

OUTPUT GAP IN COLOMBIA: AN ECLECTIC APPROACH

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Abstract

In an economy conducted under an Inflation Targeting regime, the output gap becomes one of the most important variables to guide monetary policy. Defined as the difference between observed and potential or non-inflationary output, the gap is a measure of the state of aggregate demand and, therefore, of inflationary pressures on the economy. However, this relationship might be obscured by supply and price shocks, perhaps more relevant in the case of emerging economies. This paper estimates and evaluates the output gap for Colombia between 1970 and 2003 using a wide array of methods that go from univariate approaches such as Hodrick-Prescott (HP) and Band Pass filters to multivariate or structural methods obtained by the Kalman filter technique or the production function approach. We also include some mixed procedures like the multivariate filter and the prior-consistent filter. The last one takes into account some supply and price shocks observed in the Colombian economy since 1990. An evaluation of the different estimators is made by a simulated out-of sample forecasting exercise. The results show that multivariate structural filters have a better performance than pure mechanical approaches, but the difference is marginal with respect to a prior-consistent HP filter that takes into account supply shocks. In general, the forecasting performance of all the output gaps estimators improves when we re-define core inflation to exclude some price shocks.

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1. Introduction

An accurate estimation of the degree of slackness in the economy is a central issue to define the monetary policy stance. This task is particularly crucial in Central Banks (CB) conducting Inflation Targeting (IT)². Moreover, since inflation is driven by excess demand and expectations in the long run, the other possible sources of price instability usually have only transitory effects. In this context, the estimation of the non-inflationary growth rate of output plays a crucial role not only to analysts but to policymakers.

Traditionally, the output gap has been the measure most widely used for this purpose. Computed as the difference between observed and potential (unobserved) output, the output gap is itself a non-observed variable. In this context, potential output is defined as the maximum output attained by the economy without generating inflationary pressures. Besides monetary theory, the output gap has also been used in the analysis of Business Cycle Theories. Here the potential output is defined as the level of output achieved with full employment of productive factors.

The first definition of potential output is closer to the nature of monetary policy and has become of widely used among central bankers. In terms of growth theory, it means that potential output is equivalent to the stationary state level of output. At this stage, the inflation rate converges either to zero or to the explicit target previously defined by the monetary authorities. By the same token, inflation expectations are aligned with this target. Any short-lived external shock that spurs up in the system should only have temporary effects both on inflation and output.

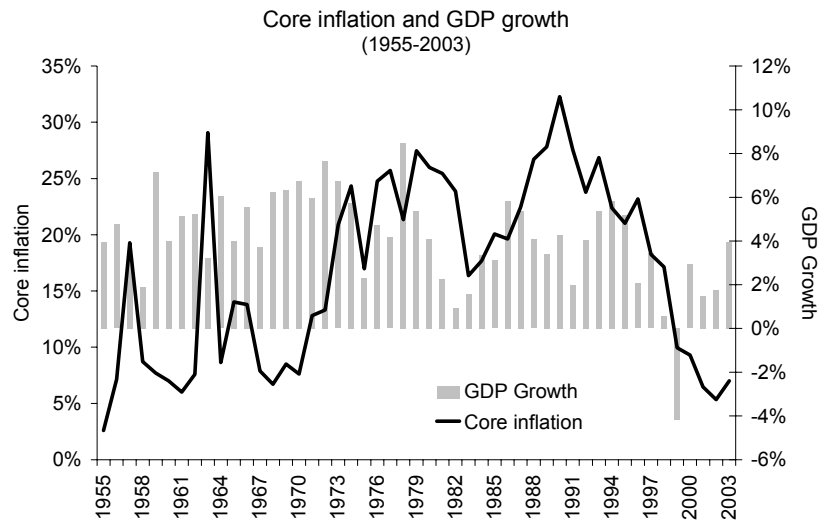
The distinction between these two definitions of potential output goes beyond formality and has practical implications for the estimation of the output gap. In the case of an emerging economy like Colombia, the existence of rigidities, mainly the markets of productive factors, frequently implies that the full employment of resources might be incompatible with price stability. Also in developing countries, inflation indicators are usually affected by transitory events not related to demand pressures. In Colombia, this

² Such is the case of the Colombian Central Bank since 1999.

has been the case with primary food prices during the last decades, but more recently with VAT reforms and government controlled prices, which represent at least 10% of the CPI basket. Thus, is not surprising that the correlation between output and inflation is not high (See **Figure 1**).

The current paper presents some estimations of the Colombian output gap using a wide range of methods. We obtained eleven estimators using different approaches. One of them (the prior consistent Hodrick-Prescott) takes into account possible supply and price shocks that may introduce noise to the estimators obtained by more traditional methods. The paper is organized as follows: The second section presents definitions of the basic concepts used throughout the paper. The third section contains a description of eight of the eleven methods used. The fourth and fifth section shows a statistical and econometric evaluation of the different estimators. In the sixth section we present proposal for a punctual estimation of the output gap using the results obtained in the later sections. Finally we present the main conclusions.

Figure 1



Source: DANE and Banco de la República, calculations by Banco de la República

2. Basic definitions

A revision of the recent research on the estimation of the output gap reveals a wide variety of methodologies. However, it is possible to identify two basic approaches: structural and non- structural methods.

Structural methods are based on economic theory, and they include methodologies such as the production function approach, SVARs, and Okun Law’s method. On the contrary, non-structural methods are purely statistical and mechanic in its nature without any explicit economic foundation. Methodologies under this second approach intend to obtain the trend component of the output series through diverse statistical methods. De-trending methods go from the very simple linear and segmented approaches to more complex ones as the Baxter and King or Band Pass (BP) filters, and the very popular Hodrick-Prescott Filter (HP).

There is also a mixed approach that combines properties of structural and non-structural methods. Known as multivariate methods, they introduce economic relationships to statistical de-trending methods. This class includes the Multivariate Hodrick-Prescott Filter, the Beveridge-Nelson Decomposition, the Unobservable Components method and multivariate structural models using Kalman Filter.

In essence, all methods rely on the assumption that the output observed at a certain period (y_t) can be expressed as:

$$y_t = y_t^C + y_t^T \tag{1}$$

Where y_t^C is the cyclical component and y_t^T is the trend component.

The decomposition shown in equation (1) is evident in the case of statistical approaches as the linear de-trending, the HP and BP filters and for all the multivariate structural methods. It may appear less obvious in the case of the production function approach. However, once it is assumed that output is at its potential level when productive factors are used at a “normal” (or non-inflationary) level, it becomes clear that potential output can be assimilated to the trend component of the observed output series. Usually, the “normal” (non-inflationary) level of productive factors is obtained using the same de-trending methods mentioned above, and they correspond to the trend component of the series.

