Estimating the COP Exchange Rate Volatility Smile and the Market Effect of Central Bank Interventions: A CHARN Approach[♣]

Juan Manuel Julio[a] , Norberto Rodríguez[b] and Héctor Manuel Zárate[c]

June 2005

Abstract

In this paper we estimated a volatility model for COP/US under two different samples, one containing the information before the "discretional interventions" started, and the other using the whole sample. We use a nonparametric approach to estimate the mean and "volatility smile" return functions using daily data. For the pre-interventions sample, we found a nonlinear expected return function and, surprisingly, a non-symmetric "volatility smile". These lack of linearity and symmetry are related to absolute returns above 1,5% and 1,0%, respectively. We also found that the "discretional interventions" did not shift the mean response function, but moved the expected returns along the line towards the required levels. In contrast, the "volatility smile" tends to increase in a non-symmetric way after accounting for "discretional interventions". The Sep/29/2004 announcement does not seem to have had any effect on the expected conditional mean or variance functions, but the Dec/17/2004 announcement seems to be related to non-symmetric effects on the volatility smile. We concluded that the announcement of discretional intervention by the monetary authority was more efficient when time and amount were unannounced.

Keywords: Volatility Smile, Exchange Rate Risk, Nonparametric Estimation, Central Bank Intervention.

JEL: C14, C22, E58, F31, E44.

♣ The authors indebt very much the valuable comments made by Silvia Juliana Mera and Munir Jalil from Banco de la República to an earlier version of this draft. Remaining errors and opinions are the sole responsibility of its authors and do not compromise the Banco de la República or its Board of Governors.

[a] Associate Econometrician, Department of Macroeconomic Models, Banco de la República, and Associate Professor, Department of Statistics, Universidad Nacional de Colombia.

[b] Associate Econometrician, Econometric Unit, Banco de la República.

[c] Chief of the Statistics Division, Economic Information and Technical Unit, Banco de la República.

1. Introduction

After a COP revaluation escalade against the USD, the Colombian central bank issued a series of intervention and regulatory announcements during the fourth quarter of 2004, aimed at controlling this trend. Figure 1 displays the COP nominal exchange rate along with the announcements and policies issued to counter the observed revaluation trend. "Discretional, central bank interventions" in the FOREX spot market started in late September 2004, but they were subject to a pre announced total buy of 1 billion USD by the end of that year. In December 17^{th} the bank announced that these "discretional" interventions were to continue indefinitely both in time and amount. From this figure it strikes that the revaluation rate actually accelerated on November and December of 2004, and that after the second announcement, the revaluation trend seems to have stopped.

Figure 1 suggests that the market reacted to the two announcements in different ways. During the fourth quarter of 2004, agents did not perceive central bank interventions as credible probably because the September 29^{th} announcement fixed the amount and time to perform the intervention. The December 17^{th} announcement dropped the amount and time constraint and seems to be more successful. However, this interpretation is still subject to the intervention size and the international environment at the two periods. In fact, the amount of intervention during the last quarter of 2004 was 1325.3 millions USD and during the first quarter of 2005 just 773.8 millions USD, which seems to confirm that the second announcement was more credible.

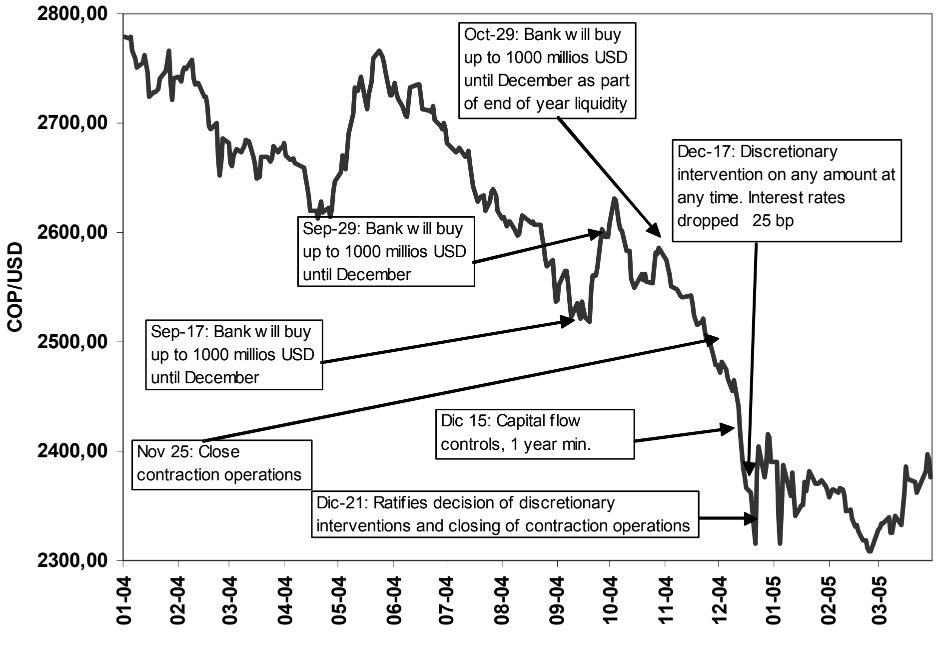

Figure 1. COP/USD Exchange Rate and Central Bank Announcements.

