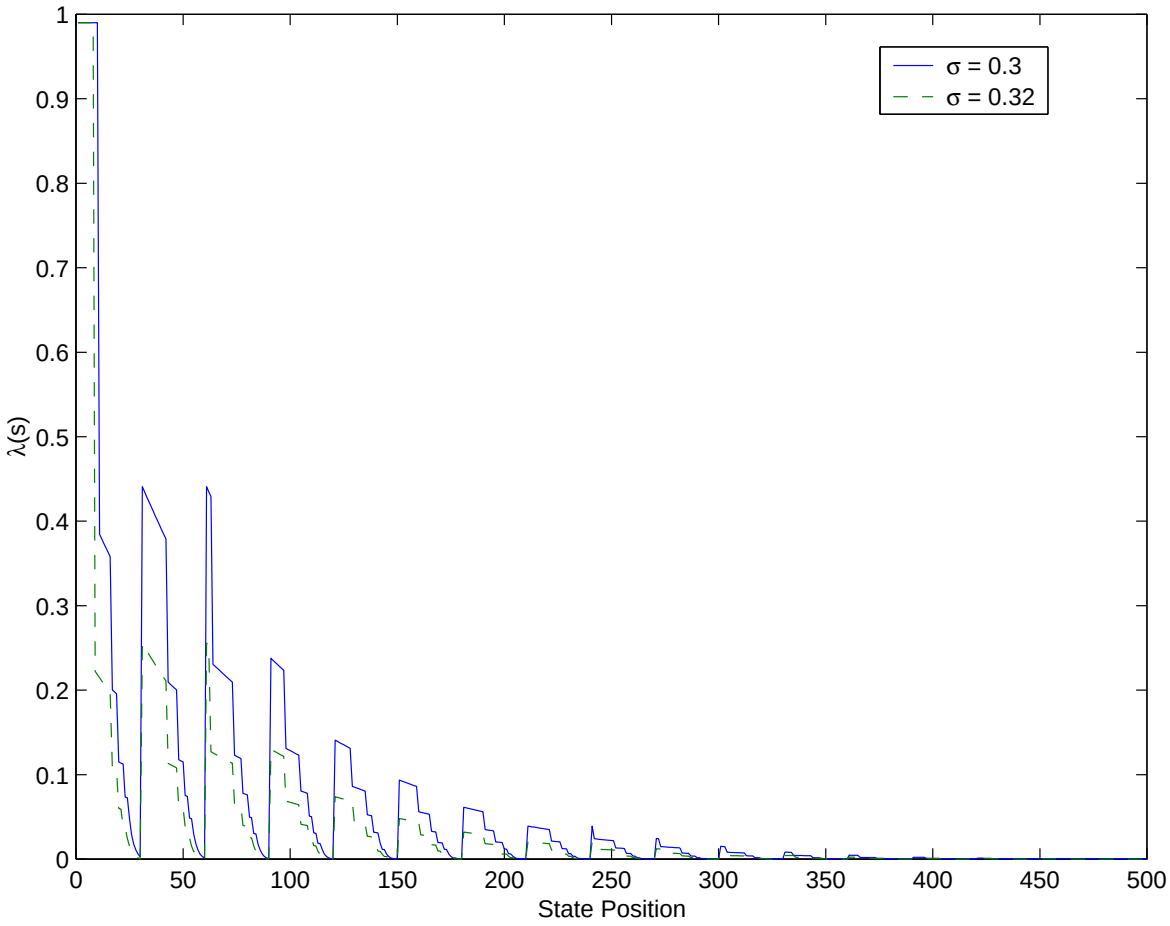


Figure 7: Effect of Higher Income Volatility

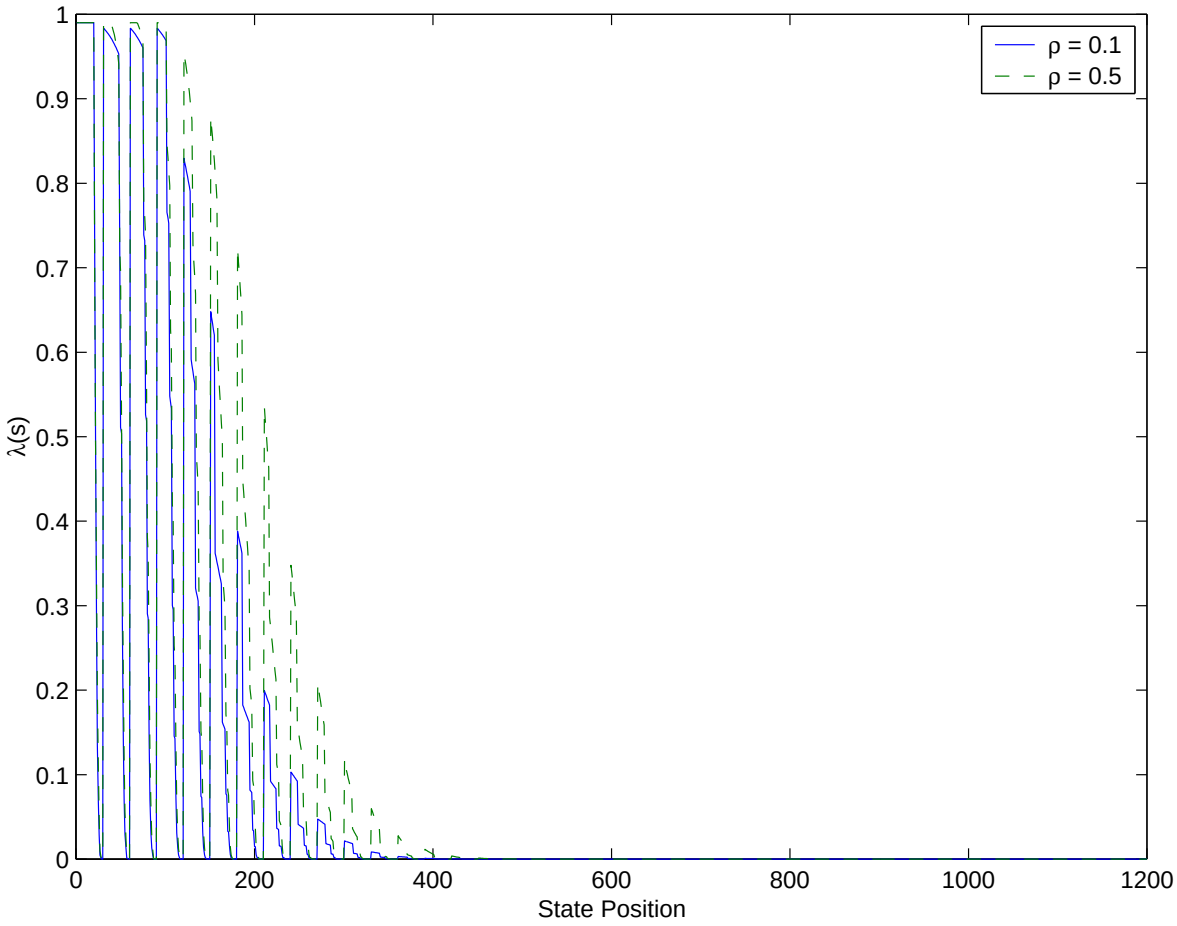