Given equation (1), the output gap can be defined as the cyclical component of the output series and is measured as a percentage of the trend component of the series, as follows:

$$gap_t = y_t^c / y_t^T \quad (2)$$

The gap definition in equation (2) assumes that the trend component of observed output can be assimilated to a “normal” level of output at which no changes in inflation should occur. In this context, “normal” means a non-inflationary output. Inflation reaches its stationary level (or the long run inflation target in an economy with IT regime) once output gap is zero. Potential output (defined as the one achieved with full utilization of productive factors) will lie above non-inflationary output, or at the best scenario (in a frictionless economy with a high degree of efficiency and minimum rigidities) it would be equivalent to that level.

The question that arises here is if the output trend is an adequate indicator for non-inflationary output. Given its nature, non-structural univariate methods do not offer a direct answer to this question since de-trending does not take account of the inflationary history at all. A different case may be adduced for de-trending under structural multivariate methods, which allow a Phillips Curve type of relationship to be considered. A similar assessment could be made for the production function approach as long as the estimation of the “normal” level of productive factors takes into account inflation.

At the same time, however, multivariate methods might erroneously estimate the output gap under supply shocks if they do not differentiate permanent from temporary deviations. Under a transitory supply shock for example, observed output shifts out of its trend path and inflation deviates from its target. Thus, a mechanical interpretation would conclude that there has been a change in the gap size and therefore monetary policy should react in accordance. But, from the standpoint of inflation analysis this is a short-lived event and, therefore, no changes either in non-inflationary output or in the output gap have occurred.

In terms of the monetary policy stance, the adequate measure of output gap is the one that accounts for deviations of output from its trend component, only when these deviations are due to demand shocks. These types of shocks tend to be permanent in

nature, affecting the output gap, and thus requiring policy makers' attention. On the contrary, transitory supply shocks should lead to changes in non-inflationary output, leaving output gap unchanged. Of course, these pre-requisites are hardly achieved by mechanical univariate methods and it might also be ignored by structural multivariate approaches run under poor specifications.

3. The methods

Since the main purpose of this paper is to compare alternative measures of output gap for Colombia, we proceeded to gather all the available estimations, under the widest array of methodologies available. The Colombian CB has already implemented most of the methods evaluated here. This has occurred during the last five years, when the CB has officially conducted monetary policy under an IT regime.

To make the comparison possible, we estimated all the output gaps using annual data from 1970 to 2003. T models were estimated on quarterly and yearly frequencies. Due to data restrictions, the estimation using the production function method was performed only with annual data.

Besides the production function approach, we use both standard univariate and multivariate methodologies. Under the first category three methodologies were used: the HP Filter, the BP Filter and a prior-consistent HP Filter. Under the second category we present estimations using the Multivariate HP Filter and the Kalman Filter.

Since the HP and BP filter are well know by the specialized reader and its application is very simple, we skip their description and concentrate only on those methods that require some model specification.

3.1 The production function model

Among the structural methods used to estimate the output gap, the most frequently used in recent literature is the production function approach (see Laxton and Tetlow 1992, de Brouwer 1998, Cerra and Saxena 2000 and Billmeier 2004). Some applications for Colombia include: Rodriguez and Prieto (1997) and Rodriguez et al (2004).

We used the model developed by the CB based in these previous works to produce our estimations. As it is usual in most of the related works, we assume that output can be represented by a constant return-to-scale Cobb-Douglas production function as follows:

$$Y_t = A_t * K_t^\beta * L_t^{(1-\beta)} \quad (3)$$

where A_t is total factor productivity or Solow residual, L_t is the effective level of labor, K_t is the capital stock and Y_t is observed GDP. We assume that the capital share (β) is 0.4; therefore, the labor share is 0.6. These values were taken from estimations obtained by Prieto and Rodriguez (1997) for Colombia.

Since total factor productivity (TFP) is not directly observed, it must be derived as the residual from equation (3):

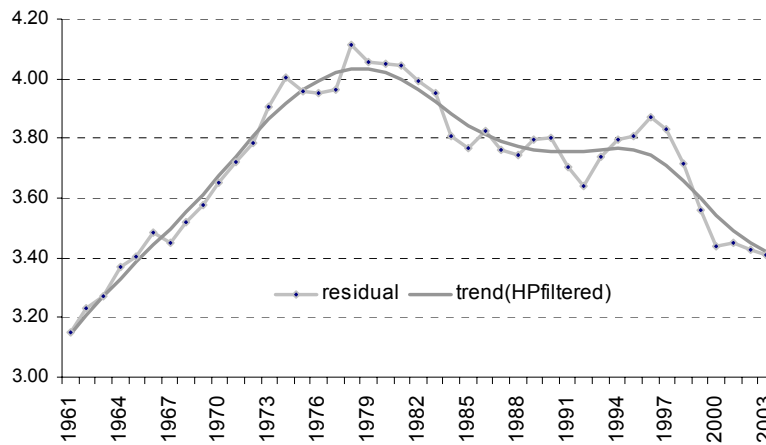
$$a_t = y_t - \beta k_t - (1 - \beta)l_t \quad (4)$$

Where lowercase variables represent logs.

Figure 2 depicts the TFP or Solow Residual for Colombia since 1960. After a continuous upward trend between 1960 and 1980, the TFP shows a declining pattern interrupted only by a short recovery during the expansion cycle in the mid-nineties. That happened despite an important investment effort in human capital along this period. An explanation could be the possible existence of frequent institutional shocks in addition to the surge of internal security threats. However, we cannot rule out problems related to poor quality data, especially concerning the capital stock series.