This new kind of "discretional" interventions adds to the Colombian central bank FOREX market intervention toolkit which used to contain only "foreign reserves management" and "short run volatility control" interventions. In fact, just two years after adopting an inflation targeting regime with floating exchange rates in 1999, the Colombian central bank announced interventions to accumulate foreign reserves and to control short run exchange rate "volatility" outbursts. Two years after that, the bank announced interventions to reduce foreign reserves, completing in this way the "foreign reserves management" intervention procedures. See Uribe and Toro (2004).

The “foreign reserves management” interventions consists of call (put) options which give the holder the right to buy (sell) foreign currency (USD) to the central bank. The amount and date of these options are set by the Board of Directors at its own discretion, and the mechanism is designed to be consistent with the achievement of the inflation target. The objective of these interventions is to manage foreign reserves holdings, a constitutional mandate for the bank, and they do not have any devaluation or exchange rate level target.

The “volatility” interventions are an automatic offering of put(call) options scheme which is triggered once the nominal daily exchange rate deviates at least 4% from its last 20 business day moving average. The amount of the auction is set by the Board of Directors at its own discretion, currently US$180 millions, and these interventions are also consistent with achieving the inflation target. The objective of these interventions is to counter volatility surprises, which may cause unwanted effects on key macro variables through the expectations channel, and, in extreme cases, undermine financial stability through currency risk. See Uribe and Toro (2004), Schaechter *et al* (2000) and Svensson (2002).

The “discretional” interventions are spot market operations. Currently the central bank buys USD from the market, on the FOREX market in an amount and timing set discretionally by the Board of Directors, according to both, the announcements and the intervention strategy chosen by the Board. The objective of these interventions, according to the Central bank press releases, is to counter the negative effects that a strong COP may have (directly or indirectly) on some Colombian economic sectors. These interventions seem to defend a wall on the nominal exchange rate to its devaluation rate.

Whether or not central bank “discretional” interventions on the FOREX market have the desired effect is a matter of debate in both the academic and central bank literature. In order to study the effectiveness of central bank interventions, researchers customarily study the conditional mean and conditional variance responses to interventions on high frequency data. These responses tell us about the effectiveness of interventions and announcements, and would give a measure of the duration of the intervention effect on the market. See Aguilar and Nydalh (2000), Dominguez (1998), Frenkel *et al* (2003), Schaechter *et al*. (2000), Toro and Julio (2005), etc.

Conventional models to evaluate the effectiveness of central bank interventions in the FOREX market rely on simple parametric or nonparametric assumptions built upon a symmetric “volatility smile”. In fact, the class of parametric models generally used for this purpose restricts to the simplest members of the ARCH, GARCH family, or to very simple nonparametric computations like the exponentially weighted moving averages EWMA. These models are characterized for having a linear mean response function and a symmetric (generally quadratic) conditional variance response function.

If the central bank does not intervene in the FOREX market this assumption may be valid provided that the market is fully efficient. In fact, central bank “discretional” interventions are recognized to be an important source of non-symmetric conditional risk. “Discretional” interventions (like the ones performed by the Colombian central bank during the last three quarters), usually pursue an implicit or explicit devaluation or exchange rate target level, wall or band, which makes the central bank go against the market. This translates into non-symmetric effects on the “volatility smile”, the market risk perception conditional on the last observed return. On the other hand, market inefficiencies may also be a source of non-symmetry. An alternative explanation of some observed market inefficiency, at least in the Colombian market, may be related to high transaction costs.

However, since central bank intervention and/or market inefficiency may be important sources of “volatility smile” non-symmetry, the use of simple or rigid models may hide relevant information on the estimated risk perception and the market effect of interventions.

In this paper we use a novel nonparametric approach to estimate the conditional mean and “volatility smile” functions, that is, the conditional mean response and conditional variance response to actual returns. The model is a member of the “Conditionally Heteroskedastic Autoregressive Nonlinear” nonparametric, CHARN, family. The models in this family are based on two equations, one for the conditional mean which explains expected returns as a function of lagged returns, and a conditional variance equation which explains the conditional variance as a function of lagged returns. Under this setup the conditional mean and conditional variance equations are unknown but assumed to be smooth and continuous. In each case, the estimated function is derived from a flexible kernel smoother, which produce unrestricted shapes. In particular, the estimated conditional mean function is not necessarily linear and the estimated “volatility smile” is not necessarily quadratic or symmetric. In this way, we are able to determine the interesting features of the “volatility smile” and mean response functions, and the market effect of central bank interventions on those functions.

Estimation of the conditional mean and “volatility smile” functions is carried out by local polynomial estimation, LPE, methods. We estimate two CHARN models, one with the sample before the “discretional” interventions started, and the other using the full sample. By comparing these two estimates, we can derive the effect of “discretional” interventions on the conditional mean and “volatility smile” functions. In addition to that, we study the effect of the two announcements regarding the “discretional, interventions.

