Disinflation Costs Under Inflation Targeting in a Small Open Economy[*]

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Abstract

Since 1991, inflation in Colombia was reduced from 25% on average to about 6% more recently. Although this performance is in line with a long run inflation target of 3%, some analysts ask whether the Central Bank should continue disinflating. In this paper we present a dynamic stochastic general equilibrium model of inflation targeting for a small open economy to answer this question. We calibrate the model to the Colombian economy and compute the welfare costs and benefits of achieving the long run inflation target. We find that the long run welfare gains are about 4.54% in terms of capital. Furthermore, accounting for the transition the welfare gains are about 1.18% in terms of capital. Our results differ from previous findings because transition costs are introduced and our environment considers the presence of real rigidities (monopolistic competition) and nominal rigidities (sticky information) in a small open economy. We also analyze the sensitivity of the results to some key parameters and conclude that higher price flexibility leads to lower gains from reducing inflation and that a country with markups around 15% receives higher gains than those countries with different levels of markups. The weight given to the inflation gap in the monetary policy rule is important, as a more aggressive Central Bank can improve welfare. Finally, we find that disinflation is more expensive in the case of a closed economy.

JEL Codes: E31, E32, E52 and F41

Keywords: Small Open Economy; Inflation Targeting; Disinflation; Compensation; Colombia.

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1   Introduction

Colombia used to be recognized for having two digit inflations. Since 1977 up to 1991 our inflation rate oscillated between 40% and 15% with a mean value of about 24% (See figure 1). Throughout this period the Central Bank was part of the government. Since 1991, with the new political constitution, the Central Bank was declared independent from the central government, and the new task of the board of governors was to keep price stability. In the year 2000 the board fixed 3% as the long run inflation target. A disinflation process started and between 1991 and 1997 inflation fell from 30% to a level around 18% (Disinflation I on figure 1). Around 1998 Colombia suffered a sudden stop which brought down inflation from around 18% to 10%. After 1998 the board of governors of the Central Bank continued the disinflation process in a slow manner, as a result, we've had an inflation rate of approximately 6% for the last two or three years. This year's target is approximately 5.5%. Assuming that the target is reached, we are still missing a 2.5% drop in order to reach the long run target.

High inflation rates are costly and there exists a large amount of literature on the economic costs of inflation. There are different ways to quantify these costs, but we focus on the welfare costs of anticipated inflation. Although we expect agents to prefer a 3% inflation rate instead of a 5.5% rate, the transition between states may be costly too. In fact some argue that under certain circumstances, those costs overweight the benefits. For instance, Ball (1998,[2]), argues that economic authorities, face a tradeoff between output and inflation because disinflation reduces production and employment during the periods of transition towards a lower inflation. So, usually disinflation processes come together with recessions that can have permanent effects on employment ("hysteresis"). So, should the Central Bank continue to disinflate even when the transition is considered?

In this paper we use a dynamic, stochastic general equilibrium model of a small open economy with nominal rigidities, in a context of inflation targeting, to study the overall welfare costs and benefits of reducing inflation from 5.5% to 3%. We take into account the path of the economy from one state to another and compute the welfare costs and benefits in terms of compensations of capital and output.

This model distinguishes from previous literature in several aspects[1]. Here we present a small open economy and not a closed economy, this may affect the results because individuals can smooth

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[1]For Colombia, a first study using the Sidrauski (1967,[16]) model shows that the welfare loss from an increase in the inflation rate from 5% to 20% is about 7% of the GDP (Carrasquilla, Galindo and Patron 1994, [20]). Later on, a study which also uses Sidrauski but with a monetary economy, under the assumption of perfect foresight and endogenous production shows that the long run welfare gain for society in terms of consumption as a portion of output without taking into account the benefits or costs of the transition, of bringing down inflation from 20% to 10% are around 3.9% of GDP (Posada 1995,[21]). Another study based in Sidrauski and Lucas (1994,[15]) without capital accumulation, explores how much do Colombians loose in the long run in terms of welfare for tolerating a 20% inflation rate, it is found that the cost is approximately 1.5% of annual consumption in relation to the ideal situation of 0% inflation (Riascos 1997,[22]). De Gregorio (1998,[8]) using a ratio between the quantity of money and GDP, finds that a decrease of 10 percentage points in the inflation rate would increase output between 0.1% and 0.26%. Once more using a Sidrauski model in which preferences are non separable functions of the service flows of non-durable goods and money holdings, Lopez (2001,[14]) finds that the welfare loss due to an increase in the inflation from 5% to 20% is no higher than 2.3% of the GDP, and the welfare loss due to an increase in the inflation rate from 10% to 20% is equivalent to about 1% of the GDP. Finally using two models of the wage price system calibrated for the Colombian case, Gomez (2003,[7]), showed how wage rigidities translate into price rigidities and that price rigidities are in turn the key element explaining the costs of disinflation.

Figure 1: Inflation
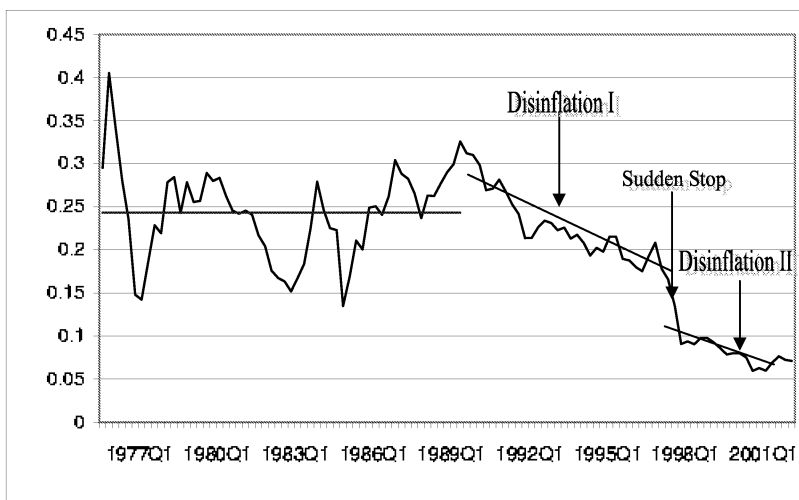
consumption by acquiring debt abroad so that effects of the nominal and real interest rates are not so harsh. We also have a specific monetary policy regime of inflation targeting while other studies are either independent of the monetary policy regime or assume a monetary growth rule (except for Gómez (2003, [7]) that also has inflation targeting). Nominal rigidities are introduced similarly to Calvo (1983, [5]) in order to obtain non-neutrality of money in the short run. A stock of habit was introduced into the utility function to obtain the observed persistence in consumption. Finally and as we mentioned above, our calculations take into account the transition from one state to the other evaluating not only the long run but also the short run costs of disinflation.

The rest of the paper proceeds as follows: in the next section we lay out the model, define the competitive equilibrium and explain the method of solution. Section 3 shows the calibration procedure. Section 4 refers to the extent to which the model replicates the salient features of Colombian data. In section 5 we calculate the benefits of an inflation rate of 3% versus 5.5%, the costs of the transition from one state to another and present a sensitivity analysis of the results to three different parameters and the case of a closed economy. The last section summarizes our findings.

2   The Model

We consider a small open economy with a representative household, two types of firms and a government. The first type of firms hire labor and capital from households and produce an homogeneous good. The second type of firms buy the homogeneous good, put a label at no cost, and end up with a differentiated good[2]. From now on we will refer to the first type of firms as "producers" and to the second as "retailers". Households consume differentiated consumption goods and pay a

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[2]One way to think about the second type of firms is as "branding" firms. They buy wheat, pack it and put a label on it. This is just a device to introduce price-stickiness into the model, See Schmitt-Grohe and Uribe (2004,[23]). This type of setup is not new in the literature, to our knowledge it was first implemented by Bernanke, Gertler and Gilchrist (1999, [4]).

liquidity cost, they also supply homogeneous indivisible labor, accumulate capital and supply it to producers. They receive lump sum transfers from the government and hold wealth as cash. Producers hire labor and capital from households as factor inputs and produce homogeneous goods. These homogeneous goods are demanded by retailers, which transform homogeneous goods into differentiated consumption goods and sell these to households. The consolidated monetary and fiscal authority issues money, makes net lump sum transfers to households, makes some unproductive expenditure and collects the liquidity costs from households[3]. All quantities are in per-capita terms if not stated otherwise[4].

2.1 The Representative Household

Households are the owners of the firms that produce the homogeneous good as well as of the retail sector firms and are consumers. Their income at period t is given by the nominal wage, nominal returns to capital, the benefits from retailers and the net lump sum transfers obtained from the government in this same period. Apart from their income they also count with a real money stock given at the beginning of the period as well as with a stock of real domestic private bonds and foreign assets[5]. Expenditure is determined by consumption, the liquidity costs and investment. At period t, they also decide the level of expected real money holdings, real domestic private bond holdings and foreign asset holdings for period $t+1$. Then the budget constraint is given by:

$$
\begin{aligned}
c_t + \Phi + m_{t+1}^d + \frac{P_t x_t}{P_t^c} + b_{t+1} + \frac{e_t F_{t+1}}{P_t^c} = \quad & \frac{W_t}{P_t^c} h_t^s + \frac{R_t}{P_t^c} k_t^s + \frac{\Pi_t^R}{P_t^c} + \frac{\Pi_t}{P_t^c} + \\
& m_t^d \frac{P_{t-1}^c}{P_t^c} + b_t \frac{P_{t-1}^c}{P_t^c}(1 + i_t) + \frac{e_t F_t}{P_t^c}(1 + i_t^f) + \tau_t \quad (1)
\end{aligned}
$$

where: c_t is real consumption, m_t^d is real money demand, x_t is real investment, W_t is the nominal wage, h_t^s is the number of hours worked per-capita, R_t is the nominal return to capital, k^s is capital supply, Π_t are the benefits from the homogeneous good producers, Π_t^R are the benefits from the retailers, τ_t are government lump sum transfers to the households, P_t^c is the price index of consumption goods and P_t is the price index of homogeneous goods, b_t are net real private domestic bonds, F_t are net foreign assets (or liabilities depending on the sign) denominated in units of the tradable homogeneous good, e_t is the nominal exchange rate (COP/USD), i_t is the domestic nominal interest rate and i_t^f is the foreign nominal interest rate denominated in dollars. M_0, k_0, b_0 and F_0 are known. As $m_t = \frac{M_t}{P_{t-1}^c}$, hence m_0 is known and the same follows for b_0. Φ is a function which determines the transaction costs, and is given by

$$
\Phi\left(c_t, m_{t+1}, x_t\right) = \kappa \left(\frac{c_t + \nu \frac{P_t}{P_t^c} x_t}{m_{t+1}} \right)^a \quad (2)
$$

where all variables are in real terms (relative to the consumption good) and ν is a parameter that determines the fraction of investment that affects the optimal choice of real money holdings.

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[3]By doing so we intend to eliminate the wealth effect.

[4]This model is not based on a previous model, it is the combination of the features of several models made by different authors and each one is mentioned when pertinent.

[5]Stock variables are given at the beginning of the period and flows are known at the end, i.e. M_t is known at the start of period t, P_{t-1} is given at the end of period $t-1$ so it's known at the beginning of period t, as $m_t = \frac{M_t}{P_{t-1}}$, real money holdings are known at the start of period t.

According to this expression, as the household consumes or invests more, its liquidity costs increase, and they decrease with the real money holdings they save for next period.

The external nominal interest rate is defined as

$$\left(1 + i_t^f\right) = (1 + i_t^*)\left(1 + \vartheta\left(\frac{F_t}{y_t}\right)\right) \tag{3}$$

where i_t^* is the international risk free nominal interest rate and ϑ is the risk premium function[6]. Notice that if the net foreign assets (F_t) are negative, then the country is a net debtor and otherwise it is a net lender. It is also assumed that the purchase power parity (PPP) is satisfied, so $P_t = e_t P_t^*$. This means that the price for the homogeneous good equals the foreign price for the homogeneous good times the exchange rate. We set $P_t^* = 1$ for all t, therefore $P_t = e_t$ and so the depreciation rate equals the inflation rate of homogeneous goods, $\pi_t = d_t$. If we define $q_t = \frac{P_t}{P_t^c}$ as the relative price of homogeneous goods to heterogeneous goods, and $\frac{P_{t-1}^c}{P_t^c} = \frac{1}{(1+\pi_t^c)}$, then the budget constraint (1) can be rewritten as

$$
\begin{aligned}
c_t + \Phi + m_{t+1}^d + q_t x_t + b_{t+1} + q_t F_{t+1} &= \frac{W_t}{P_t^c}h_t^s + \frac{R_t}{P_t^c}k_t^s + \frac{\Pi_t^R}{P_t^c} + \frac{\Pi_t}{P_t^c} + \\
&\quad \frac{m_t^d}{(1+\pi_t^c)} + \frac{b_t}{(1+\pi_t^c)}(1+i_t) + F_t q_t(1 + i_t^f) + \tau_t
\end{aligned} \tag{4}
$$

Households accumulate capital according to the following expression:

$$k_{t+1} - (1-\delta)k_t - f\left(\frac{x_t}{k_t}\right)k_t = 0 \tag{5}$$

where f is a twice continuously differentiable and concave function, which reflects investment adjustment costs in capital, and δ is the depreciation rate. The specification of the function f, is such that when the economy is in steady state, there are no adjustment costs[7].

Consumption and leisure generate utility to households, but they have a habit stock which generates dis-utility, this is

$$u(c_t, H_t, h_t, \mu_t^u) = \mu_t^u \log(c_t) - \gamma \log(H_t) - Bh_t \tag{6}$$

where H_t is the habit stock, B is a parameter, μ_t^u is an exogenous variable that represents an intertemporal preference shock[8], and

$$c_t = \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \tag{7}$$

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[6] The risk premium function is defined as $\vartheta\left(\frac{F_t}{y_t}\right) = \omega_{ss} + \omega_1 + \omega_2 * Exp\left[\omega_3\left(\frac{\frac{F_t}{y_t}}{\frac{F_{ss}}{y_{ss}}}\right) * \mu_t^\vartheta\right]$ where the subscript ss stands for the steady state value of the variable, $\vartheta'\left(\frac{F_t}{y_t}\right) < 0$ and μ_t^ϑ is an exogenous variable which logarithm follows a standard autoregressive process of order one of the form $log(\mu_{t+1}^\vartheta) = \rho_4 log(\mu_t^\vartheta) + (1-\rho_4)log\left(\overline{\mu^\vartheta}\right) + \epsilon_{t+1}$.