Figure 2

Solow Residual



The model is built on the assumption that not all capital stock available at a specific time is effectively used in the production process. In the economy, it is common to find spare capacity, so we consider convenient to adjust observed capital stock by an indicator of installed capacity utilization. Since this last variable is time dependent, its exclusion might lead to a poor estimation of the TFP rate of growth. Thus, effective capital stock is obtained according to the following equation:

$$K_t = K_{t-1}^{Obs} * ICU_t \quad (5)$$

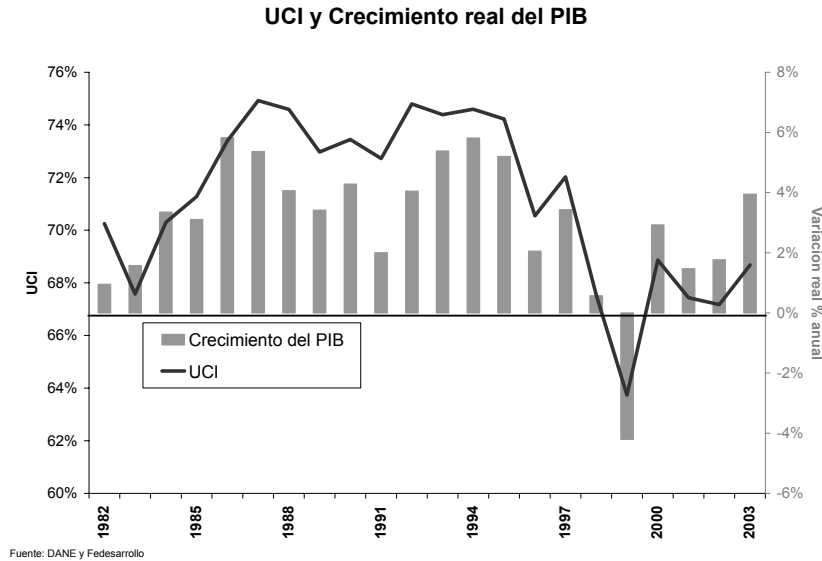
where K_t^{Obs} is the observed capital stock according to estimations done by the CB based on historical data and net total investment, and ICU_t is the percentage of installed capacity utilization (i.e.: one minus the percentage of spare capacity) according to industrial firms³. One problem that arises with this strategy has to do with the fact that data on spare capacity correspond only to the industrial sector, which does not represent more than 20% of Colombian GDP. **Figure 3** suggests a strong correlation between industrial ICU and GDP growth.

Most of the available literature on this issue does not consider adjusting the capital stock by a measure of spare capacity. It could be argued that this strategy is innocuous

³ The source is the opinion monthly poll to entrepreneurs of the industrial sector conducted by Fedesarrollo (EOE).

once we estimate potential GDP by using the trend component of the Solow residual. However, according to our experience with Colombian data, differences remained between the two alternatives.

Figure 3



On the other hand, effective labor is defined as the actual or observed labor force (*ie.*: number of employees) at a specific time. A first-best alternative for labor force could be to consider adjustments for changes in labor intensity, usually associated with increase in the share of part-time jobs or in the number of hours worked by an average full-time worker, for example. However, due to the lack of data, we were forced to choose a second-best alternative.

In turn, potential output is computed according to:

$$y_t^* = a_t^* + \beta k_t^* + (1 - \beta)l_t^* \quad (6)$$

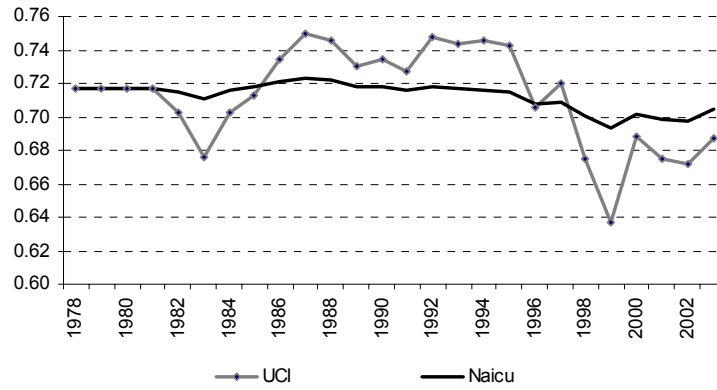
Long-run trend total factor productivity (a_t^*) is obtained by smoothing the Solow residual series from equation (4) using the HP filter. The long term or “normal” level of capital (k_t^*) is defined as:

$$k_t^* = k_{t-1}^{Obs} + NAICU_t \quad (7)$$

where $NAICU_t$ is the non-accelerating inflation capacity utilization. This variable was estimated for Colombia according to Nigrinis (2003), using a Kalman Filter methodology, and taking into account the relationship between ICU and inflation⁴. Thus the NAICU is the trend component of the industry's spare capacity series (ICU) and it can be used as a proxy for non-inflationary output⁵ (see **Figure 4**).

Figure 4

ICU and Naicu



Finally, the long term or “normal” labor is estimated as:

$$l_t^* = l_t^S + \text{Log}(1 - NAIRU_t) \quad (8)$$

Where l_t^S is the labor supply as measured by the Colombian statistical department (DANE) and $NAIRU_t$ is the non-accelerating inflation rate of unemployment. The NAIRU series used in this paper correspond to Julio (2001). In his work, the author estimates the NAIRU using a fully structural method, on a cubic spline specification for

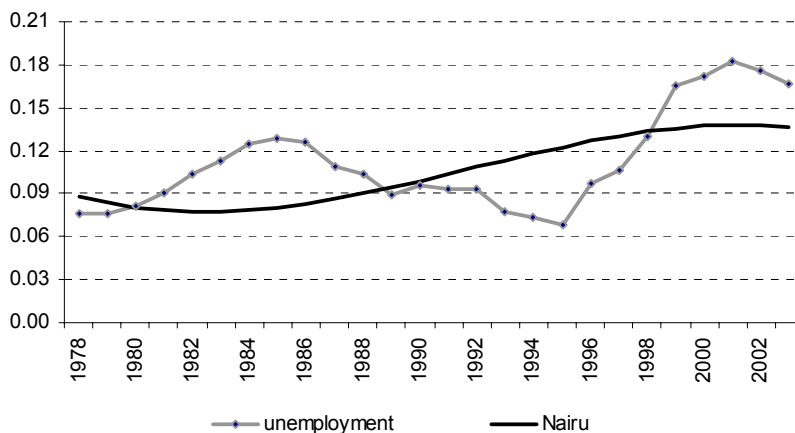
⁴ The estimation of the non-inflationary ICU (an unobserved variable) was conducted by Nigrinis (2003) using a Kalman filter approach. The model includes a Phillips Curve, among other equations. The procedure decomposes ICU (u_t) in its cyclical (g_t) and trend components (u_t^*): $u_t = u_t^* + g_t$, assuming a random walk dynamic for the trend component (following Gordon (1997) and Stock (1999)).

⁵ Later in the paper, we evaluate a proxy of output gap computed as: $gapuci_t = (1 - \frac{ICU_t}{NAICU_t})$

this variable⁶. **Figure 5** shows the NAIRU and the unemployment rate. The high value reached by the NAIRU estimates (approximately 13.7% between 2000 and 2003) force us to accept these results with caution.