We found that for the pre intervention sample, the expected return function conditional on the last observed return is basically linear with a small slope, a result consistent with lack of market efficiency. For this sample we found a surprising “volatility smile” lack of symmetry related to absolute returns above 1,0%. We also found the “discretional” interventions did not have mean response shifts, but moved expected returns along the line towards the required levels. However, the “volatility smile” tends to increase in a non-symmetric way after accounting for interventions. The Sep/29/2004 announcement does not seem to have had any effect on expected conditional mean or variance functions, but the Dec/17/2004 announcement seems to be related to non-symmetric effects on the volatility smile.

This paper is distributed in four sections aside from this introduction. In the second section we uncover the stylized facts of our database. In the third we describe ARCH-GARCH parametric family of volatility models, describe the CHARN family and compare both of them. In the fourth section we present the estimation results of our CHARN model and analyze the effect of “volatility options”. In the last section we conclude. Mathematical details may be found in Appendix A, as well as interpretation hints in Appendix B.

2. Stylized Facts about Exchange Rate Returns

Our whole data set consists of daily measurements of the COP/USD exchange rate. This sample is characterized for being drawn after the abandonment of the exchange rate band regime, and covers the span from Sep-27-1999 to Mar-31-2005. The pre-intervention sample ends at Sep-29-2004. The data correspond to the Colombian market representative exchange rate, the weighted average spot prices observed in Colombian inter-bank market for USD currency.

Table 1 contains the descriptive statistics for the sample. From this table we asses that the frequency distribution of exchange rate returns is highly non-normal as suggested by the result of the Jarque-Bera test. This result is explained by excess kurtosis and some degree of asymmetry. Positive mean and median returns indicate an average devaluation trend. Estimated probabilities to obtain returns below or above one, two and three standard deviations show a clear deviation from the normal distribution. In particular, it is worth observing that the estimated probability of obtaining returns beyond three standard deviations from the mean is higher than expected under the normal assumption, but the probability of falling beyond one standard deviation is very low in comparison with the normal.

Statistic	**Whole**	**Pre-Intervention**
N	1288	1245
Mean	0,0147	0,0207
Median	0,0002	0,0046
Maximum	2,7640	2,3672
Minimum	-3,0864	-3,0864
Std, Dev,	0,4240	0,4113
Skewness	0,1475	-0,0345
Kurtosis	8,2880	7,6632
Jarque-Bera **P-Value**	1505,4 0.0000	1128,3 0,0000
% Below 3 σ	0,6988	0,5622
% Below 2 σ	2,3292	3,4538
% Below σ	12,0342	13,0924
% Above σ	12,8106	11,8876
% Above 2 σ	3,1056	2,2490
% Above 3 σ	0,5435	0,6426

Table 1. Descriptive Statistics

These findings are confirmed by figure 2 which displays the frequency distribution and estimated density of exchange rate returns for the intra-day sample on the left and for daily observations on the right. From table 1 and figure 2 we conclude that the unconditional density of exchange rate returns is highly non-normal. This lack of normality comes from the excess kurtosis and high tails beyond three standard deviations. Excess kurtosis may be related to volatility clustering.

Table 2 contains the estimated autocorrelation functions for daily returns and quadratic returns. Surprisingly, daily exchange rate returns present some degree of first order autocorrelation, and partial autocorrelations present a decay pattern that may be consistent with a first order Moving Average process. This finding is consistent with some degree of inefficiency in the Colombian FOREX market, a usual finding in international literature on exchange rate returns for developing countries. See Andersen and Bollerslev (1998). These results point towards the existence of Conditional Heteroskedasticity.