[7] We assume that f is a quadratic function $f\left(\frac{x_t}{k_t}\right) = c_2\left(\frac{x_t}{k_t}\right)^2 + c_1\left(\frac{x_t}{k_t}\right) + c_0$. c_2 determines the concavity of the function, that is, how expensive it is on the margin to adjust the capital outside the steady state and is fixed in order to replicate investment's volatility. Parameters c_1 and c_2 are determined by the fact that there are no adjustment costs on the steady state.

[8] The log of this exogenous variable follows a standard autoregressive process of order one, $log(\mu_{t+1}^u) = \rho_3 log(\mu_t^u) + (1-\rho_3)log\left(\overline{\mu^u}\right) + \epsilon_{t+1}$.

where $c(z)$ is the consumption of a specific good z coming from the retailer z, and θ is the elasticity of consumption of each good z with respect to the whole bundle[9].

The functional form of the utility function deserves some explanation. First, the linear specification of utility involving h follows Hansen (1985, [10]) where labor is indivisible. Workers can either work some given number of hours or not at all (i.e. they can't work part time). Second, the utility function is separable in consumption and leisure. Third, agents trade employment lotteries instead of hours of work. This implies that hours worked are proportional to employment[10].

On the other hand, H represents the consumption habits of each individual:

$$H_{t+1} - H_t - \rho(c_t - H_t) = 0 \qquad (8)$$

where H_0 is given. Consumption habit today depends on last period's consumption and habit[11]. The higher habit is, the more dis-utility it is going to generate. In the present period the individual is going to have to consume more to be as satisfied as last period[12].

Then the representative household's dynamic problem is

$$\max_{\{c,h,x,k,m,H\}} E_t \left(\sum_{t=0}^{\infty} \beta^t u\left(c_t, H_t, h_t, \mu_t^u\right) \right)$$

subject to (2), (3), (4), (5), (7), (8) and the two following transversality conditions[13]

$$Lim_{t\to\infty} \beta^t \gamma_t = 0$$

$$Lim_{t\to\infty} \frac{\beta^t \lambda_t}{(1 + i_t^f)} \gamma_t = 0$$

According to this, the first order conditions of the household's problem are the following[14]:

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[9] The utility function was chosen log-linear for simplicity. A sensitivity analysis could be done with respect to the utility function but this is not our purpose. The implication that the value of the coefficient of risk aversion has on the model is the willingness of agents to smooth consumption, the higher the value of this parameter the greater the desire of agents to smooth consumption. This will therefore have effects on investment and on the current account in the case of an open economy.

[10] Each period instead of choosing manhours households choose a probability of working α. The new commodity being introduced is a contract between the firm and the household that commits to work h_0 hours with a probability α. The contract is what is being traded, so the household gets paid wether it works or not. Since households are identical all are going to choose the same α. So all households are going to offer αh_0 which is a fixed quantity. As the utility function is linear in leisure it implies an infinite elasticity of substitution between leisure in different periods. This follows no matter how small this elasticity is for the individuals in the economy. Therefore the elasticity of substitution between leisure in different periods for the aggregate economy is infinite and independent of the willingness of the individuals to substitute leisure across time.

If α increases then it means that people are willing to work more. That is, that a higher portion of people are working. Therefore the sum of hours worked is higher and with the same population (assuming there is no population growth) the number of hours worked per-capita is going to be higher.

[11] Commonly known as inward looking habit.

[12] This friction is introduced in order to obtain the persistence in consumption which is observed in the data.

[13] In the solution method the two transversality conditions are replaced by stability conditions.

[14] The first order condition concerning consumption is done with respect to c_t and not $c(z)_t$, because it was proven to be the same. See appendix 6 for demonstration.

$$u_{c_t}\left(c_t, H_t, h_t, \mu_t^u\right) + \eta_t \rho = \lambda_t \left(1 + \Phi_{c_t}\left(c_t, m_{t+1}, x_t\right)\right) \tag{9}$$

$$u_{h_t}\left(c_t, H_t, h_t, \mu_t^u\right) + \lambda_t \frac{W_t}{P_t^c} = 0 \tag{10}$$

$$\lambda_t \left(\Phi_{x_t}\left(c_t, m_{t+1}, x_t\right) + q_t\right) = \gamma_t f_{x_t}\left(\frac{x_t}{k_t}\right) k_t \tag{11}$$

$$\beta E_t \left(\lambda_{t+1} \frac{R_{t+1}}{P_{t+1}^c} + \gamma_{t+1}\left(f\left(\frac{x_{t+1}}{k_{t+1}}\right) + \frac{\partial\left(f\left(\frac{x_{t+1}}{k_{t+1}}\right) k_{t+1}\right)}{\partial\left(k_{t+1}\right)}\right) + \gamma_{t+1}(1-\delta)\right) = \gamma_t \tag{12}$$

$$\beta E_t \left(\frac{\lambda_{t+1}}{\left(1 + \pi_{t+1}^c\right)\left(1 + \Phi_{m_{t+1}}\left(c_t, m_{t+1}, x_t\right)\right)}\right) = \lambda_t \tag{13}$$

$$\beta E_t \left(\frac{\lambda_{t+1}\left(1 + i_{t+1}\right)}{\left(1 + \pi_{t+1}^c\right)}\right) = \lambda_t \tag{14}$$

$$\beta E_t \left(\lambda_{t+1}\left(1 + i_{t+1}^f\right) q_{t+1}\right) = \lambda_t q_t \tag{15}$$

$$\beta E_t \left(\eta_{t+1} + U_{H_{t+1}}\left(c_{t+1}, H_{t+1}, h_{t+1}, \mu_{t+1}^u\right) - \eta_{t+1}\rho\right) = \eta_t \tag{16}$$

and equations (4), (5) and (8). Where λ, γ and η are the lagrange multipliers associated with the budget constraint, the evolution of capital and the evolution of the stock of habit, respectively.

2.2 The Producers

This sector is competitive and the producers seek to maximize their profits by choosing the level of capital and labor, given the rental rate of capital, the nominal wage and a technology to produce output, which is sold at price P_t. The technology is assumed to be a standard Cobb-Douglas production function. Hence the problem faced by producers is to solve

$$\max_{\{k,h\}} \Pi_t = P_t A_t (k_t^d)^\alpha (h_t^d)^{(1-\alpha)} - R_t k_t^d - W_t h_t^d \tag{17}$$

where A_t is the level of productivity, the subscript d represents the specific input's demand and $log(A_t)$ will follow a standard autoregressive process of order one[15]. The first order conditions for the producers of the homogeneous good are the standard ones.

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[15]$log(A_{t+1}) = \rho_1 log(A_t) + (1 - \rho_1)log(\overline{A}) + \epsilon_{t+1}$ where $\overline{A}$ represents the average value taken by A across time.

2.3 The Retailers

The retailers, purchase homogeneous output from producers at a price P_t, and turn it into their specific brand of consumption good at zero additional cost. However, on each period retailers face a constant probability, $1 - \varepsilon$, of receiving a signal, that tells them that they can re-optimize their price, this probability behaves as in Calvo (1983, [5]). The other ε retailers follow a backward indexation rule, see Christiano, Eichenbaum, Evans (2001, [13])[16]. This probability is independent across firms and time. We assume that if a retailer doesn't receive the signal, it fixes his price according to[17]:

$$p_t^{rule}(z) = p_{t-1}^c(z)(1 + \pi_{t-1}^c) \tag{18}$$

where p_{t-1}^c is retailer's last periods price and π_{t-1}^c is the period $t-1$ rate of inflation of the aggregate consumption price index.

With probability $1 - \varepsilon$ a retailer is going to optimize and set p_t^{opt}. If this is the case the retailer's problem is the following:

Each retailer[18] (z) expected profits at period $t + j$ are given by:

$$E_t\left(\Pi^R(z)_{t+j}\right) = E_t\left(c(z)_{t+j}(p^c(z)_{t+j} - P_{t+j})\right) \tag{19}$$

The real profits of each retailer are $\Pi^R(z)_{t+j}/P_{t+j}^c$ so those firms who are allowed to adjust their price in period t will choose $p^c(z)_{t+j}$ to:

$$\max_{\{p^c(z)_t\}} E_t \sum_{j=0}^{\infty} (1 - \varepsilon)\,\varepsilon^j \Delta_{t+j} \frac{\Pi^R(z)_{t+j}}{P_{t+j}^c}$$

where the discount factor $\Delta_{t+j} = \beta^j \frac{u'(C_{t+j}, h_{t+j}, H_{t+j})}{u'(C_t, h_t, H_t)}$ is an appropriate discount factor according to the market's real interest rate, and households take it as given for their maximization problem. Notice that in period t the firm chooses a price from now on, $p^c(z)_{t+j} = p^c(z)_t$ because of the uncertainty on future price changes, in other words, the firm does the maximization taking into account that today they can re-optimize prices (with probability $(1 - \varepsilon)$) and that for j periods they are not going to re-optimize them (with probability ε^j).

From the households problem it can be shown (see appendix 1) that the demand for the consumption good $c(z)_t$ is:

$$c(z)_{t+j} = \left(\frac{p^c(z)_{t+j}}{P_{t+j}^c}\right)^{-\theta} c_{t+j} \tag{20}$$

so the maximization problem ends up being:

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[16]This indexation rule makes it possible for the model to have inflation different from zero. It also implies that in the steady state prices are going to have zero dispersion, i.e. the price that follows the backward indexation rule is equal to the optimal price. Other pricing rules are $p_t^{rule}(z) = p_{t-1}^c(z)$ or $p_t^{rule}(z) = p_{t-1}^c(z)(1 + \bar{\pi})$ where $\bar{\pi}$ is the long run inflation. These rules are studied by Dotsey, King and Wolman (1999, [6]).

[17]One way to interpret this pricing rule is to assume that on each period retailers face a constant probability $1 - \varepsilon$, of wanting to gather information about the state of the economy in order to re-optimize their price (see Mankiw and Reis, 2002, [17]). So those $1 - \varepsilon$ who gather the information, re-optimize their price according to it. In contrast the other ε retailers follow a backward indexation rule, they keep changing their prices according to past information. So in a sense this is not exactly a case of sticky prices, because as one can see everyone is changing prices but not re-optimizing. This is more a case of sticky information.

[18]Retailers are indexed by z.

$$\max_{\{p^c(z)_t\}} E_t \sum_{j=0}^{\infty} (1-\varepsilon)\varepsilon^j \Delta_{t+j} c_{t+j} \left[\left(\frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{1-\theta} - q_{t+j} \left(\frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{-\theta} \right]$$

After solving for $p^c(z)_t$, the solution becomes (see appendix 2 for derivation):

$$\frac{p_t^{opt}}{P_t^c} = \frac{\theta}{\theta-1} E_t \left[\frac{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} q_{t+j} \left(\frac{P_{t+j}^c}{P_t^c} \right)^{\theta}}{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \left(\frac{P_{t+j}^c}{P_t^c} \right)^{\theta-1}} \right] \tag{21}$$

or what is the same

$$\frac{p_t^{opt}}{P_t^c} = \frac{\theta}{\theta-1} E_t \left(\frac{\Theta_t}{\Psi_t} \right)$$

where

$$\Theta_t = \Delta_t c_t q_t + \varepsilon E_t \left((1+\pi_{t+1}^c)^{\theta} \Theta_{t+1} \right)$$

$$\Psi_t = \Delta_t c_t + \varepsilon E_t \left((1+\pi_{t+1}^c)^{\theta-1} \Psi_{t+1} \right)$$

and p_t^{opt} denotes the price of the good $c(z)_t$ set by the retailer z in the case in which he decides to optimize. Since (20) implies that the price index is also a CES aggregator, it can also be shown that the price index P_t^c is given by[19]

$$P_t^c = \left[\varepsilon(p_t^{rule})^{1-\theta} + (1-\varepsilon)(p_t^{opt})^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{22}$$

and then the aggregate inflation dynamics is given by

$$(1+\pi_t^c) = \left(\varepsilon(1+\pi_{t-1}^c)^{(1-\theta)} + (1-\varepsilon) \left(\frac{p_t^{opt}}{P_t^c} \right)^{(1-\theta)} (1+\pi_t^c)^{(1-\theta)} \right)^{\frac{1}{1-\theta}} \tag{23}$$

2.4   Consolidated Monetary and Fiscal Authority

On each period t, the government issues money, transfers a net lump sum to households and makes unproductive expenditures. It is also assumed that the government collects the liquidity costs payed by households. Seigniorage as well as the liquidity costs represent income for the government so their budget constraint is the following:

$$m_{t+1}^s - \frac{m_t^s}{1+\pi_t^c} + \Phi\left(c_t, m_{t+1}, I_t\right) = \tau_t + \left(\frac{g_t}{y_t} \right) y_t \tag{24}$$

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[19]As we know the consumption index is $c_t = \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$ which implies that the demand for the z-th good is $c(z)_{t+j} = \left(\frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{-\theta} c_{t+j}$, where P_{t+j}^c is an index of the cost of buying a unit of $c(z)_t$: $P_t^c = \left[\int_0^1 (p_t^c(z))^{1-\theta} dz \right]^{\frac{1}{1-\theta}}$. This integral can be divided into two. So, retailers can be separated into two groups, a fraction $(1-\varepsilon)$ that optimizes their price, and a fraction ε that doesn't.

where the letters with subscript s represent a supply, and g_t is real government expenditure. $log\left(\frac{g_t}{y_t}\right)$ follows a standard autoregressive process of order one[20] .

It is also assumed that monetary policy is conducted with an interest rate policy rule, of the form:

$$i_t = i + \zeta\left(\pi_t^c - \overline{\pi}^c\right) + \xi\left(y_t - y^{ss}\right) \tag{25}$$

where i is the steady state nominal interest rate level, $\overline{\pi}^c$ is the inflation target[21], and y^{ss} corresponds to the steady state level of output (this is the level of output in absence of shocks)[22]. y_t is determined by the production technology described in the last subsection. ζ and ξ are parameters that determine the importance that the monetary authority gives to inflation and output respectively when using the nominal interest rate as the policy instrument.

2.5   Competitive Equilibrium

To characterize the competitive equilibrium, the following definitions are used:

Definition: A price system is a positive sequence $\{W_t, R_t, p_t^{rule}, p_t^{opt}, P_t^c, P_t, e_t, i_t, i_t^f\}_{t=0}^{\infty}$.