Figure 5

Unemployment rate and Nairu



By defining potential output as in equation (6) we try to approximate a measure of non-inflationary output, consistent with a steady state rate of inflation. Most of the reviewed literature (for example Billmeier 2004, Cerra and Saxena 2000) only offers a halfway approximation to this variable. Although most papers consider a definition of effective labor similar to the one used above, they ignore the roll of ICU and NAICU in the estimation of effective capital.

3.2 The Prior-Consistent (PC) Filter

This univariate filtering methodology is similar to the Hodrick-Prescott filter, with the advantage that it allows for the imposition of priors on the properties of the series (such

⁶ The main features of the model are given by the following equations:

$$\Delta \pi_t^A = \mathcal{S}_t^T \beta + \alpha_0 \Delta \pi_{t-1} + \alpha_1 \Delta \pi_{t-1}^F + \gamma \mathcal{U}_t + \varepsilon_t$$

$$Nairu_t = -\mathcal{S}_t \beta / \gamma$$

as the levels, rates of change or variances)⁷. For example, if there is specific information about a particular behavior of the trend series in a specific point in time that is not being captured by some other filtering method, we can introduce a prior in the PC filter that accounts for it and obtain a filtered series that behaves in that particular way.

Estimates of potential output from the PC filter are derived by minimizing the squared deviations of observed output, y_t , from potential output, \bar{y}_t , subject to a constraint that penalizes squared deviations in the change in potential output relative to some prior estimate of the change in that variable, which is denoted as $\Delta\bar{y}_t^*$. Specifically, fixing a sample of size T, the PC filter solves for the sequence $\{\bar{y}_t\}$ that minimizes the objective function:

$$\sum_{t=1}^T (y_t - \bar{y}_t)^2 + \lambda \sum_{t=2}^T [(\bar{y}_t - \bar{y}_{t-1}) - \Delta\bar{y}_t^*]^2 \quad (9)$$

This methodology might be especially useful under frequent and unexpected supply or inflation shocks like is the case of the Colombian economy. These situations are hardly taken into account by structural methods and might lead to noisy and inaccurate estimations. Priors may be used to correct this noise. In our estimations, we have set priors for 1987, 1991, 1997 and 1999. The respective values are 0.0005, 0.009, -0.004 and -0.06. For these years, we have some evidence of the presence of supply or price shocks associated to energy shortages and VAT or import tariffs changes, among other events.

3.3 *The Multivariate HP Filter*

A relatively easy way to give some economic structure to a pure de-trending statistical method is the multivariate HP filter. In this paper we present an estimation of output gap using this methodology implemented by Julio (2004), according to Laxton and Tetlow (1992) and to a later application by de Brouwer (1998). The economic structure is given by the following equations:

⁷ For further details on this methodology see Laxton et al (1998).

$$u_t^G = \gamma gap_t + \varepsilon_{U,t} \quad (10)$$

$$icu_t^G = \delta_1 gap_t + \delta_2 gap_{t-1} + \varepsilon_{ICU,t} \quad (11)$$

$$\Delta\pi_t = \alpha_1 \Delta\pi_{t-1} + \alpha_2 \Delta\pi_t^F + \alpha_3 \Delta\pi_t^m + \alpha_4 \Delta\pi_{t-1}^m + \gamma gap_t + \varepsilon_{\pi,t} \quad (12)$$

Where: $u_t^G = u_t - nairu_t$ (Unemployment gap);

$icu_t^G = icu_t - naicu_t$ (Capacity utilization gap);

$gap_t = y_t - y_t^T$ (Output gap);

$\Delta\pi_t = \pi_t - \pi_t^E$

Equations (10), (11) and (12) correspond to the Okun's Law equation, the capacity utilization equation and the Phillips curve, respectively. The dependent variable in the Phillips Curve is CPI inflation (π_t), whereas in the RHS we have food price inflation (π_t^F) and import price inflation (π_t^m) besides the output gap and lagged values for CPI inflation.

The estimation technique used by Julio (2004) proceeds as de Brouwer's (1998). Equations (10) and (11) use NAIRU and NAICU estimations developed by Julio (2001) and Nigrinis (2004) respectively. This procedure replicates the one used under the production function approach shown above. Again, the sample period is 1970 – 2003 and the estimation is performed on a quarterly and yearly frequency but only the later is presented. The results for equations (10) and (11) are:

$$u_t^G = \underset{(0.020)}{0.0054} - \underset{(0.000)}{0.618} gap_t + \varepsilon_{U,t}$$

$$icu_t^G = -\underset{(0.0001)}{0.5698} gap_t + \underset{(0.000)}{0.7385} gap_{t-1} + \varepsilon_{ICU,t}$$

To estimate the Phillips Curve we assume that expected inflation is formed according to the following equation:

$$\pi_t^E = 0.5\pi_{t-1} + 0.5\pi_{t+1} \quad (13)$$

Thus, expected inflation has both forward and backward looking elements. This functional form is taken from the central model employed by the monetary authorities to define the policy stance. One of the reasons to choose this specification has to do with the fact that values obtained using (13) show the highest correlation with expected inflation obtained from quarterly CB surveys.

Following standard procedures, the residuals $\varepsilon_{U,t}$, $\varepsilon_{U,t}$, $\varepsilon_{ICU,t}$ and $\varepsilon_{\pi,t}$ augment the HP loss function. Potential or non-inflationary output is the series that minimizes this function (see de Brouwer 1998 for further details).

3.4 *A multivariate system using the Kalman Filter*

In the last couple of years, the Colombian Central Bank has been experimenting with the Kalman filter methodology to estimate the output gap. In this paper we present the most recent results based on a system of 9 equations as developed by Perez et al (2004)

⁸. The equations are:

$$\pi_t^c = \alpha_1\pi_{t-1}^c + (1 - \alpha_1)\pi_{t-1}^M + \alpha_2y_t + \varepsilon_t^{\pi^c} \quad (14)$$

$$y_t = \beta_1y_{t-1} - \beta_2z_t^r + \beta_3z_t^{Iz} + \varepsilon_t^y \quad (15)$$

$$Y_t^p = Y_{t-1}^p + g_{t-1} + \varepsilon_t^{Y^p} \quad (16)$$

$$g_t = \gamma_0g_{t-1} + (1 - \gamma_0)\bar{g}_{t-1} + \varepsilon_t^g \quad (17)$$

$$\bar{g}_t = \bar{g}_{t-1} \quad (18)$$

$$y_t = Y_t - Y_t^p \quad (19)$$

$$r_t = \bar{r}_t + z_t^r \quad (20)$$

⁸ An earlier unpublished paper on this topic using Colombian data is the one by Misas and Oliveros (2002). In their work, the authors estimated a system of five equations, including a Phillips Curve and an IS curve. The current paper presents an extension of this former model developed by Pérez et al (2004). In relation to former works, this one makes some improvements in the specification of the Phillips and IS curves and introduces a new equation accounting for the real interest rate gap.

$$\bar{r}_t = \bar{r}_{t-1} + \varepsilon_t^{\bar{r}} \quad (21)$$

$$z_t^r = \varepsilon_t^{z^r} \quad (22)$$

Where⁹:

$\pi_t^c = 4(P_t^c - P_{t-1}^c)$, with P_t^c as the log of the CPI excluding food prices,

$\pi_t^m = 4(P_t^m - P_{t-1}^m)$, with P_t^m as the log of the Producer Price Index (PPI) for import, goods,

Y_t : GDP.