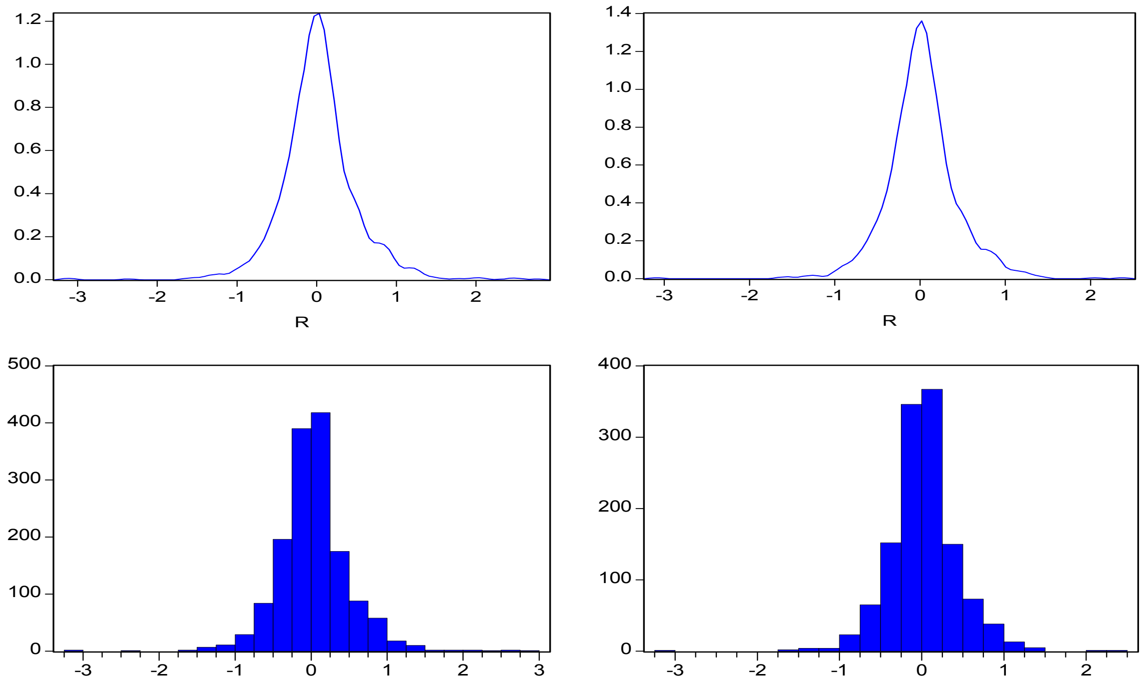

Figure 2. Kernel Estimated Density and Histogram. Whole Sample on left Panel.

	RETURNS				SQUARED RETURNS			
Lag	**AC**	**PAC**	**Q**	**PV**	**AC**	**PAC**	**Q**	**PV**
1	0,23	0,23	80	0,0	0,31	0,31	141	0,0
2	-0,08	-0,15	91	0,0	0,34	0,27	312	0,0
3	0,04	0,10	93	0,0	0,22	0,08	387	0,0
4	0,01	-0,04	93	0,0	0,16	0,01	424	0,0
5	-0,05	-0,03	97	0,0	0,17	0,06	466	0,0
6	0,00	0,02	97	0,0	0,16	0,06	502	0,0
7	0,01	-0,01	97	0,0	0,07	-0,05	509	0,0
8	0,06	0,08	103	0,0	0,10	0,02	525	0,0
9	0,10	0,07	118	0,0	0,07	0,02	533	0,0
10	0,06	0,03	122	0,0	0,06	0,00	538	0,0

Table 2. Auto-correlations for Daily Database

Figure 3 displays the daily exchange rate returns, which reveal volatility clustering and some unusual behavior at the end of the sample. It also seems to show a positive shift in variance from the second semester of 2002. Figure 4 shows the squared daily exchange returns, which confirm a high degree of conditional heteroskedasticity and volatility clustering, and also a positive variance shift from the second semester of 2002.

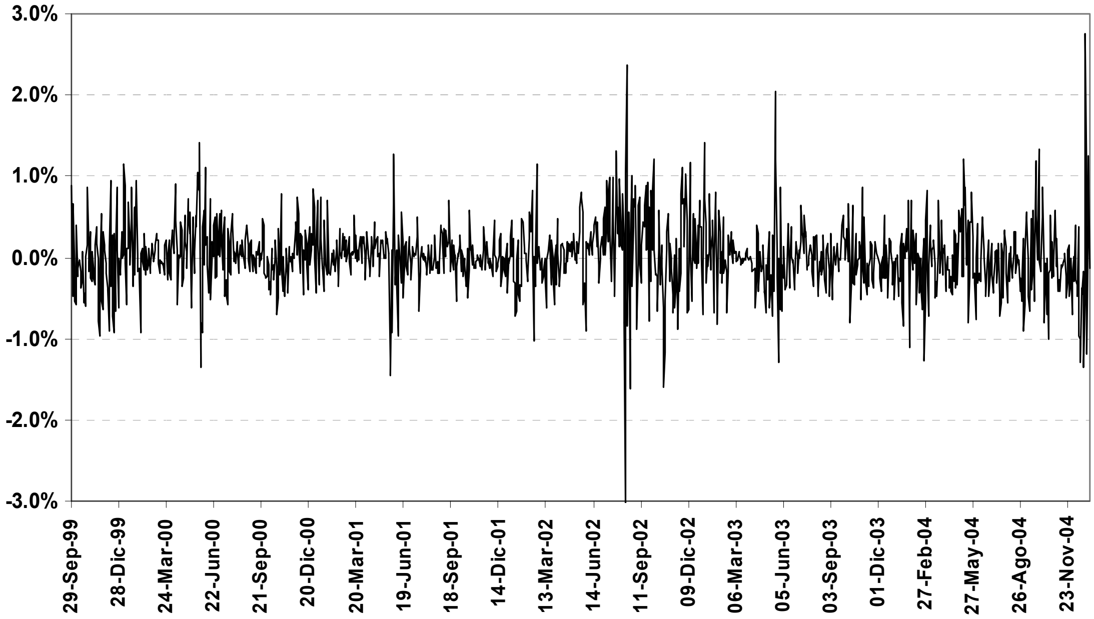

Figure 3. Daily Exchange Rate Returns.

In summary, the distribution of returns tends to be highly leptokurtic and somewhat biased, therefore highly non-normal, as usually found in financial returns distributions. There is some degree of conditional mean returns and strong evidence in favor of conditional heteroskedasticity. These findings point to conditional mean - conditional heteroskedasticity type of models for the Colombian daily exchange rate returns. These findings strongly suggest the use of nonparametric methods.