Definition: $\{A_t, \mu_t^{\vartheta}, \mu_t^u, \frac{g_t}{y_t}, P_t^*\}_{t=0}^{\infty}$ are taken as exogenous sequences. m_0, k_0, b_0, F_0, $H_0 > 0$ are also taken as given. An equilibrium is a price system, a sequence of consumption $\{c_t\}_{t=0}^{\infty}$, investment $\{x_t\}_{t=0}^{\infty}$, capital $\{k_t\}_{t=1}^{\infty}$, number of hours worked per-capita $\{h_t\}_{t=0}^{\infty}$, habit stock $\{H_t\}_{t=1}^{\infty}$, domestic real private bonds $\{b_t\}_{t=1}^{\infty}$, net foreign assets $\{F_t\}_{t=1}^{\infty}$ and a positive sequence of real money $\{m_t\}_{t=1}^{\infty}$ in order that:

1. Given the price system and net lump sum transfers, household's optimal control problem is solved with $\{m_t^d = m_t^s = m_t\}_{t=1}^{\infty}$, $\{k_t^d = k_t^s = k_t\}_{t=1}^{\infty}$, $\{b_t = 0\}_{t=1}^{\infty}$, $\{h_t^d = h_t^s = h_t\}_{t=0}^{\infty}$, $\{c_t\}_{t=0}^{\infty}$ and a level of $\{F_t\}_{t=1}^{\infty}$ such that $(1 + i_t) = \left(1 + i_t^f\right)(1 + d_t)$ is satisfied.

2. The government's budget constraint (24) and policy rule (25) are satisfied for all $t \geq 0$.

3. $Y_t = C_t + I_t + G_t + F_{t+1} - \left(1 + i_t^f\right)F_t$ for all t.

This last condition is the standard resource restriction in a small open economy (See appendix 5 for derivation).

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[20]$log\left(\frac{g_{t+1}}{y_{t+1}}\right) = \rho_2 log\left(\frac{g_t}{y_t}\right) + (1 - \rho_2)log\left(\frac{\overline{g}}{\overline{y}}\right) + \epsilon_{t+1}$ where $\left(\frac{\overline{g}}{\overline{y}}\right)$ represents the average value taken by $\frac{g}{y}$ across time.

[21]Notice that this target is in terms of the inflation of the prices of heterogeneous goods.

[22]It's not the level of output in the absence of frictions because transaction costs are still present in the steady state.

2.6 Solving the Model

In order to solve the model, we first state the first order nonlinear dynamic system that characterizes the competitive equilibrium. In order to calculate the steady state we transform the system equations into their deterministic steady state representation and solve using numerical methods. Then we log-linearize around the deterministic steady state. At this stage the system is expressed in terms of relative deviations from the steady state.

After solving the model using the method of King, Plosser and Rebelo (2001,[12]) we obtain matrices $\mathbf{M}$ and $\mathbf{H}$ which generate the dynamic solution by iterating on the following two equations:

$$\mathbf{Y}_t = \mathbf{H}\mathbf{x}_t \qquad (26)$$
$$\mathbf{x}_{t+1} = \mathbf{M}\mathbf{x}_t + \mathbf{R}\eta_{t+1}$$

where $\mathbf{Y}$ is a vector composed by control, co-state and flow variables, $\mathbf{x}$ is a vector of endogenous and exogenous states, $\mathbf{H}$ characterizes the policy function and $\mathbf{M}$ the state transition matrix. η_{t+1} is an innovation vector and $\mathbf{R}$ is a matrix composed of zeros, ones or a parameter instead of a one. This matrix determines which variables are hit by the shock and in what magnitude.

3 Calibration

We now proceed to calibrate the model. There are some parameters that are uncontroversial, while others deserve some explanation. Parameter B is calibrated to obtain $h = \frac{1}{3}$ in steady-state. The capital share within the production function is set at $\alpha = \frac{1}{3}$ which approximately corresponds to the capital share in income. The capital stock time series in Colombia is a constructed one, which assumes a quarterly depreciation rate of 0.012, so we set $\delta = 0.012$. The parameter θ that determines the degree of competition in the differentiated goods market, is set to 5 in order to obtain a markup of 25% according to the most recent research on market structure available in Colombia[23]. The parameter ε that determines the degree of price stickiness is set to 0.75 in order to have prices changing every one year, this was the estimation obtained by Bejarano (2004, [3]) for Colombia. β, which in equilibrium is equal to $\frac{1}{1+r}$ is fixed at 0.984 according to Vasquez (2003,[24]) who estimated the annual long term interest rate for Colombia in 6.81% which corresponds to 1.6% quarterly. The inflation target $\bar{\pi}$ is fixed at 5.5% (annual rate) according to the target set for this year by the Central Bank. i was fixed according to $\bar{\pi}$ and r. We set the international interest rate $i_t^* = 0.03$. The parameters ζ corresponding to the weight given by the monetary authority to the inflation was in 1.7 according to Melo and Riascos (2004,[18]), although they estimated the rule with a lag on the interest rate and the parameter ξ was fixed in 0^{24}.

The parameter ω_{ss} of the risk premium function was calibrated according to the spread between i and i_t^*. We calibrate the rest of parameters of the risk premium function, ϑ, to match the long term total external debt to GDP ratio, which for Colombia is about 30%.

Investment adjustment costs where calibrated so that in the steady state there are no adjustment costs, $f\left(\frac{x}{k}\right) = c_2\left(\frac{x}{k}\right)^2 + c_1\left(\frac{x}{k}\right) + c_0 = \left(\frac{x}{k}\right)$ and $f'\left(\frac{x}{k}\right) = 1$. For a given c_2, this two conditions determine c_1 and c_0. So, c_2 is fixed to replicate investment's volatility which according to the Hodrick and Prescott filter is 18.8% for Colombia.

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[23]See Arango et al. (1991,[1])

[24]The model is very unstable for values different from zero in this parameter.

Since there is no information about the parameters that determine the evolution of habit over time, we calibrate them to replicate some stochastic properties of the consumption time series in Colombia: φ is set to replicate its volatility as close as possible (which is of 1.4% for Colombia according to data filtered with Hodrick and Prescott) and ρ is fixed to obtain the observed persistence of consumption's cyclical component.

We pay special attention to the parameter a in the transaction cost function, which determines the elasticity of the quantity of money demanded to consumption and interest rate. The first order conditions of the model allow us to obtain an approximation to the money demand of this economy. So we decided to estimate the values of a and κ. Using equations (14) and (13) we solve deterministically for m_{t+1} and obtain:

$$m_{t+1}^{1+a} = \frac{a\kappa(c_t + \nu q_t x_t)^a}{\frac{i_{t+1}}{1+i_{t+1}}} \tag{27}$$

Applying logs to equation (27) we obtain:

$$log(m_{t+1}) = \frac{1}{1+a}log(a\kappa) + \frac{a}{1+a}log(c_t + \nu q_t x_t) - \frac{1}{1+a}log(i_{t+1}) + \frac{1}{1+a}log(1+i_{t+1})$$

and we estimate it in order to solve for the coefficients a, κ and ν. We used non-linear ordinary least squares with the following three restrictions: $a > 1$, $0 < \nu < 1$ and $\kappa > 0.0645$. The restriction on a is to avoid the case of a linear function, the one on ν is straight forward and in principle κ should be $\kappa > 0$ but 0.0645 is the minimum value for which we were able to obtain the solution. What we found was a corner solution on κ, so our results were $a = 1.858$, $\kappa = 0.0645$ and $\nu = 0.025$. M1 was used for m, for q which can be defined as the real exchange rate (recall that in the model $q = \frac{e_t}{P_t^c} = \frac{P_t}{P_t^c}$) we used the spot's market nominal exchange rate times the U.S. core CPI (CPI minus food and energy) divided by Colombia's CPI, and for i we used the CD's 90 days interest rate.

We finally describe the parameters related to the exogenous shocks. We focus only on the productivity shock since it is the only one used in our simulations. For the productivity shock, A, we performed a standard Solow residual computation to obtain an autocorrelation coefficient of $\rho_1 = 0.83$. The standard deviation is calibrated to reproduce as closely as possible the observed output's volatility (using a Hodrick and Prescott filter it is 1.62%)[25]. Finally, the standard deviation of the forcing variable A is set to reproduce as closely as possible the observed output's volatility which was found to be 1.62% according to Hodrick and Prescott's filter.

The autocorrelation of the remaining shocks, government expenditures, preferences and risk premium were found to be 0.773, 0.8 and 0.69 respectively[26]. As we mentioned above this parameters are not considered in our simulation exercise.

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[25]Using labor, capital and product quarterly data from 1984:1 until 2003:4, and expressing the production function in logarithms one can solve for $log(A_t)$ in order to obtain a time series for A. From this new data we found an average value $\overline{A} = 1.19$ (in levels). The parameter ρ_1 was found by running the following regression $log(A_t) = \rho_1 log(A_{t-1}) + (1 - \rho_1)log(\overline{A}) + \epsilon_t$ where ϵ is an error term. We performed a Wald's test to prove the null hypothesis $\rho_1 + (1 - \rho_1) = 1$ and we obtained a F-statistic value of 0.2156 and a P-value of 0.6437, so our null hypothesis is accepted, and A can actually follow a standard autoregressive process of order one as stated before.

[26]The autocorrelation ρ_2 of the variable $\frac{g_t}{y_t}$ is found by doing the following: we take the ratio between real total government expenditure and real GDP, we calculate the mean of this series and find $\frac{\overline{g}}{y} = 0.15$, then we estimate an autoregressive process and find $\rho_2 = 0.773$ and that the standard deviation of the error is 0.0063.

4 Validating the Model

The validation of the model can be found in detail in Hamann, Julio, Restrepo and Riascos (2004, [9]), where in order to asses the extent to which the calibrated model replicates salient and/or interesting features of the actual economy, they follow a frequency domain methodology proposed by Diebold et al. (1998,[19]).

The salient features of the data that the model has to mimic are the following: inflation and output gap are dominated by periodic movements between 2 and 25 quarters with a peak between 10 and 12 quarters; the cross spectrum and coherence show results in the same direction and the population coherence does not seem to be dominated by a particular set of frequencies.

The theoretical model frequency analysis shows some persistence both in the univariate spectra as well as in the cross spectrum, with monotone spectrum for output gap and cross spectrum. The inflation spectrum peaks at a frequency of $0,10\pi$, that is, for periodic movements between 9 and 10 quarters. The model's theoretical coherence presents clear dominance in frequencies between $0,05\pi$ and $0,45\pi$, that is periodic movements between 2 and 20 quarters, with a maximum coherence at $0,12\pi$, that is periodic movements between 8 and 9 quarters.

The results are that the comparison between sample and theoretical spectra and cross spectra reveal important similarities. The theoretical spectra and cross spectra fall into the sample uncertainty bands for frequencies beyond $0,05\pi$, that is for periodic movements of inflation and comovements of inflation and output gap of up to 20 quarters, that is 5 years, and for periodic movements of output gap of up to 10 quarters (2 and a half years). For shorter frequencies the spectra and cross spectra of the model are significantly different from the sample ones. The model's coherence falls into the uncertainty bands for most of the frequencies but the ones surrounding the peak of the model's coherence, and very long run periodic movements.

They conclude that the model's theoretical spectra and cross spectra do not differ statistically from the respective population quantities for, at least, frequencies beyond $0,05\pi$, which correspond to periodic movements of up to at least 10 quarters. Population's coherence is not statistically different from the model's coherence at most of the frequencies, it is only statistically different at the peak of the model's theoretical coherence and for very short frequencies (very long run period movements).

From these conclusions we can see that the model captures the relevant cyclical movements of inflation and output, so we are comfortable that the results obtained for the benefits or costs of inflation are going to capture that desired information.

For the preference shock we take the consumer sentiment survey made by "Fedesarrollo" and specifically use the consumer confidence index. We assume that by construction the index has media zero, this is because consumers are asked if they feel positive or negative about something and the negative answers are subtracted from the positive ones, so in steady state opinions should be divided in half. As the media of the process was assumed to be zero, then we run the regression of the autoregressive process without intercept and we find the autocorrelation ρ_3 of the variable μ_t^u to be $\rho_3 = 0.8$ and the standard deviation of the error 0.07.

The autocorrelation ρ_4 of the variable μ_t^ϑ is found by doing the the following: a daily series of the EMBI was used as a proxy of the variable μ_t^ϑ. As our model is quarterly then we find the quarterly geometric average of the series. We know that we are assuming that this variable has $\overline{\mu^\vartheta} = 1$, and so the intercept of the autoregressive process is zero, so we find the logarithm of our quarterly series and subtract its mean from it. Then we estimate an autoregressive process and find $\rho_4 = 0.69$ and that the standard deviation of the error is 0.0245.

5 The Welfare Costs of Disinflation

The Central Bank of Colombia has a long run inflation target of 3% and in average, last three years target has been of approximately 5.5%. Therefore to reach this goal and get to a steady state with an inflation level of 3%, the bank should start a disinflation process. Before moving on to the quantification of the benefits or costs (compensation) of disinflation, we would like to spend some time on the analysis of both steady states and the transition dynamics followed by the variables in order to get to the new steady state.

Figures 2 and 3 show both steady states and the transition dynamics followed by the variables in order to get to the new steady state, when the calibration for the economy with an inflation rate of 5.5% holds, and the only thing that changes is the annual inflation rate target, all given that the Central Bank has total credibility and it does what it announces. The dashed-dotted line corresponds to the value of the variable at the 5.5% inflation steady state and the dashed line to the 3% inflation steady state, while the solid line corresponds to the transition. In the short run (on impact), we observe an investment boom and a considerable increase in employment. A rapid increase in indebtness followed by a smooth stabilization and an immediate depreciation of the nominal exchange rate. Note that what we call nominal exchange rate is the same as the inflation of the homogeneous goods, therefore the immediate depreciation is due to an immediate increase in the inflation of this goods; this is because we've got an instantaneous increase in the real return to capital which is greater than the decrease in wages pushing prices up. Another interpretation for the same phenomenon is that as agents have perfect foresight, they know that debt is going to start rising from then on, so price of debt increases depreciating the nominal exchange rate (this last interpretation has to be treated with precaution because we don't have an exact nominal exchange rate in the model). Finally we observe an output but no consumption boom in the short run.

In the mid and long run in order to bring down inflation, the monetary authority rises nominal interest rates. As this occurs the intertemporal price of consumption changes. Households will expect to have lower future nominal and real interest rates, so in the future they are going to prefer to consume more. In order to be able to finance this consumption, households are going to increase their supply of labor and number of hours worked per capita are going to increase (this is what causes real wages to decrease on impact). This increase is going to generate a rise in output, and this increase in output generates more demand for labor than the one that already exists, causing wages to increase in the long run. The increase in the supply and interest rates make inflation start to fall[27].