Y_t^p : “potential” or non-inflationary GDP.

z_t^{lz} : the bilateral real exchange rate gap obtained using an HP filter.

z_t^r : the real interest rate gap.

Equation (14) corresponds to an Augmented Phillips Curve. In the current specification, it relates consumer inflation (excluding food) to excess demand represented by output gap lagged one period, lagged inflation, and import goods inflation in order to capture exchange rate pass-through effects. A reduced form of aggregate demand or IS curve is represented by equation (15). It shows output gap as a function of its lagged value, the real interest gap (equation (20)) and the real exchange rate gap. Potential output follows a random walk with drift process according to equation (16) with g_t as its growth rate, which, in turn, follows an autoregressive process of order one. In the long run, the output gap growth rate converges to its equilibrium level \bar{g} as shown in equations (17) and (18). Equations (19) and (20) are identities that define the output gap and the real interest rate and equation (21) defines the long run interest rate as a random walk. Finally, equation (22) defines the real interest rate gap as noise, allowing the interest rate to be treated as an endogenous variable.

To estimate the parameters and unobserved state variables of the model, Pérez (2004) uses a Kalman filter algorithm. As usual, to estimate the model a state-space form has to be defined. Measurement equations are (14), (19) and (20), while the rest of equations correspond to the state equation. In matrix form, the measurement equation is:

⁹ The price indexes and GDP have been subject to seasonally adjustments using a standard X11 procedure.

$$\begin{bmatrix} Y_t \\ \pi_t^c \\ r_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} Y_t^p \\ g_t \\ \bar{g}_t \\ y_t \\ \bar{r}_t \\ z_t^r \\ \pi_t^c \end{bmatrix} \quad (23)$$

Accordingly, the transition equation is

$$\begin{bmatrix} Y_t^p \\ g_t \\ \bar{g}_t \\ y_t \\ \bar{r}_t \\ z_t^r \\ \pi_t^c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_0 & 1-\gamma_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & -\beta_2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & 0 & 0 & \alpha_1 \end{bmatrix} \begin{bmatrix} Y_{t-1}^p \\ g_{t-1} \\ \bar{g}_{t-1} \\ y_{t-1} \\ \bar{r}_{t-1} \\ z_{t-1}^r \\ \pi_{t-1}^c \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \beta_3 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1-\alpha_1 \end{bmatrix} \begin{bmatrix} z_{t-1}^z \\ \pi_{t-1}^m \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{Y^p} \\ \varepsilon_t^g \\ 0 \\ \varepsilon_t^y \\ \varepsilon_t^{\bar{r}} \\ \varepsilon_t^{z^r} \\ \varepsilon_t^\pi \end{bmatrix} \quad (24)$$

As with the rest of methods, the estimation period was 1970-2004. The estimation was performed both on quarterly and yearly frequencies but we only present results for the second exercise. The model as described faces some short-comings. First, the gap of the interest rate gap (equation 22) is not based on any fundamental or structural relation. In practice, we assume that the interest rate behaves as a random walk. This is a poor attempt to convert the interest rate in an endogenous variable. Second, the rest of equations excluding Phillips and IS curves are ad-hoc equations. Third, in general, there is poor micro-foundation in the model specification.

Regarding the estimation method, the number of parameters is high (6) and this might lead to a dimensionality curse problem. In order to reduce this risk, error values for some of the equations were imposed. In general, given its structure, the system showed to be unstable and sensible to the selection of the initial values. Besides two estimations obtained under the standard procedures (k and kr as we will show later), we also obtained a third one (kmmt) estimating the state vector running only the Kalman filter with the parameters used in the CB's core model (MMT).

4. Descriptive analysis

The statistical assessment we present in the following sections is performed for yearly estimations. **Table 1** shows basic statistics for eleven output gap measures finally used in the empirical analysis. In addition to the indicators introduced above (hp, hpp, mvhp, bp, k, kmmt, kr PnF, icugap, nairugap), in this section we include a new one obtained from monthly opinion polls. This indicator - ddgap – measures the demand situation according to manufacturing firms. Contrary to the former ten indicators, this one is qualitative in its nature.

Most of the output gap estimates have been obtained for the period 1970 – 2010. In the case of the Kalman filter estimations, the starting year is 1972, whereas for icugap and ddgap the starting years are 1980 and 1990 respectively. We run the model with forecasted data in order to overcome the instability of the estimates, usually present at the end of the sample (See Miller 2003, Brouwer 1985). We assumed a GDP growth rate of 4% between 2004 and 2010. This rate appears to be high given the poor performance of the Colombian economy in the last years; however it is not too different from what was observed before 1998. In addition, it is in accordance to what many analysts are currently forecasting for 2004 and 2005. Forecasted values for other required variables as inflation or exchange rate are taken from the CB's core model and the Government's financial program.

Table 1
Output Gap Measures: Descriptive Statistics 1/

	Output Gap				GDP trend 2/	
	MEAN	MIN	MAX	S.D.	growth	S.D.
hp	0.001	-0.04	0.05	0.025	3.9%	1.1%
hpp	-0.005	-0.06	0.03	0.024	3.9%	1.5%
mvhp	-0.001	-0.05	0.06	0.028	3.8%	1.1%
bp	0.000	-0.03	0.03	0.018	3.8%	1.6%
k	0.002	-0.05	0.04	0.029	3.9%	1.5%
kmmt	0.006	-0.06	0.08	0.039	3.8%	1.1%
kr	-0.010	-0.10	0.04	0.034	3.2%	1.0%
PnF	-0.002	-0.06	0.07	0.035	3.9%	1.7%
icugap	0.000	-0.08	0.04	0.034	n.a.	n.a
nairugap	-0.005	-0.04	0.04	0.022	n.a.	n.a
ddgap	0.000	-0.47	0.48	0.272	n.a.	n.a

1/ Sample period: 1970 - 2003

2/ Potential or non-inflationary GDP

Keys:

hp: Hodrick-Prescott; **hpp**: Hodrick-Prescott with priors; **mvhp**: Multivariate HP;

bp: Band-Pass filter; **k**: Kalman filter; **kmmt**: Kalman filter using core model (MMT) parameters;

kr: Kalman filter with IPC ex. Food and regulated prices; **PnF**: Production Function; **icugap**: gap based on ICU; **nairugap**: gap based on nairu; **ddgap**: demand situation indicator.