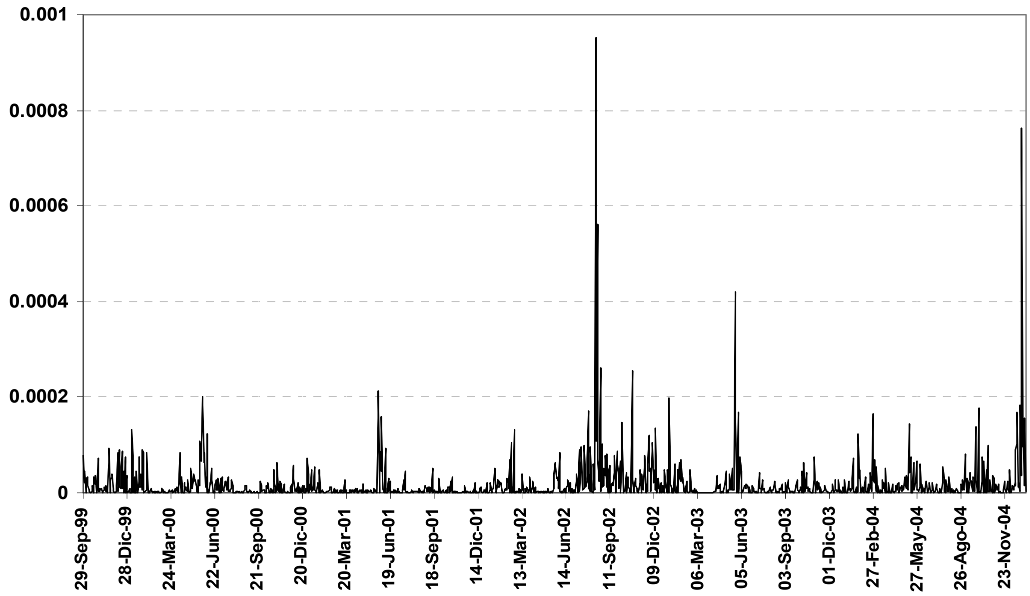

Figure 4. Squared Daily Returns.

3. Alternative Methodologies to Model Financial Returns

In this section we describe alternative methodologies which are currently used to model financial returns. The families we are interested in are those specified by three elements: First, the mean equation which describes the behavior of return levels. Second, the conditional variance equation which describes the evolution of the conditional variance. And third, the residual distribution specification.

In general the mean equation takes the form:

$$r_t = f(\beta_0, \mathbf{X}_t'\beta, \mathbf{Z}_t'\delta) + u_t\sigma_t, \quad t = 1,2,3,...,T \tag{1}$$

where r_t is the level of the financial return at time t, f is a function to be specified, β_0 is the long-run mean return level, $\mathbf{X}_t$ and $\mathbf{Z}_t$ are vectors of exogenous and dummy variables that may or may not affect the long-run mean return, β and δ are vectors of parameters, and u_t is an unobservable zero mean and unit variance random noise.

The conditional variance equation can be described as

$$\sigma_t^2 = h(\bar{h}, \mathbf{X}_t'\gamma, \mathbf{Z}_t'\lambda) \tag{2}$$

where the conditional variance is defines as $\sigma_t^2 = V\left[r_t \mid r_{t-1}, r_{t-2}, ...\right]$, h is a function to be specified, $\bar{h}$ is the long-run level of conditional variance, and a set of exogenous and dummy variables which may or may not affect the long-run level of conditional variance. The residual distribution corresponds to the behavior of the random noise u_t.

In the first sub-section we describe some of the most common members of the ARCH-GARCH family. In the second sub-section we describe the CHARN type of models. Finally, in the last subsection we compare these models emphasizing on the linearity and symmetry of the mean and variance-response functions.

3.1 Parametric Models for Financial Returns

The Auto-Regressive Conditional Heteroskedastic, ARCH, is a family of nonlinear models whose member try to capture "volatility clustering", that is, the tendency of large absolute returns to follow large absolute returns and small absolute returns to follow small absolute returns, therefore producing some degree of autocorrelation in conditional variances.

The $ARCH(q)$ model equations can be written as

$$r_t = \beta_0 + \sum_{j=1}^{p} \beta_j X_{jt} + \sum_{k=1}^{m} \delta_k Z_{kt} + u_t = \beta_0 + \mathbf{X}_t'\beta + \mathbf{Z}_t'\delta + u_t$$

$$\sigma_t^2 = \alpha_o + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 \quad t = 1,2,3,...,T$$

$$u_t \sim iid\ N\left(0, \sigma_t^2\right)$$

which includes q lags of the squared mean residuals u_{t-i}^2 in the variance equation.

Since the random error is assumed to follow a Gaussian process, $u_t \sim iid\ N(0,\sigma_t^2)$, parameter estimation is usually carried out by maximum likelihood. The log-likelihood for a sample of T observations from an $ARCH(q)$ model is given by

$$l = -\frac{T}{2}\ln(2\pi) - \frac{1}{2}\sum \ln \sigma_t^2 - \frac{1}{2}\sum (r_t - \beta_0 - \mathbf{X}_t'\beta - \mathbf{Z}_t'\delta)^2 / \sigma_t^2$$

Although the application of these kinds of models is extensively used for volatility modeling in the financial sector, they have several drawbacks. First, the unknown number of lags, q, in the variance function has to be specified in advance and might be very large, leading to a non-parsimonious model. Second, since the conditional variance must be strictly positive, non-negativity constrains on the parameters must be imposed, which in turn complicates the estimation procedure. And third, variance functions are assumed to be symmetric.