Notice that prices are not falling, they are just growing at a lower rate. This is because the inflationary pressures on the homogeneous goods coming from the prices of the production factors start to fall, and when the inflation rate of the homogeneous good starts to fall, generates a fall

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[27]Our model has shown that the hysteresis hypothesis mentioned by Ball (1998, [2]) doesn't seem to show in our case. We can see from figures 2 and 3 that neither output or employment fall beneath their old steady state level during the transition. By the contrary both increase, and actually the increase in the labor supply is one of the elements that makes the transition to have a smaller welfare. What we observe here is the opposite to the hysteresis hypothesis: a long run increase in output and employment. Ball mentions that hysteresis is more likely to occur in the case of countries which have strong social security institutions, or which have very long lasting unemployment insurance. But this is not the case of our model, here we observe that because of the microfoundations of the labor market, households receive a salary wether they work or not (See 2.1), so implicitly, this works as an unemployment insurance period to period.

on the marginal costs of retailers making their prices grow at a lower rate.

As the inflationary tax is being reduced, then the government increases lump sum taxes in order to finance a given level of expenditure (see figure 3), and therefore households disposable income is going to be diminished. Real return to capital increases due to the rise in the number of hours worked per capita, so households are going to invest more. Notice that it is possible that real return to capital increases from one state to the other because the real interest rate is not constant; our risk premium function depends on the debt to output ratio. Therefore we've got households with lower disposable income and higher levels of consumption and investment, so in order to finance this, they are going to increase their levels of indebtness (this is possible due to the openness of the economy, it is possible that in the case of a closed economy consumption's behavior is different).

As the level of indebtness increases, external nominal interest rate also increases. In the long run, the nominal interest rate starts to fall as inflation does, and so does the external nominal interest rate (not shown), this is because the rise in output is greater than that of debt.

5.1   Two Different Steady States

At this stage we would like to quantify the compensation for moving from a 5.5% long-run inflation to a long-run inflation of 3%. In order to calculate this, we do the following: the steady state values of the variables are known, so we know the value of the utility in each period of time. As we know that while one is on the steady state the situation is going to hold for infinity, we can tell that the welfare for an inflation rate of 3% is going to be

$$W_L = \frac{1}{1-\beta} u\left(c_3, H_3, h_3\right)$$

where W_L stands for welfare with low inflation and the subscript 3 stands for the steady state value of a variable corresponding to a world with an inflation rate of 3%. By doing the calculation we find that $W_L = 10.3931$. The same applies for the world with an inflation rate of 5.5%:

$$W_H = \frac{1}{1-\beta} u\left(c_{5.5}, H_{5.5}, h_{5.5}\right)$$

where W_H stands for welfare with high inflation and the subscript 5.5 stands for the steady state value of a variable corresponding to a world with an inflation rate of 5.5%. In this case we find $W_H = 10.0489$. So as $W_H < W_L$ we can see that it is actually better to be in a world with an inflation rate of 3%.

The values of W_L and W_H show that there are welfare gains of lowering long-run inflation. But as these magnitudes are in terms of utilities, they are not telling us much, so in order to have an idea of the magnitude of those gains, we calculate what would be the compensation in terms of capital and output, in order to have agents indifferent between an inflation rate of 3% and 5.5%. This compensation in terms of capital and/or output would be the long-run compensation for being in a world with an inflation rate of 3% instead of 5.5%.

To calculate this compensation, we assume that households receive an amount of capital which is reflected in an instantaneous change in output[28], and parting from this new level of capital which

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[28]State variables are the only ones that can be deviated from their steady state level, controls and flows are jumping variables, and therefore react to changes in the state variables, so we don't have any control over them. Capital is a state variable while output is a flow variable, that's why we variate capital instead of output.

Figure 2: Disinflation (a)
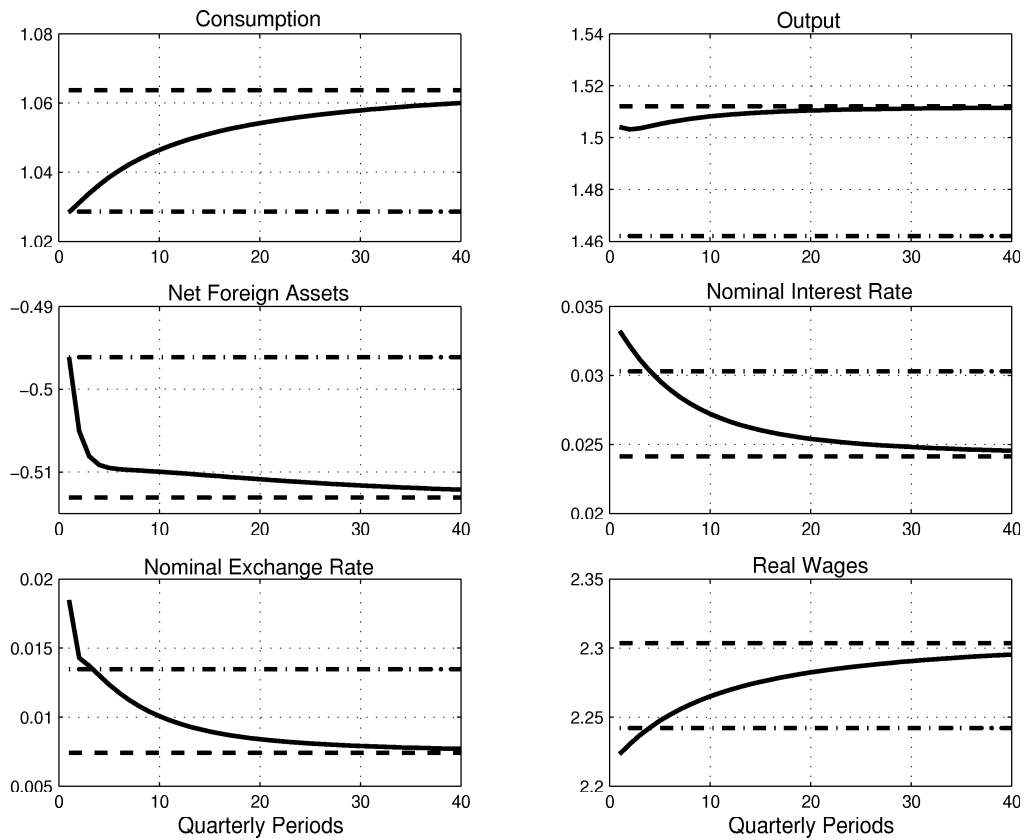
Note: The x-axes shows quarterly periods. And the y-axes shows the corresponding variable in levels. The dashed-dotted line corresponds to the variable level in the steady state with 5.5% inflation. The dashed line is the variable level in the steady state with 3% inflation. The solid line is the transition followed by each of the variables to move from one state to the other.

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Figure 3: Disinflation (b)
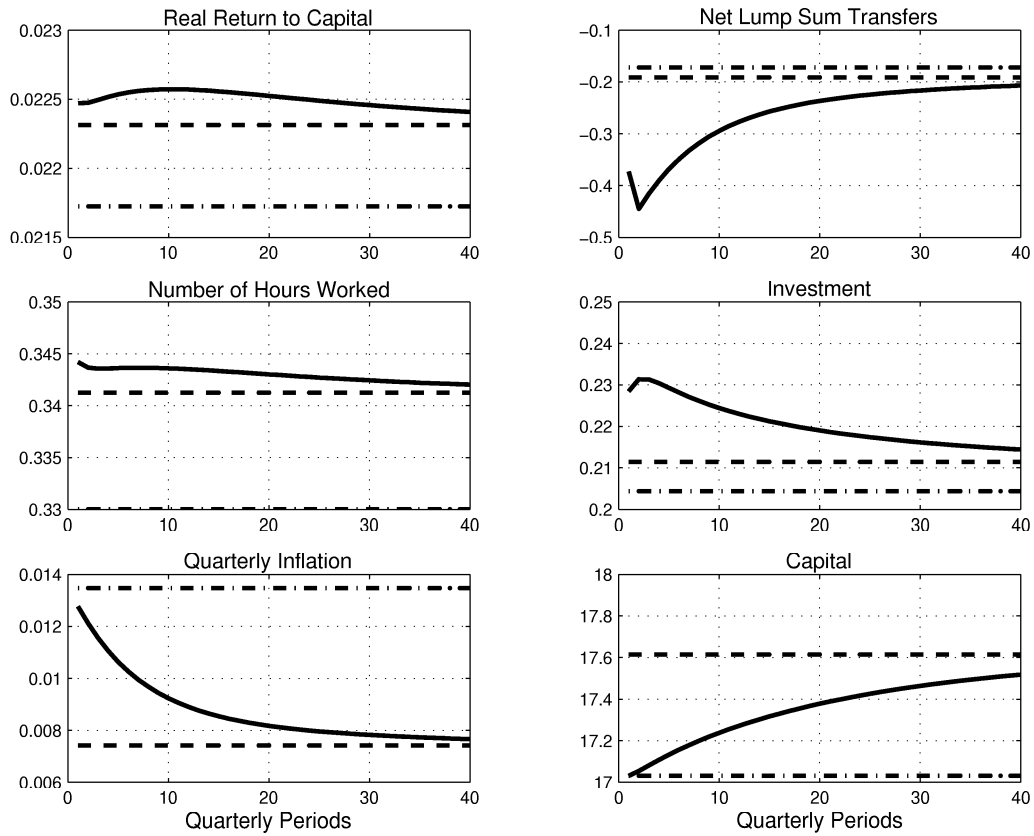
Note: The x-axes shows quarterly periods. And the y-axes shows the corresponding variable in levels. The dashed-dotted line corresponds to the variable level in the steady state with 5.5% inflation. The dashed line is the variable level in the steady state with 3% inflation. The solid line is the transition followed by each of the variables to move from one state to the other.

is eventually going to return to its steady state level, we calculate the present value of the utility function in each of the periods[29]:

$$W_{H1} = \sum_{t=0}^{\infty} \beta^t u\left(c_t, H_t, h_t\right)$$

where W_{H1} stands for the welfare of receiving an amount of capital in $t = 1$ and returning to the *original* steady state. The subscript t stands for the value that each variable takes in a given period of time given that it is returning to its original value. After calculating W_{H1} we compare its value with that of W_L and if $W_{H1} = W_L$ then we stop and know that the percentage increase that was given to households is the compensation received if one goes instantaneously to a state with an inflation rate of 3% (instantaneous compensation, IC), otherwise the amount given to households is increased and the same calculations are done until we find that $W_{H1} = W_L$. In other words, IC tells us that being in a world with an inflation rate of 3% is as good as if today one receives an extra IC percent of capital, and then one would be indifferent between moving to the new state or staying in the actual one.

In this case we find that given the actual calibration of the model the IC is 4.54% in terms of capital and 0.16% in terms of output[30]. This tells us that moving to a lower inflation in the long run, is the same, as if the individuals were compensated today with a capital stock 4.54% higher (or 0.16% higher output). This calculation assumes that agents are just left from one moment to another in a world with lower inflation, in other words, they do not have to make the transition between steady states. Since this transition may be costly for the agents, it is likely that agents are not willing to move (do the transition).

5.2   Transition Dynamics Towards the New Steady State

Now we calculate the compensation received for doing the transition from one state to the other. To do this we use the decision rules of the model corresponding to an inflation rate of 3%, and assume that the endogenous state variables deviate from their steady state in that percentage that does their steady state value in the world with an inflation rate of 5.5%. The endogenous state variables start in their original value and start to converge to the steady state value corresponding to an inflation rate of 3% (the model's calibration corresponding to the steady state with an inflation rate of 5.5% is maintained and we only change the inflation target). As the state variables move, all the other variables move in order for the system to be in equilibrium in every single period[31]. So

$$W_T = \sum_{t=0}^{\infty} \beta^t u\left(\mu_t^u, c_t, H_t, h_t\right) \tag{28}$$

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[29]To do these calculations we assumed that we were standing in the world with the inflation rate of 5.5%. This means that the dynamic followed by the variables was based in the state space representation given by the model with a steady state inflation of 5.5%.

[30]In order to find this percentage change in output, we take the initial change in capital, and as output is a jumping variable, when the change in capital takes place output moves instantaneously. And we calculate the percentage deviation of this initial value from the steady state corresponding to an inflation rate of 5.5%.

[31]We are aware that here we are using linear rules, so there is room for an approximation error.

where W_T corresponds to the welfare of doing the transition from one state to the other[32]. Computing 28 we obtain $W_T = 10.1428$. As $W_L > W_T > W_H$ households still obtain benefits from doing the transition. Any household that has perfect foresight and is able to calculate its utility in every period of time, is going to know that it's worth doing the transition. So in order to know how good the transition is, we want to know how much capital a household living in a world with an inflation rate of 5.5% should receive in order to make $W_T = W_H$. That percentage of capital and/or output is going to be the compensation received by households for doing the transition from a state corresponding to an inflation rate of 5.5% to 3%.

To calculate this compensation we follow a similar procedure to the one we use to calculate the IC. The capital's steady state level corresponding to a world with an inflation rate of 5.5% is deviated from its steady state in a positive amount. Households are going to receive an amount of capital in period $t = 1$ and are going to return to their original steady sate. So their welfare is going to be W_{H1} instead of W_H. We calculate W_{H1} and compare it with W_T, if $W_T = W_{H1}$ then that percentage of capital and/or output that is given or taken from households is the compensation received for doing the transition from one state to the other (transition compensation, TC). If $W_T \neq W_{H1}$ we increase or decrease the amount of capital given or taken respectively from households and repeat the procedure.

From doing the calculations we found that the TC is 1.18% in terms of capital and 0.04% in terms of output. In other words if today one receives 1.18% more capital, one would be indifferent between doing the transition towards the state with an inflation rate of 3% and staying in the actual one.

5.3 Sensitivity Analysis

We now study the properties of the dynamic response of the model to some key parameters: the degree of price stickiness, the markup value in absence of price rigidities, the degree of responsiveness of the Central Bank to the inflation gap and the degree of openness of the economy.

5.3.1 More Flexible Prices

In our benchmark calibration we have $\varepsilon = 0.75$, so that retailers adjust prices every year. Now we show how the dynamics of the model, welfare, IC and TC change as retailers adjust prices more and more frequently, this is $\varepsilon = 0.5, 0.3$ and 0.1.

When doing the sensitivity analysis for the case of more flexible prices, we want to observe two things: one, is how the welfare measures W_L, W_H and W_T change with the grade of price rigidity, and second, how do IC and TC behave in terms of capital and output.

From table 1 we can see that all W_L, W_H and W_T decrease when price rigidity increases. This means that when prices are more flexible welfare increases. Figure 11 in appendix 4 shows the changes in the dynamics when varying the parameter ε, and we can see that as prices become more

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[32]Although the essence of the model is stochastic, this characteristic is not being used in any of the calculations. It can be introduced by using the simulations, but our hypothesis is that our results are not going to change much. As agents have a risk aversion coefficient equal to one, this model economy is going to have a relatively small volatility, so the information added is not going to make a big difference. It is still convenient to have the stochastic part of the model if one wants to add a second order approximation. However we did use the stochastic properties of the model for the validation procedure.