As it has been found for other countries' data (Billmeier 2004, Miller 2003), in our case most output gap estimates are centered on zero. However, for some indicators obtained using structural methods (**kr**, **nairugap**, **hpp**) this is less clear cut. The explanation, as in de Brouwer's (1995), probably lies in the downtrend shown by inflation since 1995. Under these circumstances, the output gap had to be somewhat negative in a more permanent manner.

Similarities among estimates are also present in the case of volatility. Excluding **ddgap**, which cannot be compared to the other estimates given its nature, all the measures have a standard deviation between 2% and 4%. However, significant differences arise when we look at the extreme peaks and troughs. Maximum points go from +3% (in **bp**) to +8% (in **kmmt**), whereas minimum points vary from -3% (again in **bp**) to -10% (in **kr**).

These important differences are evident when a visual inspection is performed (see Figures in the Appendix). Although all output gap estimates followed closely the economic cycle in the last 30 years and have a similar profile (as shown by high correlations in **Table 2**), notable mismatches occur on specific years. Thus, for example, in 1991 **hp** and **bp** filters show a negative gap whereas none of the other estimates (including **icugap** and **ddgap**) do it. This is notorious for structural or semi-structural multivariate estimates (**k kr**, **kmmt**, **nairugap** and **mvhp**) since the Colombian economy was hit by supply shocks (energy and food shortages) in 1991 that may have reduced potential growth without leading to a negative gap. In other words, in spite of the lower and even negative growth rate for this period, inflationary pressures remained in place. In fact, in that year the inflation rate was historically high.

Table 2

Output Gap Measures: Correlations 1/

	bp	ddgap	PnF	hp	hpp	mvhp	icugap	nairugap	k	kmmt	kr
bp	1.000	-0.102	0.553	0.745	0.356	0.608	0.145	0.290	0.491	0.413	0.480
ddgap	-0.102	1.000	-0.647	-0.390	-0.865	-0.566	-0.902	-0.737	-0.722	-0.630	-0.736
PnF	0.553	-0.647	1.000	0.900	0.802	0.963	0.763	0.943	0.976	0.942	0.978
hp	0.745	-0.390	0.900	1.000	0.577	0.964	0.503	0.788	0.873	0.883	0.865
hpp	0.356	-0.865	0.802	0.577	1.000	0.741	0.871	0.824	0.871	0.772	0.875
mvhp	0.608	-0.566	0.963	0.964	0.741	1.000	0.671	0.912	0.968	0.972	0.962
icugap	0.145	-0.902	0.763	0.503	0.871	0.671	1.000	0.829	0.811	0.736	0.823
nairugap	0.290	-0.737	0.943	0.788	0.824	0.912	0.829	1.000	0.965	0.960	0.970
k	0.491	-0.722	0.976	0.873	0.871	0.968	0.811	0.965	1.000	0.975	0.999
kmmt	0.413	-0.630	0.942	0.883	0.772	0.972	0.736	0.960	0.975	1.000	0.970
kr	0.480	-0.736	0.978	0.865	0.875	0.962	0.823	0.970	0.999	0.970	1.000

1/ Samile: 1970 - 2003

A striking difference is also evident in 1998 when direct measures still tend to show a positive gap whereas indirect measures (ddgap, icugap and nairugap) show negative or null magnitudes. In this year, the economy was falling into its deepest recession in decades but inflation did not recede, partly because of strong exchange rate depreciation. Multivariate methods do not fully account for this fact in spite of controlling by the exchange rate; however they performed better than univariate methods (hp and bp).

Surprisingly, 2003 gap size is similar for most of the methodologies used. In all cases but bp, the estimates give a negative value. And for all these estimators but kmmt and ddgap, the size is around $-3,0\%$. These similar results are obtained once the models are estimated with forecasted data. When models are estimated with observed data only, more significant differences in magnitude and in sign tend to show up.

Regarding potential or non-inflationary growth, all methodologies achieve very similar results. Between 1970 and 2003 the estimated GDP trend growth is close to 3.9% for seven out of the eight methods (**Table1**). This outcome seems robust to different assumptions on output growth from 2004 to 2010¹⁰ and it confirms what other works have found with different data (for example Miller 2003 for Peru). All models also show a similar degree of volatility for potential GDP growth. In all cases, standard deviation lies between 1.0% and 1.6%. Compared to other papers' findings, these numbers are small, which probably reflects the historically stable growth pattern of the Colombian economy. This stability, however, does not imply that potential GDP growth rate has been constant in time.

¹⁰ Other estimations of GDP trend with different growth rate forecasts using HP filter did not show substantial sensibility changes.

5. Empirical evaluation

In the context of monetary policy and IT, probably the best way to assess the performance of an output gap estimate is to establish how much it helps to explain inflation¹¹. With that purpose in mind, we have defined a model of inflation based on the specification found in the CB's core model (MMT) for the Phillips Curve. The equation to be tested corresponds to a Phillips Curve augmented by expectations. It also takes into account movements in the exchange rate and international prices. The equation is:

$$\pi_t^C = \alpha_0(\pi_{t-1}^M - \Delta z_t) + \alpha_1 \pi_{t-1}^{SA} + (1 - \alpha_0 - \alpha_1)\pi_t^E + \alpha_2 gap_t + \varepsilon_t \quad (25)$$

where π_t^C is CPI inflation excluding food prices (core inflation); π_t^M is imports inflation; Δz_t is long term real exchange rate depreciation; π_t^E is inflation expectations and gap_t is the GDP gap.

The equation was estimated with yearly data only. Given this, a maximum of one lag was considered for all variables. Preliminary tests showed the best fits when the model was estimated with the contemporary output gap. The original model, estimated on quarterly data, has only one quarter lag in this variable. As dependent variable we use an official measure of core inflation (non-food inflation). Thus, we can exclude one of the most frequent sources of instability and transitory shocks in prices for Colombian data.