A natural extension of ARCH models that avoids over-fitting and provides a more parsimonious model is a Generalized ARCH model, $GARCH\left(p,q\right)$. This model allows the conditional variance σ_t^2 to depend also on previous lags, so that the conditional variance equation becomes

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{l=1}^{p} \xi_l \sigma_{t-l}^2$$

where conditional variance depends on a long-term average value, lagged squared residuals and lagged conditional variance.

The assumption of conditional normality for u_t may be replaced by conditional student distributed random noise in ARCH and GARCH models.

Even though this model is parsimonious and avoids over-fitting, non-negativity constrains still have to be imposed on the parameters of the variance equation. Moreover, these models do not allow for direct feedback between the conditional variance and the conditional mean, and do not provide sufficient flexibility to model asymmetric volatility effect, a fact exposed by Cleveland (1979), where it was found that negative innovations tend to have larger impacts on the conditional volatility of future observations than positive innovations, which is known as "leverage effect".

However, there are members of the $GARCH$ family which allow for asymmetric volatility effects. For instance, the so called Exponential GARCH, $EGARCH$ model whose conditional variance equation is given by

$$Log(\sigma_t^2) = \omega_t + \sum \beta_k g(r_{t-k})$$

where ω_t and β_k are deterministic coefficients and $g(r_t) = \theta r_t + \tau(|\,r_t\,| - E\,|\,r_t\,|)$.

In this model there is no need for non-negativity constrains on the parameters, and more importantly, it allows for asymmetries in the variance equation. However, empirical studies have shown that EGARCH models over-weights the effects of large positive shocks on volatility which results in poorer fits than standard GARCH models.

Other alternatives for asymmetric modeling are the Threshold GARCH Models, TGARCH also known as GJR-GARCH models. In order to fit one of these models, we divide the range of innovations into a positive and a negative interval, and then approximate the conditional variance by a piecewise linear function, allowing for differing effects of positive and negative innovations. The conditional variance equation of a TARCH$\left(q\right)$ model can be written as

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \bar{\alpha}_i \varepsilon_{t-i}^2 I(\varepsilon_{t-i} < 0)$$

where $I(\cdot)$ is an indicator function.

On the other hand, if we specify the return level equation as a function of stepwise (piecewise constant) functions, we obtain a qualitative threshold $ARCH$ model, QTARCH whose mean equation, for the one-lag QTARCH(1), can be written as

$$r_t = \sum_{j=1}^{J} m_j I(r_{t-1} \in A_j) + \sum_{j=1}^{J} s_j I(r_{t-1} \in A_j)\varepsilon_t \qquad (3)$$

where m_j and s_j>0 are scalars, A_j, $j = 1,...,J$ is a partition of the support of the lagged returns, r_{t-1}. These models characterize for having an abrupt mean transition whenever the innovations cross a threshold. See Gourieroux and Manfort (1992) for details.

There are several reasons to expect non-linearity in the volatility function of exchange rate returns, particularly because of central bank interventions and the form of its policy reaction function. In fact, since discretional central bank interventions tend to defend a wall, band or target, they tend to have a non-symmetric effect on the bid ask spread, which translates on future non-symmetric trades. See Bossaerts and Hillion (1991). On the other hand, since the discretional central bank reaction function tends to be non-symmetric, the devaluation rate may reflect this lack of symmetry. See Neuman (1984). However, the nonlinear parametric models we have presented so far have several drawbacks on this respect. Specifically, these models require a priori choices of parametric mean and conditional volatility functions which do not always capture the relevant features of the process. In addition, the mean and volatility functions are fixed, and so are their responses to innovations. Thus, the choice of a nonlinear parametric model implies a trade-off between flexibility and parsimony. Finally, in order to capture the effects we set out to, parametric models tend to be non-parsimonious.

3.2. Nonparametric CHARN Models for Financial Returns

Nonparametric models provide flexibility in the mean and conditional variance functions, as well as in its response to innovations. These models rely on flexible, nonlinear and nonparametric mean and conditional variance functions, and do not make explicit assumptions on the residual distribution. In general the mean equation may be written as

$$r_t = m(r_{t-1}) + \sigma(r_{t-1})u_t \qquad t = 1, 2..., T \qquad (4)$$

where $\{u_t\}$ is a zero mean and unit variance white noise sequence such that $\mathrm{Cov}(u_t, r_{t-1}) = 0$, $m : R \longrightarrow R$ and $\sigma : R \longrightarrow (0,\infty)$ are unknown functions. Under these conditions $E[r_t \mid r_{t-1} = x] = m(x)$.

The conditional variance equation is written as

$$\sigma^2(x) = V[r_t \mid r_{t-1} = x] \qquad (5)$$

If we make no assumptions on the white noise sequence distribution, we obtain the so called Conditional Heteroskedastic Auto-Regressive Nonlinear Model, $CHARN$, which can be interpreted as a limiting $QTARCH(1)$ case when the number of partitions goes to infinity in equation 3.