Table 1: W_L, W_H and W_T when Prices are more Flexible

	$\varepsilon = 0.1$	$\varepsilon = 0.3$	$\varepsilon = 0.5$	$\varepsilon = 0.75$	$\varepsilon = 0.8$	$\varepsilon = 0.85$	$\varepsilon = 0.9$
W_T	10.6056	10.5627	10.4812	10.1428	9.934	9.5032	8.1157
W_H	10.5819	10.5318	10.4369	10.0489	9.8139	9.3391	7.8739
W_L	10.6563	10.6296	10.5803	10.3931	10.2899	10.1015	9.6472
$W_L - W_H$	0.0744	0.0978	0.1434	0.3442	0.476	0.7624	1.7733

flexible (ε decreases) the change from one steady state to the other is more dramatic, changes in consumption and output are greater.

As in this framework welfare is measured by the present value of all future utilities, and utility is composed mainly by consumption and leisure, it can be observed in graphs 4 and 5 that consumption increases as price rigidity falls and although leisure decreases, the effect of consumption is dominating; causing utility to increase. The reason why leisure increases as prices become more rigid, is because the rigidity makes the relative price of consumption over leisure to increase, so households substitute consumption for leisure. But as mentioned before, as the effect of consumption dominates, welfare is lower when prices are more rigid.

On the other hand, if we look at both the IC and TC in terms of capital from table 2 we can see that as prices become more rigid they increase. This means that economies with higher grades of price rigidity are going to obtain more gains when having lower or bringing down inflation. Price rigidity emphasizes the distorting effects of the inflation tax, this is why lower grades of price rigidity conduct to lower gains of moving towards a lower inflation or having one. By looking at the last row of table 1 we can also see, that the difference between the utility of being in a steady state with an inflation rate of 3% and a steady state with an inflation rate of 5.5% is greater for higher levels of price rigidity, so households gain more if they move from one state to the other. This can also be seen by looking at at the behavior of consumption in figure 4.

Now, if we look at the IC and TC in terms of output (see table2), we can see that they are increasing with price rigidity. But we can also see that they are negative for most levels of ε. This would imply that there are costs (negative compensation) of both having lower rates of inflation and moving towards one. But in our previous analysis it has been shown that there are certainly benefits in terms of capital. So what is the puzzle?

In a general equilibrium framework, where labor is an endogenous variable, output may not be the best measure of welfare. If one looks at figure 6, which corresponds to the behavior of several variables when capital is increased by IC when $\varepsilon = 0.1$, and where the reference line is the dashed-dotted line which corresponds to the steady state with an inflation rate of 5.5%, one can see that although output is falling, leisure and consumption are increasing, which are the main variables determining welfare. Output is falling just because the effect of the fall in the number of hours worked is dominating over the effect of capital.

So as in this framework we consider that output is not the appropriate measure of welfare, from now on we will refer to the compensation in terms of capital although our results are not going to be fully comparable with those of previous literature.

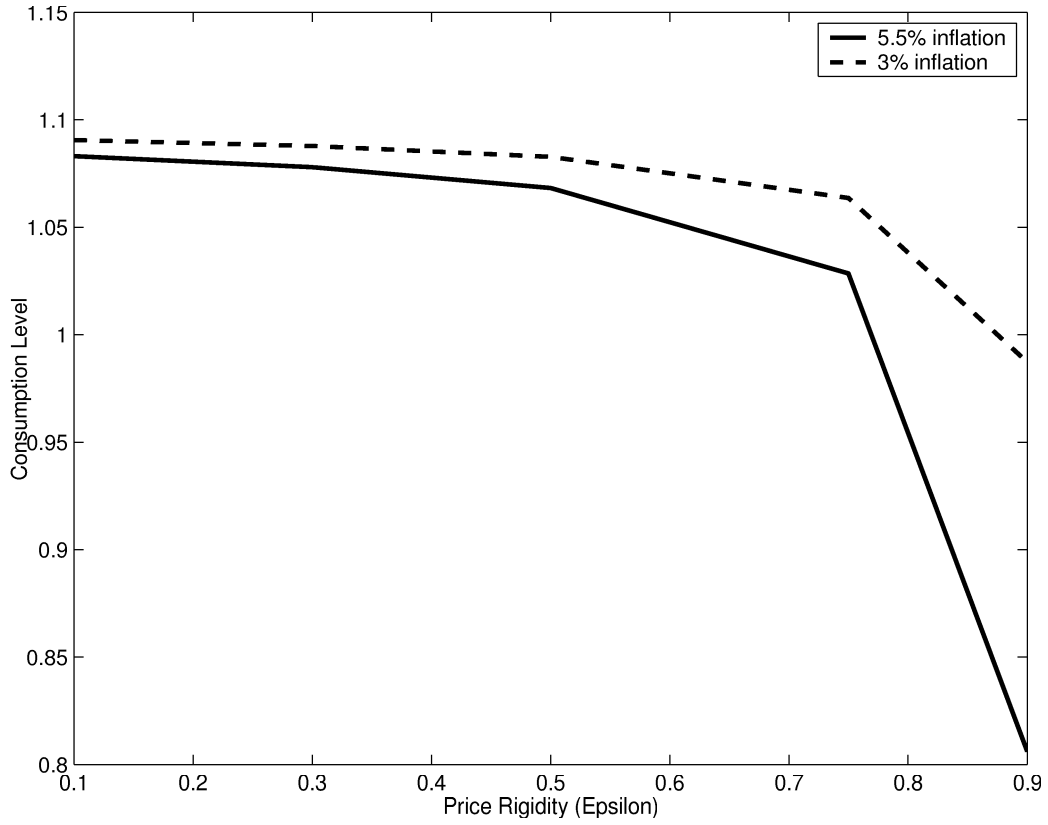Figure 4: Consumption as Price Rigidity Decreases

Table 2: More Flexible Prices

		$\varepsilon = 0.1$	$\varepsilon = 0.3$	$\varepsilon = 0.5$	$\varepsilon = 0.75$	$\varepsilon = 0.8$	$\varepsilon = 0.85$	$\varepsilon = 0.9$
IC	In terms of capital	1.11%	1.4%	1.98%	4.54%	6.23%	9.91%	48.96%
	In terms of output	-0.016%	-0.0134%	-0.00124%	0.16%	0.34%	0.9%	414%
TC	In terms of capital	0.28%	0.38%	0.55%	1.18%	1.52%	2.09%	6.65%
	In terms of output	-0.00415%	-0.0036%	-0.00034%	0.04%	0.0821%	0.19%	56.32%

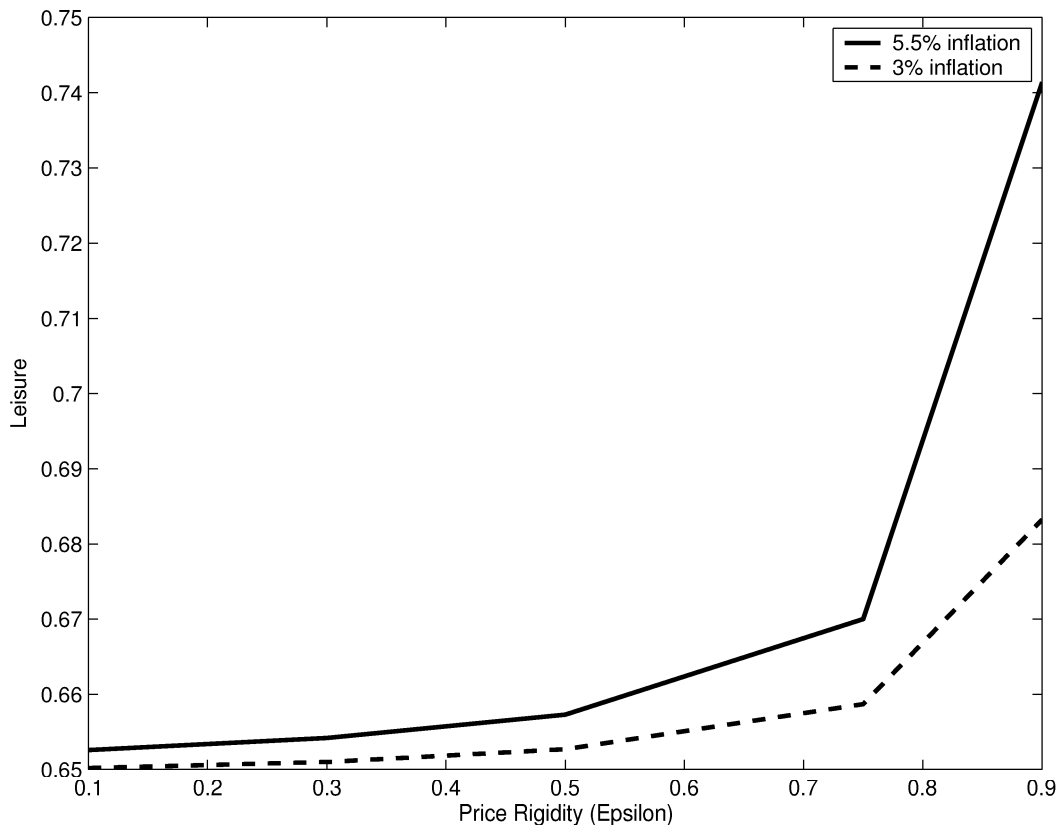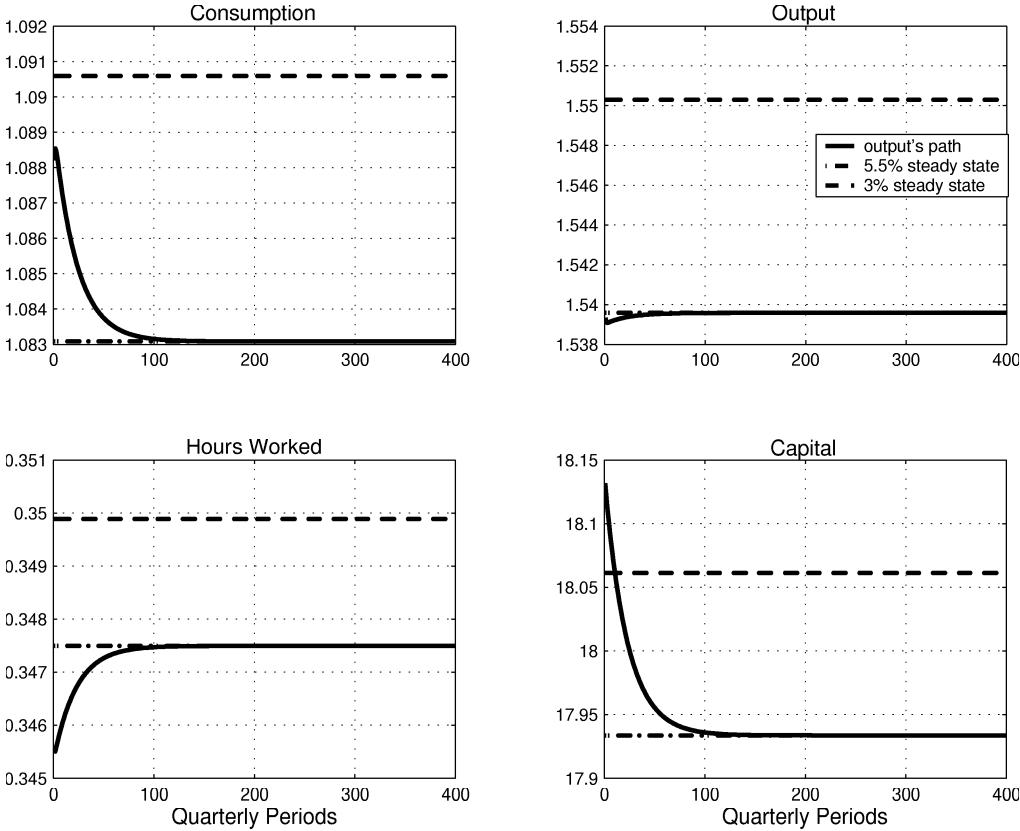Figure 5: Leisure as Price Rigidity Decreases

Figure 6: Behavior of Consumption, Leisure and Output when Capital is Increased by IC

Note: The x-axes correspond to the value of the variables in levels.

Table 3: W_L, W_H and W_T when markups are smaller

	$\theta = 5$	$\theta = 6$	$\theta = 7.6667$	$\theta = 11$
W_T	10.1428	10.5408	10.9420	11.2549
W_H	10.0489	10.4357	10.8170	11.0761
W_L	10.3931	10.8230	11.2824	11.2549
$W_L - W_H$	0.3442	0.3873	0.4654	0.1788

Note: $\theta = 5$ corresponds to a markup of 25%, $\theta = 6$ to 20%, $\theta = 7.6667$ to 15% and $\theta = 11$ to 10%

Table 4: Smaller Markup

	$\theta = 5$	$\theta = 6$	$\theta = 7.6667$	$\theta = 11$
IC	4.54%	4.87%	5.56%	5.02%
TC	1.18%	1.27%	1.44%	1.28%

5.3.2 Smaller Markups

Our benchmark calibration of the model corresponds to a markup of 25% in the absence of price rigidities. We now show how the dynamics of the model, welfare, IC and TC change as θ increases to $\theta = 6, 7.66$ and 11 which correspond to a decrease in the markup from 25% to 20% to 15% to 10% respectively. Figure 12 in appendix 4 shows the changes in the dynamics when varying the parameter θ.

From table 3 we can see that welfare increases as the markup goes down (or theta goes up) in the case of the transition (W_T) and the world with an inflation rate of 5.5% (W_H). The case of 5.5% inflation can be explained by looking at figures 7 and 8 where consumption is higher when there are smaller markups and leisure is higher when there are higher markups (for higher markups the relative price of consumption over leisure increases), but once more, consumption is dominating over leisure. The gap between the two steady states consumption level, increases as markups decrease. The situation is not so clear in the case of welfare for an inflation rate of 3% (W_L), we can see that it reaches a maximum for a markup of 15%, and therefore the greatest gain in terms of welfare of going from one state to the other is at this same markup level.

Also, when the values of θ are varied, the IC and TC change. Table 4 shows the results in terms of capital for the case when the markup is smaller.