Inflation forecasts obtained from equation (25) and based on ten of the output gap estimates were compared and confronted to the forecasts obtained from an altered model excluding the gap variable. The evaluation period spans from 1991 to 2003 and it was made using a standard simulated out-of-sample procedure. After estimating equation (25) for the period 1970 through 1990, a one-year-ahead forecast for inflation in 1991 is made. This forecast is compared to observed inflation, yielding a forecast

¹¹ Billmeier (2004) using data for Finland and other EC countries, and de Brouwer (1998) present a similar exercise but with a different model specification.

error. This procedure is repeated for every year until 2002, when the last inflation forecast can be compared to observed data. Thus, we can obtain a forecast error series for each output gap estimate and for the model version with no gap.

Table 3 compares the results for each output gap indicator using some well-known statistics such as the RMSE, MAE and Theil's U. Additionally, **Table 3** also displays a success ratio statistic (SR) that measures how well the model anticipates the sign of the acceleration of inflation (the higher the better) and a weighted average of all the statistics upon which a ranking of gaps is conducted. The results marginally favor the inclusion of almost any output gap measure as explanatory variables of Colombian inflation since 1990. The ranking function as well as each individual statistic gives better outcomes once most of the output gap estimators are included in the model. Among all alternative indicators, the best predictors of inflation are hpp, k and hp. In general, multivariate structural methods and univariate methods performed just as well.

Table 3
Output Gap Estimators - Colombia
Econometric evaluation using core inflation (CPI excluding food prices) in the Phillips Curve

	RMSE	RMSPE	MAE	MAPE	U-THEIL	SR	Weighted Average	Ranking
hp	0.0215	17%	0.0175	14%	0.649	62%	0.131	3
hpp	0.0210	16%	0.0169	12%	0.601	62%	0.113	1
mvhp	0.0227	17%	0.0187	14%	0.674	54%	0.157	7
bp	0.0220	17%	0.0172	14%	0.662	62%	0.137	6
k	0.0213	17%	0.0180	13%	0.659	62%	0.135	5
kmmt	0.0254	20%	0.0214	16%	0.779	46%	0.213	11
kr	0.0201	15%	0.0169	13%	0.578	54%	0.120	2
PnF	0.0216	17%	0.0184	14%	0.657	62%	0.135	4
icugap	0.0282	19%	0.0251	17%	0.713	62%	0.159	8
nairugap	0.0219	18%	0.0194	15%	0.691	46%	0.179	10
no gap	0.0222	20%	0.0170	14%	0.753	62%	0.171	9

The former results were obtained using non-food consumer inflation as the dependent variable in equation (25). However we know that those results could lead to misjudging the role of gap estimates, given the numerous shocks (both in prices and output) that inflicted the economy since 1990, both in prices and output. One of these shocks deals with the yearly adjustment in government-controlled prices (regulated prices as utilities, public transportation and gas prices). In the last decade, they have been one of the most important sources of headline inflation since they have risen faster than the rest of prices, due to the need of balancing fiscal accounts. In order to account for this shock,

we estimated the Phillips Curve using CPI inflation excluding food and regulated prices. From a practical perspective, we are introducing a new measure for core inflation.

Table 4 shows the results obtained from a new application of the simulated out-of-sample procedure already explained using the new price index. The ranking function improves substantially the role of output gap as an explanatory variable of inflation. Differences between the gap models and the no-gap version become more relevant for absolute statistics such as RMSE or MAE than in the previous exercise, mainly for the best estimators (kr, nairugap). Additionally, structural methodologies tend to outperform mechanical approaches. This is partially true for the production function approach whose performance fall short of expectations.

Table 4

Output Gap estimators - Colombia
Econometric evaluation using CPI inflation excluding food and controlled prices in the Ph. Curve

	RMSE	RMSPE	MAE	MAPE	U-THEIL	SR	Weighted Average	Ranking
hp	0.0211	20%	0.0157	14%	0.699	54%	0.168	8
hpp	0.0198	17%	0.0150	12%	0.584	54%	0.123	3
mvhp	0.0209	18%	0.0163	13%	0.641	54%	0.146	6
bp	0.0221	23%	0.0164	16%	0.783	54%	0.200	10
k	0.0195	17%	0.0154	12%	0.598	54%	0.129	4
kmmt	0.0226	20%	0.0183	15%	0.704	46%	0.186	9
kr	0.0169	14%	0.0132	11%	0.486	54%	0.085	1
PnF	0.0205	19%	0.0162	14%	0.655	46%	0.166	7
icugap	0.0261	19%	0.0223	16%	0.662	62%	0.141	5
nairugap	0.0204	18%	0.0171	15%	0.616	62%	0.121	2
no gap	0.0227	25%	0.0175	17%	0.873	62%	0.220	11

6. The output gap in 2003

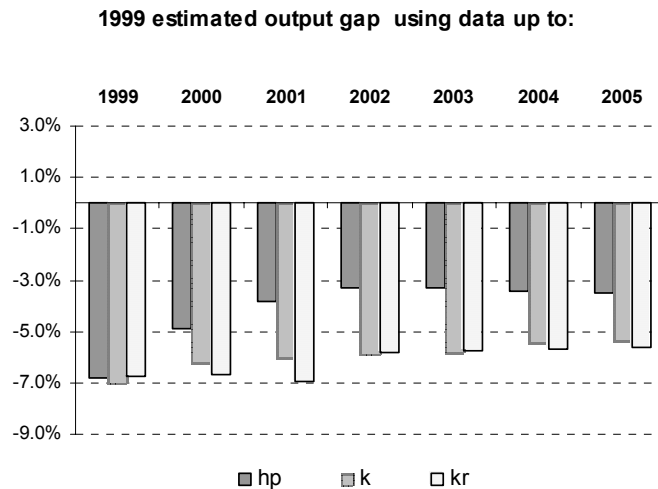
The former exercises show that it is possible to cautiously use several of the output gap measure to assess the state of demand and the economic cycle. In general, most of the gap estimators showed a similar profile and were related to inflation as theory anticipated. Most of the discrepancies among them have to do with scale rather than trend effects, confirming other authors' findings for different countries. Even under the presence of mismatches in sign or trend in a particular year, having more than two alternative measures helps to take a position on the gap situation.

However, assessing for the gap's profile and its relative position with respect to the whole series might not be enough. The monetary policy stance is determined by the output gap size and sign at a specific period and not only by its profile. This is

especially true if what guides the analysis is a Phillips Curve type of model like the one introduced above. A positive gap adds to inflation and might lead to a raise in interest rates. A negative gap might lead to an opposite reaction from the CB.