In order to get the estimator for the conditional mean and variance functions, there are two alternatives based on a multi-step procedure. The first alternative is based on the following steps: i) Estimate $\hat{m}(r_{t-1})$, ii) Estimate the equation $r_t^2 = g(r_{t-1}) + \xi_t$, yielding an estimator $\hat{g}(r_{t-1})$ for the second moment, iii) Estimate the conditional variance function $\hat{\sigma}^2(r_{t-1}) = \hat{g}(r_{t-1}) - \hat{m}^2(r_{t-1})$. The only possible problem that may arise is the presence of negative values for $\hat{\sigma}^2(r_{t-1})$. The second alternative is as follows: i) Estimate $\hat{m}(r_{t-1})$ using some nonparametric technique, ii) Estimate the Heteroskedastic residuals $\hat{\varepsilon}_t = r_t - \hat{m}\left(r_{t-1}\right)$, and demean them, $e_t = \hat{\varepsilon}_t - \overline{\varepsilon}$. Then estimate $e_t^2 = \sigma^2\left(r_{t-1}\right) + \eta_t$, leading to the estimator $\hat{\sigma}^2(r_{t-1})$, which characterizes for having $\hat{\sigma}^2(r_{t-1}) > 0$ for all t.

An important feature of nonparametric strategies based on interval specific information, is that they provide asymptotically unbiased measures, and therefore approximately serially uncorrelated measurement errors. See Andersen *et al* (2002).

Local Polynomial Estimation

Local regression strategy provides methods for fitting regression functions to measurements of two or more variables in which one is a response and others are explanatory. A function is fitted to the data to explain how the response depends on the factors without explicit assumptions on the functional form or the residual distribution. This fact makes this technique be classified as nonparametric.

Local polynomial (LP) fitting, also known as Local Weighted Regression, has gained acceptance as an attractive method for estimating the regression function and its derivatives. The advantages of this nonparametric estimation method are its simplicity, intuitiveness, ease of computation, and the fact that it achieves automatic boundary corrections. Moreover, the resulting estimators have important statistical properties. This procedure is a generalization of the Nadaraya-Watson which is a LP estimator of degree 0.

From early papers on LP, Stone (1977) and Cleveland (1979), many relevant contributions of this methodology have appeared in statistics and econometric literature, ranging from those based on the independence assumption between observations to those that allow differing degrees of dependency. A list of references on this topic can be found in Fan and Gijbels (1996).

The regression model relates the points, x_t, and the responses, r_t, through the function $r_t = m(x_t) + \varepsilon_t; t = 1,...,T$ where $m(x)$ is a smooth regression function differentiable enough (up to order $p+1$, p being the local polynomial degree). It is assumed that ε_t are homoscedastic zero mean unobserved random noise, and r_t is the strictly stationary endogenous variable. Additional assumptions about mixing conditions are imposed, and can be found in Hardle (1990).

Consider any point x, in the space of explanatory variables. It is assumed that there exists a neighborhood containing x, in which the regression surface is well approximated by a polynomial, locally fitted in a moving fashion. This procedure produces a smooth response function as required.

Estimations of the conditional mean and conditional variance in equations 4 and 5 are obtained as a result of the following sequential minimization problem

$$\hat{m}(r) = \min_{\beta_j} \sum_{t=1}^{T} \left\{ r_t - \sum_{j=0}^{p} \beta_j \left(r - x_0\right)^j \right\}^2 K_h\left(r - x_0\right)$$

$$\hat{\sigma}_t^2(r) = \min_{\phi_j} \sum_{t=1}^{T} \left\{ \hat{u}_t^2 - \sum_{j=0}^{p} \phi_j \left(r - x_0\right)^j \right\}^2 K_h\left(r - x_0\right)$$

Appendix A contains the estimation mathematical details.

3.3. Comparison

There are several reasons to expect non-linearity in the mean and conditional volatility functions of exchange rate returns. In particular, it has been found that inefficient markets and/or central bank interventions may lead towards non-symmetric "volatility smiles". In fact, if the market is not efficient, agents are not able to discount market information on future returns, leading to a higher degree of dependency on past returns. In turn, central bank interventions aimed at maintaining an implicit or explicit target, band or wall on the exchange rate level or the devaluation rate, tend to go against the market, which induces unexpectedly high or low returns after a market driven return, which in general produces non-symmetric increases/decreases on the "volatility smile".

Given that in our sample there are explicit central bank interventions, and the Colombian FOREX market is far from being efficient (as clearly seen by the small number of market participants), we expect a non-symmetric "volatility smile" and perhaps some non-linearity in the conditional mean function. Therefore, we will prefer the novel CHARN techniques over the conventional models for our study.

4. CHARN Model Estimation Results

According to our previous discussion, a CHARN model approach may uncover interesting features of the data regarding non-symmetry of the volatility response function, non-linearity of the mean response function and the leverage effect. In this section we describe the estimation results of this kind of model for the pre intervention and whole samples, and use these estimators to study the effect of central bank interventions on the "volatility smile". The first sub section describes the estimation results while the second describes the estimated effect of the interventions in mean and volatility functions. In the second, in a graphical way we study the effect of the two announcements, Sep/29/2004 and Dec/17/2004 regarding the "discretional" interventions.