From figure 12 in appendix 4 we can see that as the competitiveness of the market increases the change from one state to the other is more dramatic, and in terms of consumption and output there are higher gains of moving to a lower inflation when the markup is lower. However from table 4 it is not clear in which direction is the effect going. From the table we can see that a markup of 15% ($\theta = 7.6667$) maximizes the IC and TC in terms of capital as well as the difference between W_L and W_H (see table 3, last row). So countries with markup levels around 15% are going to obtain greater gains from lower inflations. Once more, here we are focusing the analysis on the results in terms of capital because of the arguments explained before.
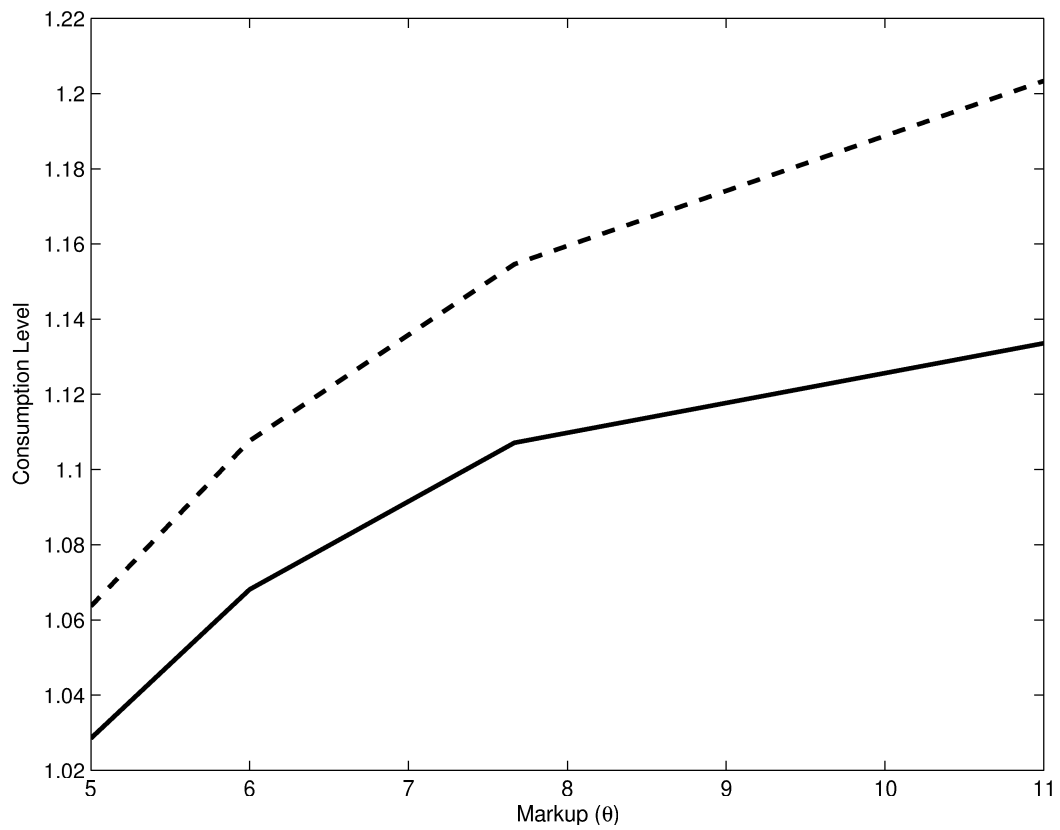
Figure 7: Consumption as Markups Become Smaller

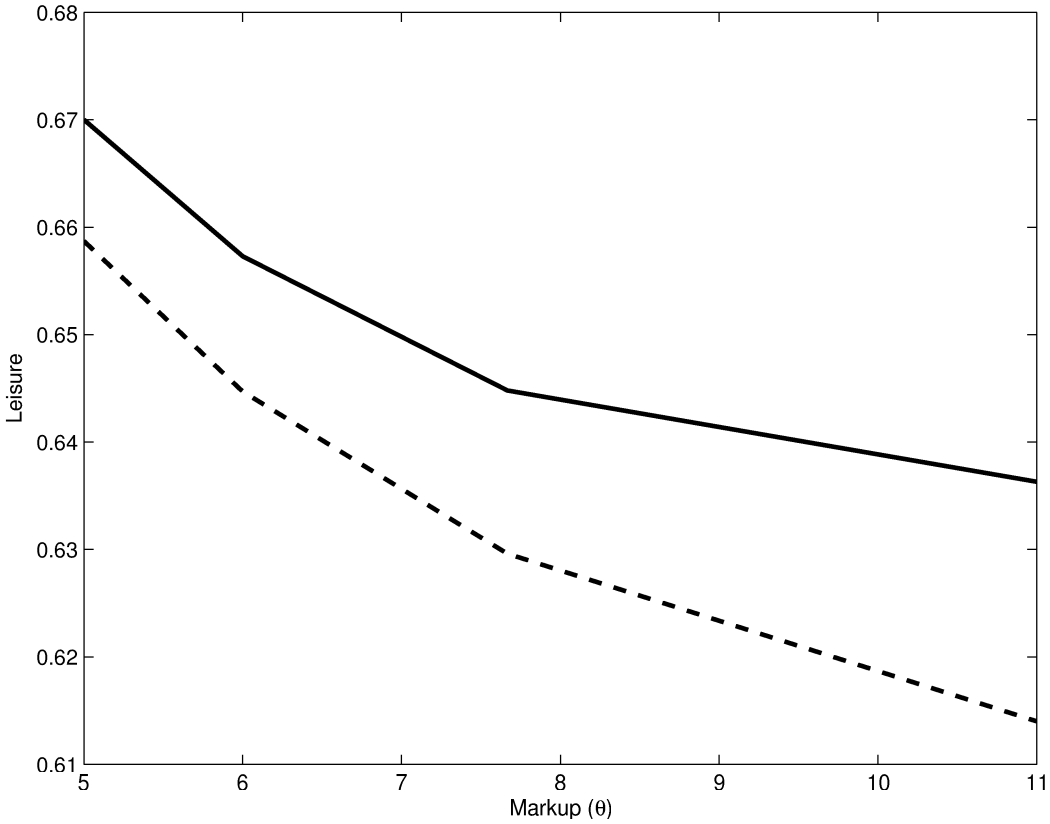Figure 8: Leisure as Markups Become Smaller

Table 5: W_L, W_H and W_T when ζ changes

	$\zeta = 1.5$	$\zeta = 1.7$	$\zeta = 2.3$	$\zeta = 2.7$	$\zeta = 3$
W_T	10.1400	10.1428	10.1500	10.1527	10.1539
W_H	10.0489	10.0489	10.0489	10.0489	10.0489
W_L	10.3931	10.3931	10.3931	10.3931	10.3931
$W_L - W_H$	0.3442	0.3442	0.3442	0.3442	0.3442

Table 6: A changing ζ

	$\zeta = 1.5$	$\zeta = 1.7$	$\zeta = 2.3$	$\zeta = 2.7$	$\zeta = 3$
IC	4.47%	4.54%	4.62%	4.64%	4.65%
TC	1.1%	1.18%	1.3%	1.33%	1.34%

5.3.3 Alternative Central Banks

We want to study what happens in the case when the Central Bank (CB) varies the weight it gives to inflation in the monetary policy rule (recall that in our model, the CB gives no importance at all to output ($\xi = 0$), so the changes are going to be made in ζ; $i_t = i + \zeta \left(\pi_t^c - \overline{\pi}^c \right) + \xi \left(y_t - y^{ss} \right)$. For values of ξ different from zero, the model has shown to be very unstable)[33].

From table 5 it can be seen that the more importance is given to inflation, the higher welfare is during the transition process (W_T); while there is no difference between W_H and W_L. This is because the parameter ζ doesn't affect the steady state of the model, it only affects the transition dynamics. So in terms of welfare it is better for households to have a CB that worries a lot for inflation in the case that a disinflation is going to take place. If households could choose a type of CB to do the disinflation, they would choose that which does it more rapidly.

Table 6 shows that there are always positive compensations for going instantaneously and doing the transition to a steady state with a lower inflation rate.

So if a CB is planning to disinflate, they should be very careful about the importance they give to their target. The more strict they are, the more benefic is going to be the transition towards the new steady state.

5.3.4 The Case of a Closed Economy

As it has been mentioned previously, the possibility of households to acquire debt outside the country, might be the reason why consumption is able to increase between the two steady states or at least the reason why it increases in the amount it does. So in order to test this hypothesis,

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[33]In our case it is not necessary to calculate a potential output defined as that output that would result in absence of shocks and price rigidities and where the difference between this potential output and the observed one would be an output gap that serves to measure inflationary pressures. This is because the monetary authority assigns a value of zero to the parameter accompanying the output gap in the policy rule. What we want to study are just the effects of having a monetary authority that is disinflating, and not one that is disinflating and at the same time eliminating the nominal rigidities from the market.

Table 7: Open vs. Closed Steady State Levels

Type of Economy	Inflation Rate	Consumption	Hours Worked	Output	Capital
Open	5.5%	1.037	0.330	1.462	17.030
Open	3%	1.269	0.404	1.789	20.851
Closed	5.5%	1.028	0.327	1.449	16.871
Closed	3%	1.063	0.338	1.499	17.451

Table 8: Open vs. Closed IC and TC

| Type of Economy	IC	TC
Open	106.75%	40.8%
Closed	4.5%	1.21%

we close our model economy and measure the steady states of certain variables for both levels of inflation, and calculate the IC and TC. In order to make a fair comparison, we use a small open economy for which net foreign assets are zero on the steady state, otherwise, if we allow any level of indebtness the close economy is going to be wealthier and for obvious reasons disinflation is going to be more costly for a small open economy. Table 7 shows the steady state levels of consumption, hours worked, output and capital for both steady states and the two types of economy, the small open economy and the closed economy, under the assumption that the closed economy has the same calibration parameters as the small open economy.

From this table we can see that the steady state levels of all variables are higher in the case of an open economy. So comparing consumption of one economy with the other in a same level of inflation, we can see that households that live in a small open economy have more possibilities of higher consumption. But as hours worked are also increasing when going from one inflation rate to the other for both economies, one would say that it is not that clear that the higher possibilities of consumption are given by the possibility to acquire debt abroad. But remember that as we are comparing steady state values and debt level is zero on the steady state in the small open economy, the relevance of being able to acquire debt abroad in order to smooth consumption is only going to be relevant during the transition.

When we calculate W_L, W_H and W_T for the closed economy we find: $W_T = 10.290$, $W_H = 10.195$ and $W_L = 10.545$. For the small open economy we find $W_T = 16.298$, $W_H = 15.012$ and $W_L = 18.376$. So there is a benefit of doing the transition in both economies because $W_T > W_H$ and welfare is higher in a small open economy.

By looking at table 8 we can see that both IC and TC are higher for the open economy. So disinflations are more costly in closed economies. By looking at figures 9 and 10, we can compare consumption's behavior. We can see that the increase in consumption in the close economy is smaller than in the open one, and the transition is much more smoother in the last one. From figure 10 we can also see that during the transition households actually increase debt, so our hypothesis has been proven. An open economy allows households to acquire debt in order to

Figure 9: Close Economy Consumption Path
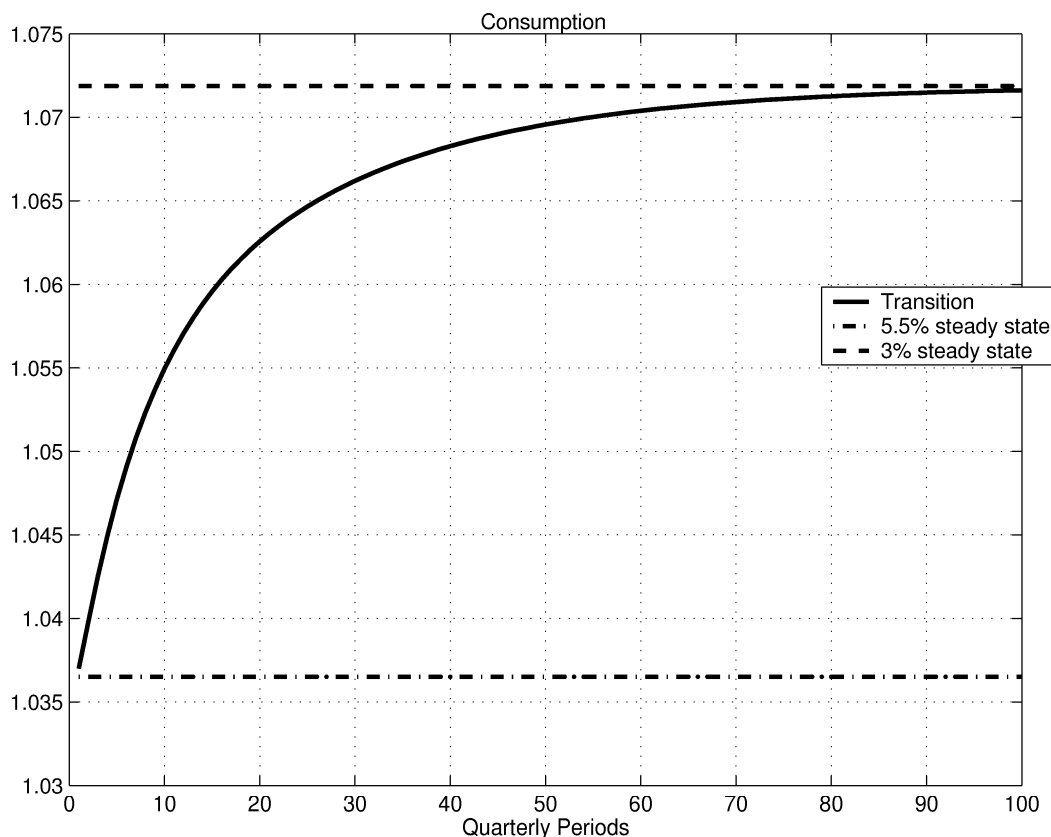
smooth consumption during the transition process so that the effects of the nominal and real interest rates are not so harsh.

6 Final Remarks

This paper evaluates quantitatively the benefits of reducing inflation from 5.5% to 3% in Colombia. We do that in the context of a SDGE model of a small open economy with nominal rigidities. Monetary policy is conducted under an inflation targeting strategy and the CB has full credibility. We find that the compensation for going instantaneously to a state with an inflation rate of 3% (IC) is 4.54% in terms of capital and the compensation for doing the transition from one state to the other (TC) is 1.18% in terms of capital. Therefore if one takes this model economy as a policy analysis and decision taking tool, one would say that it is worthwhile for Colombia's Central Bank to continue trying to bring down inflation.