In order to diminish the error in output gap estimations for 2003, we propose alternative scenarios depending on potential GDP growth. We re-estimate the gap magnitude for each estimator, starting with the year 1999. We chose this year in particular because it is the deepest trough in all the series and it is already sufficiently far away in time. This last point is important because we want to avoid the end of sample problem, common to most gap estimates. We can expect the 1999 gap to remain unaltered with new incoming information. **Figure 6** illustrates how 1999 output gap stabilizes around -3.2% with information from 2002 onwards using the HP filter and around -5.5% using the Kalman filter approach (k and kr).

Figure 6



Assuming similar results for the rest of estimates, we use the 1999 gap as a starting value to estimate its 2003 magnitude under different assumptions for 2000 – 2003 potential growth. We can approximate the 2003 gap as a function of the 1999 gap using the next equation:

$$gap_{2003} = gap_{1999} + \Delta y - \Delta y^T \tag{26}$$

Where Δy is the average growth rate for observed GDP between 2000 and 2003 and Δy^T is the equivalent for trend or potential GDP. **Table 5** presents the 2003 gap assuming average potential GDP growth of 0%, 1%, 2% and 3%.

Table 5
Where was the Output Gap in 2003?

	Output Gap in 1999	Output Gap in 2003 if non inflationary growth was ^{1/} :				average non-inflat. growth: 2000 - 2003
		0.0%	1.0%	2.0%	3.0%	
hp	-3.2%	6.9%	2.9%	-1.1%	-5.1%	2.4%
hpp	-5.9%	4.2%	0.2%	-3.8%	-7.8%	1.5%
mvhp	-3.8%	6.3%	2.3%	-1.7%	-5.7%	2.5%
bp	-3.0%	7.1%	3.1%	-0.9%	-4.9%	1.7%
k	-5.4%	4.7%	0.7%	-3.3%	-7.3%	2.1%
kmmt	-2.8%	7.3%	3.3%	-0.7%	-4.7%	3.2%
kr	-5.7%	4.4%	0.4%	-3.6%	-7.6%	2.1%
PnF	-6.4%	3.7%	-0.3%	-4.3%	-8.3%	1.8%

^{1/} We refer to average growth between 2000 and 2003 for potential (or trend) GDP growth.

Assuming an average potential growth of 2% between 2000 and 2003, the output gap size was at least -1% in 2003. The eight estimates average is even higher (-2.4%). There is no big difference between structural or semi-structural approaches and pure statistical methods.

Does this mean we are free of uncertainty? No. Regarding non structural estimators, changes in the 2003 gap are unlikely with new future information. However we might be failing to estimate the real non-inflationary gap. The case is more complex for semi-structural or structural methods, because their gap estimations take into account, among others, the inflation output relationship. However, the 2003 gap might change in the future if there are structural changes in the economy that were not considered for the forecasts used to run the models.

7. Summary and conclusions

In this paper we have tried to evaluate alternative estimators of the output gap for Colombian data, using a wide range of methodologies. We emphasize in those that allow us to take into account the output-inflation relationship, such as the multivariate HP filter, the Kalman filter and the production function approach. In the later case, this task is achieved by adjusting labor and capital with NAIRU and NAICU measures. We

also introduce a prior-consistent filter, using a HP methodology that enables us to control directly for supply shocks (very common for Colombian data).

In general, there are no marked differences among the alternatives measures concerning their trends. However, there are important differences in the size and sign of the output gap, especially in years close to the turning points of the series. This might difficult the analysis and the decision making process for policy makers in such periods.

We tested the ability of the different estimators to forecast inflation using a Phillips Curve model. Almost all the estimators of the output gap used in this paper improved the accuracy of the inflation forecast. The evaluation exercise also showed that filters accounting for structural relationships as Kalman (k) and Kalman with modified core inflation (kr) outperform pure statistical approaches like the Band-Pass (bp) and Hodrick-Prescott (hp) filters. Among univariate methods, a HP filter controlling by supply shocks using priors (hpp) offered very good results. Similarly, redefining core inflation to exclude shocks in regulated prices also produced better results for all the filters, but in particular structural filters like Kalman filter (k, *nairugap* and *naicugap*).

At this point, a warning has to be made concerning estimations using the multivariate Kalman Filter approach (k, *kmmt*, *kr*). Their final results were obtained under stringent restrictions and showed high instability, and depending on the initial values. Its implementation was also time- consuming, which might limit future updates. On this matter, it should be pointed out that a less time-consume method as the prior-consistent HP filter (hpp) could produce similar results to the ones obtained under more complex methodologies. In addition, a redefinition of core inflation that excludes transitory movements in prices may help to attain better results without the allocation of additional resources.

Other less traditional indicators as those obtained from opinion polls (*ddgap*) and from spare capacity (*icugap*) were closely correlated to more traditional estimators and performed moderately well. Among them, the best performer – *nairugap* – is the broader indicator. The reduced economic scope of the other two – only covering manufacturing industries – limits its power to detect inflationary pressures.

The methodologies do not give a conclusive answer to the question about the size and sign of the output gap at a specific year. However, it is possible to achieve some consensus as we showed with the estimation of the gap for Colombia in 2003 to reduce the instability of estimations at the end of the sample. It greatly helps to define a forecasted path for the required variables (of at least four years in models run on yearly data). In our case, adding several years of forecast to the sample also seemed to reduce dispersion among estimators.

A case has to be made for the production function approach. Although its forecasting performance falls in the middle range of the evaluation, this is still a powerful and easy handled instrument to determine the evolution of the output gap since it is the only method that gives us the chance to keep track of productive factors and technological changes. The lower than expected performance attained may be related to poor data; but with a more intensive research that can be partially overcome. Research should point to improving the capital stock indicator, excluding durable unproductive goods (as housing, for example); or including imported intermediate goods in the production function as well as some indicator for human capital. Also, an intriguing phenomenon where research is urgently needed concerns the declining trend shown by total factor productivity in Colombia since 1980. The production function estimations are very sensitive to the assumptions made on the TPF trend.

In general, we believe that econometric methods, either univariate or multivariate, can help to put some numbers to the output gap estimation, but we cannot find a tool that may fully replace economic analysis. This conclusion is especially valid in the presence of shocks and when structural changes (of institutional nature for example) are not unusual in the mid-term. Finally, we do not rule out a combination of estimators as a pragmatic and inexpensive way of dealing with the estimation of the output gap for policy making purposes.

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APPENDIX