Figure 8: Effect of Higher Income Persistence

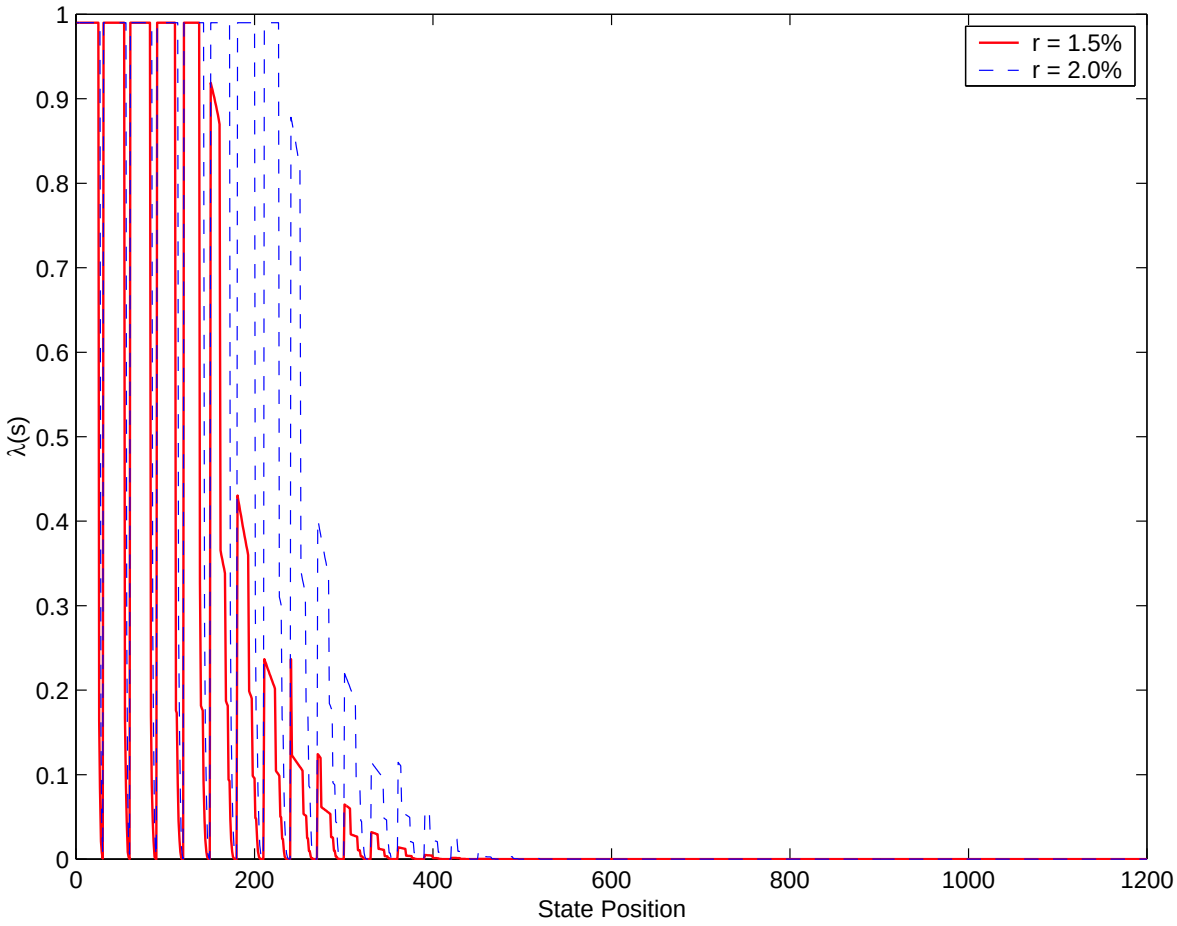


Figure 9: Effect of Higher Default Risk Free Rate