A country with a higher level of price rigidity is going to receive more benefits in the case of having a lower inflation and/or disinflating. There are higher benefits of reducing inflation for a country with markups around 15%. So a country with higher price rigidity and markups around 15%, should do a higher effort in order to bring down inflation. So the result found by Gómez (2003, [7]), where price rigidities are the key element explaining the costs of disinflation, seems to

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Figure 10: Open Economy Consumption and Net Foreign Assets Path
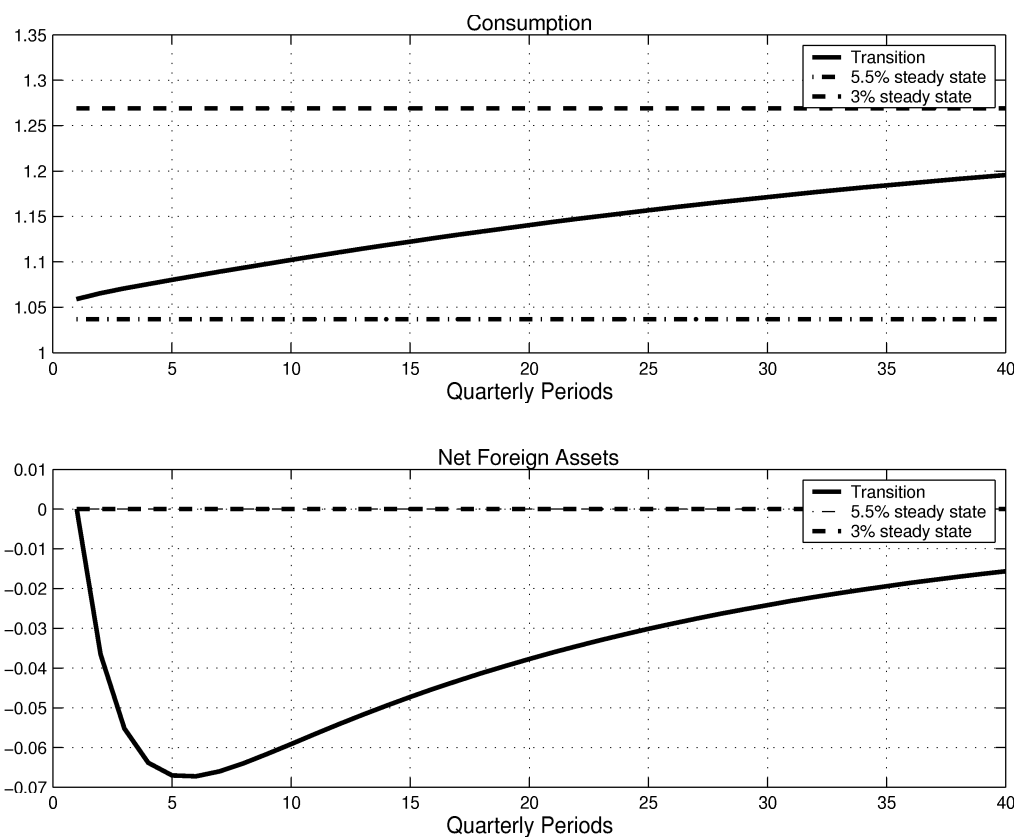
be valid in this context as well.

As already mentioned our results show that there are positive compensations for doing the transition and that there is no sacrifice ratio in terms of output. This is not what usually has been observed or found in previous studies. It is often observed that disinflations cause recessions, or the so called hysteresis hypothesis. Although this doesn't have to be the rule as shown by Hofstetter (2004, [11]), it is possible that our model fails to replicate this features and that there are short and long run benefits of disinflations, because of the absence of a friction in the labor market. As in our model wages are flexible, it is likely that this rigidity is missing, so that the number of hours worked don't adjust when disposable income changes. Probably this is why unemployment doesn't fall and why output increases in the amount it does. The introduction of this friction is left for future work.

The importance given by the CB to deviations from the inflation target, has shown to be very important in terms of the transition welfare. If a CB is planning to disinflate they have to be very careful about the weight they give to inflation in their policy rule. On the other hand disinflations are more costly in closed economies. This is because an open economy allows households to acquire debt in order to smooth consumption during the transition process so that the effects of the nominal and real interest rates are not so harsh.

Something very important that we have to highlight, is that according to our methodology and results, output is not an appropriate measure for the benefits or costs of a disinflation when labor is an endogenous variable. So our findings are not really comparable with those of previous literature. We are also aware of the fact that we are using a first order approximation, and that therefore our results might include an approximation error, it is left for future work to do the second order approximation and compare the results with the actual ones.

Finally here we need to take into account that the only thing that causes inflation to be costly is the inflation tax, and that we are not in the presence of distortive taxes. If we incorporate this taxes into our model, we would probably find that disinflation is more expensive than in this framework. This is because as the government is no longer collecting as much inflationary tax they have to increase other taxes in order to continue financing their expenditure. So although households are going to feel a relief from the inflation tax, they are going to be affected by higher distortive taxes. This modification to the model is left for future work, as well as a sensitivity analysis to different specifications of the monetary policy rule used by the Central Bank.

References

[1] Juan Pablo Arango, Orlando Gracia, Gustavo Hernández, and Juan Mauricio Ramírez. Reformas comerciales, márgenes de beneficio y productividad en la industria colombiana. *Planeación '&' Desarrollo*, XXIX:59–78, 1998.

[2] Laurence Ball. Los costos de la inflación. (36-37):63–76, 1999-2000.

[3] Jesús Bejarano. Estimación estructural y análisis de la curva de phillips neo-keynesiana para colombia. banrep, 2004.

[4] Ben Bernanke, Mark Gertler, and Simon Girlchrist. *Handbook of Monetary Economics*, volume 1C, chapter 21: The Financial Accelerator in a Quantitative Business Cycle Framework, pages 1341–1393. Elsevier Science B.V., 1999.

[5] Guillermo A. Calvo. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12(3):383–398, September 1983.

[6] Michael Dotsey, Robert G. King, and Alexander L. Wolman. State-dependent pricing and the general equilibrium dynamics of money and output. *qje*, pages 655–690, May 1999.

[7] Javier Gómez. Wage indexation, inflation inertia, and de cost of disinflation. *Ensayos sobre política económica*, (43):67–83, 2003.

[8] José De Gregorio. Sobre los determinantes de la inflación y sus costos. *Ensayos sobre política económica*, (36-37):27–62, 1999-2000.

[9] Franz Hamann, Juan Manuel Julio, Paulina Restrepo, and Alvaro Riascos. Inflation targeting in a small open economy: the colombian case. Banco de la República, 2004.

[10] Gary D. Hansen. Indivisible labor and the business cycle. *Journal of Monetary Economics*, 16:309–327, 1985.

[11] Marc Hofstetter. Desinflations in latin america and the caribbean: A free lunch. Johns Hopkins University and Universidad de los Andes, 2004.

[12] Robert G. King, Charles I. Plosser, and Sérgio T. Rebelo. Production, growth and business cycles: Technical appendix. June 2001.

[13] Martin Eichenbaum Lawrence J. Christiano and Charles Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. Working paper 01-07, Federal Reserve Bank of Cleveland, 2001.

[14] Martha López. Seigniorage and the welfare cost of inflation in Colombia. (39):115–130, 2001.

[15] R. Lucas. On the welfare costs of inflation. Working paper 394, Center for Economic Policy Research, Stanford University, 1994.

[16] Sidrauski M. Rational choice and patterns of growth in a monetary economy. *American Economic Review*, 57:534–544, 1967.

[17] Gregory Mankiw and Ricardo Reis. Sticky information versus sticky prices: A proposal to replace the new keynesian phillips curve. January 2002.

[18] Luis F. Melo and Alvaro J. Riascos. Sobre los efectos de la política monetaria en Colombia. Technical Report forthcomming, Ensayos Sobre Política Económica, Banco de la República, 2004.

[19] Francis X. Diebold-Lee E. Ohanian and Jeremy Berkowitz. Dynamic equilibrium economies: A framework for comparing models and data. *Review of Economic Studies*, 65(3):433–451, 1998.

[20] Hilde Patrón, Arturo Galindo, and Alberto Carrasquilla. Costos en bienestar de la inflación: teoría y una estimación para Colombia. Borrador Semanal de Economía 3, Banco de la República, 1994.

[21] Carlos E. Posada. El costo de la inflación (con racionalidad y previsión perfectas). Technical Report 30, Banco de la República, 1995.

[22] Alvaro Riascos. Sobre el costo en bienestar de la inflación en colombia. Borrador Semanal de Economía 82, Banco de la República, 1997.

[23] Stephanie Schmitt-Grohe and Martin Uribe. Optimal simple and implementable monetary and fiscal rules. Working paper 10253, National Bureau of Economic Research, 2004.

[24] Diego M. Vásquez. Mecanismos de cobertura para el riesgo de tasa de interés real de los bancos hipotecarios colombianos. Borrador Semanal de Economía 237, banrep, Jan 2003.

Appendix 1: Demand for the differentiated consumption good

The following is the problem that has to be solved in order to find the demand function:

$$\max_{c(z)_t} P_t^c * c_t$$

s.t.

$$\int_0^1 p^c(z)_t c(z)_t dz$$

or what is the same

$$\max_{c(z)_t} P_t^c \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

s.t.

$$\int_0^1 p^c(z)_t c(z)_t dz$$

deriving with respect to $c(z)$

$$P_t^c \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{1}{\theta-1}} c(z)^{\frac{-1}{\theta}} = p^c(z)_t$$

$$\frac{P_t^c}{p^c(z)_t} \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{1}{\theta-1}} = c(z)^{\frac{1}{\theta}}$$

$$\left(\frac{P_t^c}{p^c(z)_t} \right)^\theta \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} = c(z)$$

as

$$c_t = \left[\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

then

$$c(z)_t = \left(\frac{P_t^c}{p^c(z)_t} \right)^\theta c_t$$

Appendix 2: Optimal price chosen by retailers

$$\max_{p^c(z)_{t+j}} E_t \sum_{j=0}^{\infty} (1-\varepsilon)\varepsilon^j \Delta_{t+j} c_{t+j} \left[\left(\frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{1-\theta} - q_{t+j} \left(\frac{p^c(z)_{t+j}}{P_{t+j}^c} \right)^{-\theta} \right]$$

In period t the firm is going to choose a price for the whole horizon of time so $p^c(z)_{t+j} = p^c(z)_t$ (they choose prices from now on):

$$\max_{p^c(z)_{t+j}} E_t \sum_{j=0}^{\infty} (1-\varepsilon)\varepsilon^j \Delta_{t+j} c_{t+j} \left[\left(\frac{p^c(z)_t}{P_{t+j}^c} \right)^{1-\theta} - q_{t+j} \left(\frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta} \right]$$

deriving with respect to $p^c(z)_t$

$$E_t \sum_{j=0}^{\infty} (1-\varepsilon)\varepsilon^j \Delta_{t+j} c_{t+j} \left[\frac{(1-\theta)}{P_{t+j}^c} \left(\frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta} - \theta \frac{q_{t+j}}{P_{t+j}^c} \left(\frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta-1} \right] = 0$$

$$E_t \sum_{j=0}^{\infty} (1-\varepsilon)\varepsilon^j \Delta_{t+j} c_{t+j} \theta \frac{q_{t+j}}{P_{t+j}^c} \left(\frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta-1} + E_t \sum_{j=0}^{\infty} (1-\varepsilon)\varepsilon^j \Delta_{t+j} c_{t+j} \frac{(1-\theta)}{P_{t+j}^c} \left(\frac{p^c(z)_t}{P_{t+j}^c} \right)^{-\theta} = 0$$

$$(1-\varepsilon) \left(p^c(z)_t \right)^{-\theta-1} \theta E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \frac{q_{t+j}}{\left(P_{t+j}^c \right)^{-\theta}} + (1-\varepsilon)(1-\theta) \left(p^c(z)_t \right)^{-\theta} E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \left(P_{t+j}^c \right)^{\theta-1}$$

$$\left(p^c(z)_t \right)^{-\theta-1} \theta E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \frac{q_{t+j}}{\left(P_{t+j}^c \right)^{-\theta}} = (\theta-1) \left(p^c(z)_t \right)^{-\theta} E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \left(P_{t+j}^c \right)^{\theta-1}$$

rewriting for $p^c(z)_t$ to obtain the optimal price

$$p_t^{opt} = \frac{\theta}{\theta-1} E_t \left[\frac{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} q_{t+j} \left(P_{t+j}^c \right)^{\theta}}{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \left(P_{t+j}^c \right)^{\theta-1}} \right]$$

dividing both sides by P_t^c and multiplying and dividing by $\frac{1}{(P_t^c)^{\theta}}$

$$\frac{p_t^{opt}}{P_t^c} = \frac{\theta}{\theta-1} E_t \left[\frac{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} q_{t+j} \left(\frac{P_{t+j}^c}{P_t^c} \right)^{\theta}}{\sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} \left(\frac{P_{t+j}^c}{P_t^c} \right)^{\theta-1}} \right] \tag{29}$$

From the numerator: $E_t \sum_{j=0}^{\infty} \varepsilon^j \Delta_{t+j} c_{t+j} q_{t+j} \left(\frac{P_{t+j}^c}{P_t^c} \right)^{\theta}$, so we define

$$E_t \Theta_{t+1} = E_t \left(\Delta_{t+1} c_{t+1} q_{t+1} \left(\frac{P_{t+1}^c}{P_t^c} \right)^{\theta} \right) + \varepsilon E_t \left(\Delta_{t+1+1} c_{t+1+1} q_{t+1+1} \left(\frac{P_{t+2}^c}{P_{t+1}^c} \right)^{\theta} \right)$$

$$+ \varepsilon^2 E_t \left(\Delta_{t+1+2} c_{t+1+2} q_{t+1+2} \left(\frac{P_{t+3}^c}{P_{t+1}^c} \right)^{\theta} \right) + ...$$

and

$$\Theta_t = \Delta_t c_t q_t \left(\frac{P_t^c}{P_t^c}\right)^\theta + \varepsilon E_t \left(\Delta_{t+1} c_{t+1} q_{t+1} \left(\frac{P_{t+1}^c}{P_t^c}\right)^\theta\right) + \varepsilon^2 E_t \left(\Delta_{t+2} c_{t+2} q_{t+2} \left(\frac{P_{t+2}^c}{P_t^c}\right)^\theta\right) + ...$$

so

$$
\begin{aligned}
E_t \left(\left(P_{t+1}^c\right)^\theta \Theta_{t+1}\right) &= E_t \left(\Delta_{t+1} c_{t+1} q_{t+1} \left(P_{t+1}^c\right)^\theta\right) + \varepsilon E_t \left(\Delta_{t+1+1} c_{t+1+1} q_{t+1+1} \left(P_{t+2}^c\right)^\theta\right) \\
&\quad + \varepsilon^2 E_t \left(\Delta_{t+1+2} c_{t+1+2} q_{t+1+2} \left(P_{t+3}^c\right)^\theta\right) + ...
\end{aligned}
$$

$$\left(P_t^c\right)^\theta \Theta_t = \Delta_t c_t q_t \left(P_t^c\right)^\theta + \varepsilon E_t \left(\Delta_{t+1} c_{t+1} q_{t+1} \left(P_{t+1}^c\right)^\theta\right) + \varepsilon^2 E_t \left(\Delta_{t+2} c_{t+2} q_{t+2} \left(P_{t+2}^c\right)^\theta\right) + ...$$

$$\left(P_t^c\right)^\theta \Theta_t = \Delta_t c_t q_t \left(P_t^c\right)^\theta + \varepsilon E_t \left(\left(P_{t+1}^c\right)^\theta \Theta_{t+1}\right)$$

dividing both sides of the equation by $\left(P_t^c\right)^\theta$:

$$\Theta_t = \Delta_t c_t q_t + \varepsilon E_t \left(\left(\frac{P_{t+1}^c}{P_t^c}\right)^\theta \Theta_{t+1}\right)$$

$$\Theta_t = \Delta_t c_t q_t + \varepsilon E_t \left(\left(1 + \pi_{t+1}^c\right)^\theta \Theta_{t+1}\right)$$

In a similar way, from the denominator of 29 $E_t \sum_{j=0}^\infty \varepsilon^j \Delta_{t+j} c_{t+j} \left(\frac{P_{t+j}^c}{P_t^c}\right)^{\theta-1}$ one can obtain:

$$\Psi_t = \Delta_t c_t + \varepsilon E_t \left(\left(1 + \pi_{t+1}^c\right)^{\theta-1} \Psi_{t+1}\right)$$

Appendix 3: The Complete Model

$$c_t + x_t + g_t + F_{t+1} - \left(1 + i_{t+1}^f\right) F_t = A_t k_t^\alpha h_t^{1-\alpha}$$

$$u_{c_t}\left(c_t, H_t, h_t\right) + \eta_t \rho = \lambda_t \left(1 + \Phi_{c_t}\left(c_t, m_{t+1}, x_t\right)\right)$$

$$u_{h_t}\left(c_t, H_t, h_t\right) + \lambda_t q_t A_t (1 - \alpha) k_t^\alpha h_t^{-\alpha} = 0$$

$$\beta E_t \left(\lambda_{t+1} q_{t+1} A_{t+1} \alpha k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} + \gamma_{t+1} \left(f\left(\frac{x_{t+1}}{k_{t+1}}\right) + \frac{\partial \left(f\left(\frac{x_{t+1}}{k_{t+1}}\right) k_{t+1}\right)}{\partial \left(k_{t+1}\right)}\right) + \gamma_{t+1}(1 - \delta)\right) = \gamma_t$$

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$$\beta E_t \left(\frac{\lambda_{t+1}}{\left(1 + \pi_{t+1}^c\right) \left(1 + \Phi_{m_{t+1}} \left(c_t, m_{t+1}, x_t\right)\right)} \right) = \lambda_t$$

$$\beta E_t \left(\frac{\lambda_{t+1} \left(1 + i_{t+1}\right)}{\left(1 + \pi_{t+1}^c\right)} \right) = \lambda_t$$

$$\beta E_t \left(\lambda_{t+1} \left(1 + i_{t+1}^f\right) q_{t+1} \right) = \lambda_t q_t$$

$$\beta E_t \left(\eta_{t+1} + U_{H_{t+1}} \left(c_{t+1}, H_{t+1}, h_{t+1}\right) - \eta_{t+1}\rho \right) = \eta_t$$

$$\lambda_t \left(\Phi_{x_t} \left(c_t, m_{t+1}, x_t\right) + q_t \right) = \gamma_t \left(c_1 + \frac{2c_2 x_t}{k_t} \right)$$

$$k_{t+1} - (1 - \delta)k_t - f\left(\frac{x_t}{k_t} \right) k_t = 0$$

$$H_{t+1} - H_t - \rho(c_t - H_t) = 0$$

$$i_t = i + \zeta \left(\pi_t^c - \overline{\pi}^c \right) + \xi \left(y_t - \overline{y} \right)$$

$$\left(1 + i_t^f\right) = (1 + i_t^*) \left(1 + \vartheta \left(\frac{F_t}{y_t} \right) \right)$$

$$P_t^c = \left[\varepsilon (p_t^{rule})^{1-\theta} + (1 - \varepsilon)(p_t^{opt})^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

$$\left(1 + \pi_t^c\right) = \left(\varepsilon (1 + \pi_{t-1}^c)^{(1-\theta)} + (1 - \varepsilon) \left(\frac{p_t^{opt}}{P_t^c} \right)^{(1-\theta)} (1 + \pi_t^c)^{(1-\theta)} \right)^{\frac{1}{1-\theta}}$$

$$p_t^{rule} = p_{t-1}^c (1 + \pi_{t-1}^c)$$

$$\frac{p_t^{opt}}{P_t^c} = \frac{\theta}{\theta - 1} \frac{\Theta_t}{\Psi_t}$$
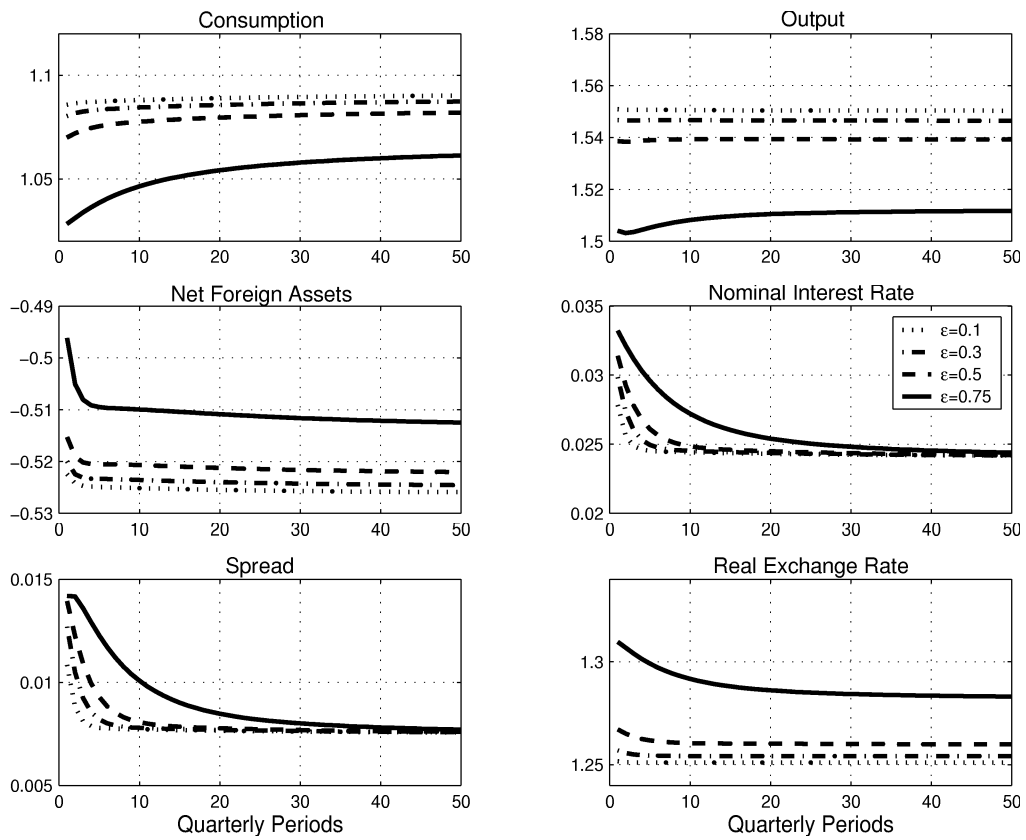
$$\Theta_t = \Delta_t c_t q_t + \varepsilon E_t \left((1 + \pi_{t+1}^c)^\theta \Theta_{t+1} \right)$$

$$\Psi_t = \Delta_t c_t + \varepsilon E_t \left((1 + \pi_{t+1}^c)^{\theta - 1} \Psi_{t+1} \right)$$

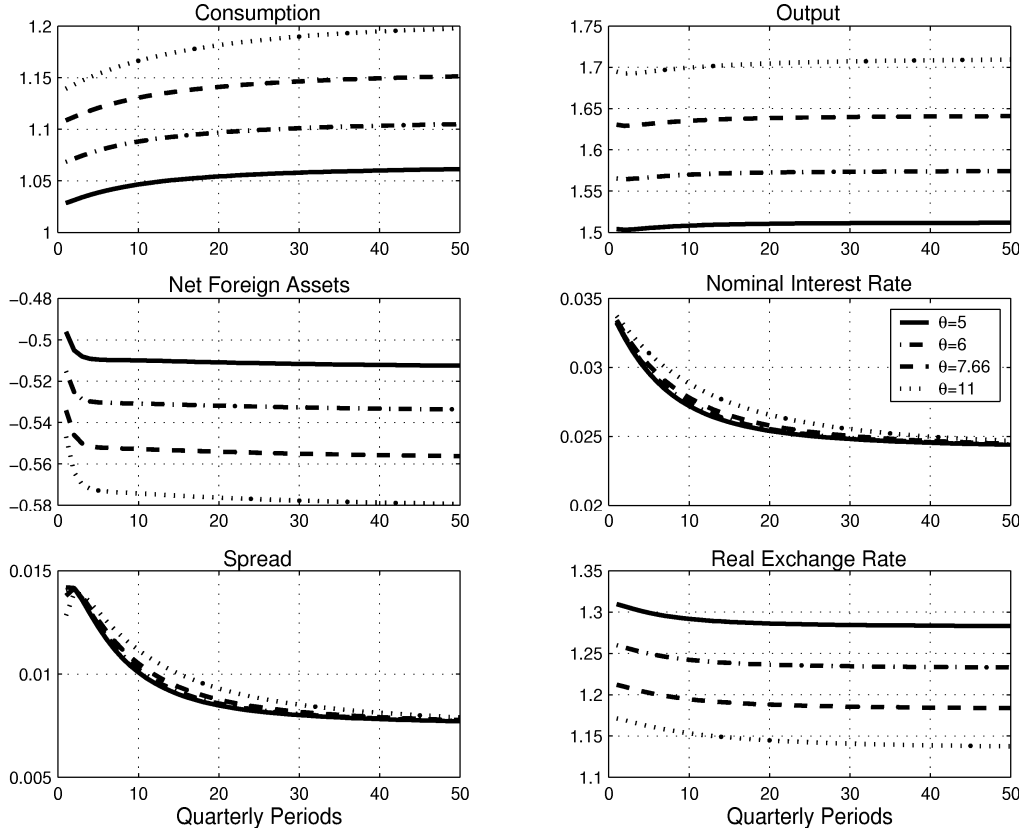# Appendix 4: Figures for Sensitivity Analysis of Impulse Responses

This appendix presents the figures used for sensitivity analysis of section 5.3.

Figure 11: Transition Dynamics when Prices are More Flexible

Note: the graph lines correspond to the transition path followed by variables in levels

Figure 12: Transition Dynamics when Markup is Lower

Note: the graph lines correspond to the transition path followed by variables in levels

Appendix 5: Derivation of the Resource Constraint

Take the monetary and fiscal authority's budget constraint 24 and solve for τ_t. Then substitute τ_t in 4 to obtain:

$$
\begin{aligned}
c_t + \Phi + m_{t+1}^d + q_t x_t + b_{t+1} + q_t F_{t+1} &= \frac{W_t}{P_t^c} h_t^s + \frac{R_t}{P_t^c} k_t^s + \frac{\Pi_t^R}{P_t^c} + m_{t+1} - \frac{m_t^d}{(1+\pi_t^c)} + \Phi \\
&\quad + \frac{m_t^d}{(1+\pi_t^c)} + \frac{b_t}{(1+\pi_t^c)}(1+i_t) + F_t q_t (1+i_t^f)
\end{aligned}
$$

canceling out terms

$$
c_t + q_t x_t + b_{t+1} + q_t F_{t+1} = \frac{W_t}{P_t^c} h_t^s + \frac{R_t}{P_t^c} k_t^s + \frac{\Pi_t^R}{P_t^c} + \frac{b_t}{(1+\pi_t^c)}(1+i_t) + F_t q_t (1+i_t^f)
$$

as we know that the real profits of retailers are

$$
\frac{\pi_t^R}{P_t^c} = c_t \left(1 - \frac{P_t}{P_t^c}\right)
$$

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we can substitute them in the previous equation to obtain

$$q_t x_t + q_t c_t + b_{t+1} - \frac{b_t}{(1 + \pi_t^c)}(1 + i_t) + q_t F_{t+1} - F_t q_t(1 + i_t^f) = q_t A_t k_t^\alpha h_t^{1-\alpha}$$

as we know that in equilibrium

$$b_{t+1} - \frac{b_t}{(1 + \pi_t^c)}(1 + i_t) = 0$$

then

$$q_t x_t + q_t c_t + q_t F_{t+1} - F_t q_t(1 + i_t^f) = q_t A_t k_t^\alpha h_t^{1-\alpha}$$

canceling out q_t we obtain the resource constraint of the economy

$$F_{t+1} - F_t(1 + i_t^f) = A_t k_t^\alpha h_t^{1-\alpha} - x_t - c_t$$

Appendix 6: Equivalence of the First Order Condition with Respect to Consumption

To simplify the explanation (and without changing the final results), lets suppose that the lagrangean of households is given by:

$$\mathcal{L} = \sum_{j=0}^{\infty} \beta^t \left[\frac{\theta}{\theta - 1} log \left(\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right) - \lambda_t \left(\left(\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right)^{\frac{\theta}{\theta-1}} + k_{t+1} - (1 - \delta) k_t - r k_t \right) \right]$$

deriving $\mathcal{L}$ with respect to $c(z)_t$

$$\frac{\partial \mathcal{L}}{\partial c(z)_t} = \beta^t \frac{\theta(\theta - 1)}{(\theta - 1)\theta \left(\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right)} c(z)_t^{-\frac{1}{\theta}} - \beta^t \lambda_t \frac{\theta}{\theta - 1} \left(\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right)^{\frac{1}{\theta-1}} \frac{\theta - 1}{\theta} c(z)_t^{-\frac{1}{\theta}} = 0$$

which is equal to

$$\frac{1}{\left(\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right) c(z)_t^{\frac{1}{\theta}}} = \lambda_t \frac{\left(\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right)^{\frac{1}{\theta-1}}}{c(z)_t^{\frac{1}{\theta}}}$$

and to

$$\frac{1}{\left(\int_0^1 c(z)_t^{\frac{\theta-1}{\theta}} dz \right)^{\frac{\theta}{\theta-1}}} = \lambda_t$$

which we know that is equal to

$$\frac{1}{c_t} = \lambda_t$$

so it has been proved that it is the same to derive with respect to c_t than to $c(z)_t$.