4.1. Pre Intervention Sample Estimation Results

Table 3 contains the estimation summary for our database based on the non-intervention sample. From this table we can observe that the degree of the local polynomial fitted for the mean and variance equations are 1 and 2 respectively, which shows a clear, but expected, difference of behavior at a local level. The degree of the polynomial is chosen to be the one that minimizes the Akaike information criteria. The estimated smoothing parameter for the mean function is 0,759 and for the variance function 0,996. The choice of this smoothing parameter was automatically estimated by the use of cross-validation so that an adequate balance of smoothness/bias is found. Given the amount of data, the cross-validation method chose a high degree of smoothing as can be seen in the number of points in the local neighborhood. This choice seems to be the result of striking an information balance for different sub intervals of the returns support. In fact, due to the leptokurtosis property, sample information is not balanced along the returns support, and thus 99% of the sample lies in an interval between -1,5% and 1,5%, leaving almost no sample information for the rest of the support.

Name	Exp Return	Volat Smile
Fit Method	Direct	Direct
Number Obs.	1245	1245
Deg Local Poly.	1	2
Smooth Parameter	0,759	0,996
Points Local Neighborhood	945	1240
RSS	0,020	0,000
Trace[L]	5,094	4,471
GCV	0,000	0,000
AICC	-10,039	-19,283
AICC1	-12499,000	-24007,000
Delta1	1239,067	1240,132
Delta2	1238,370	1239,850
Equiv. Number Parameters	4,256	4,075
Lookup DF	1239,764	1240,413
RSE	0,004	0,000

Table 3. Estimation Summary Pre Intervention Database

Figure 5 displays the scatter-plot of observed returns and lagged returns along with the estimated conditional mean function for the pre intervention sample. The fit is poor as is commonly found in financial returns. However, the estimated mean response is not horizontal but is composed of three straight line segments. The segment located in the middle goes from lagged returns in the -1,5% to 1,5% interval and covers more than 99% of the sample. The remaining two segments cover extreme revaluations or devaluations, and since they are based on a few data points, its estimates are highly unreliable although unbiased.

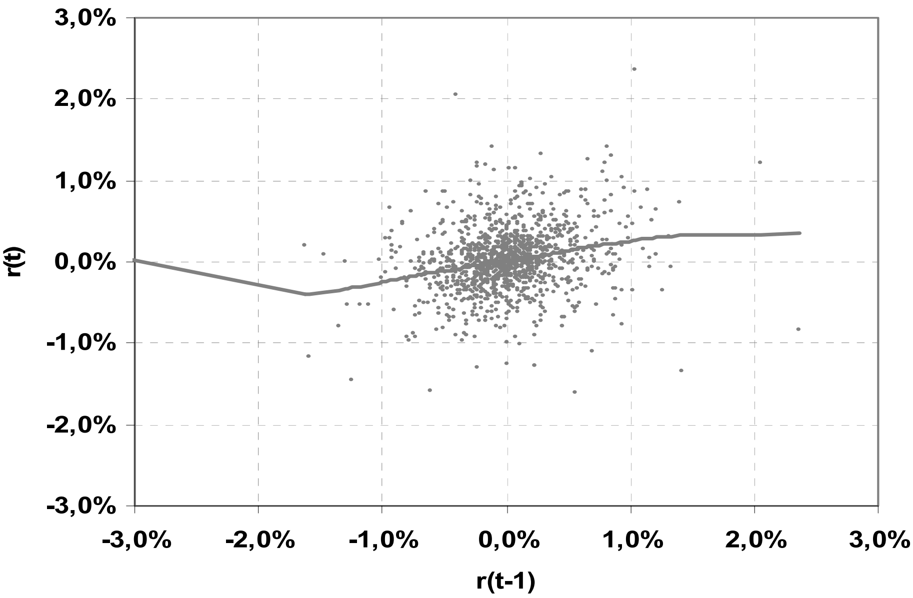

Figure 5. Conditional Mean Response Function under no Intervention.

The estimated segment at the center, the most important and more efficiently estimated of the three, increases with a very low slope of 0,23 and crosses the origin. This fact may be the result of some degree of market inefficiency since it implies that there is a small forecastable component of exchange rate returns, which may be used to pursue statistical arbitrages. However, this arbitrage may yield lower returns than other trading strategies or arbitrages present in the market. An alternative explanation to this lack of efficiency may be the widespread use of the same kind of technical analysis, a likely event in Colombia due to the market size, and the very existence of transaction costs.

Figure 6 displays the squared returns and estimated volatility conditional on last returns. This figure reveals also a poor fit, a common finding when modeling high frequency financial data. A striking result is the lack of symmetry in the "volatility smile". This result shows that on average uncertainty on future returns is higher after devaluation than after a revaluation of the same size. However, this finding is not uniform along the whole range of lagged returns. For the most common levels of lagged returns, -1,0% to 1.0%, the estimated smile is basically symmetric. For absolute returns between 1,0% and 1,5% the non-symmetry is more clear, revealing a higher conditional variance for devaluations. These results are reliable since for these levels of returns there is still considerable sample information. However, for absolute returns above 1,5% this result is not reliable since it is based on very few data points.

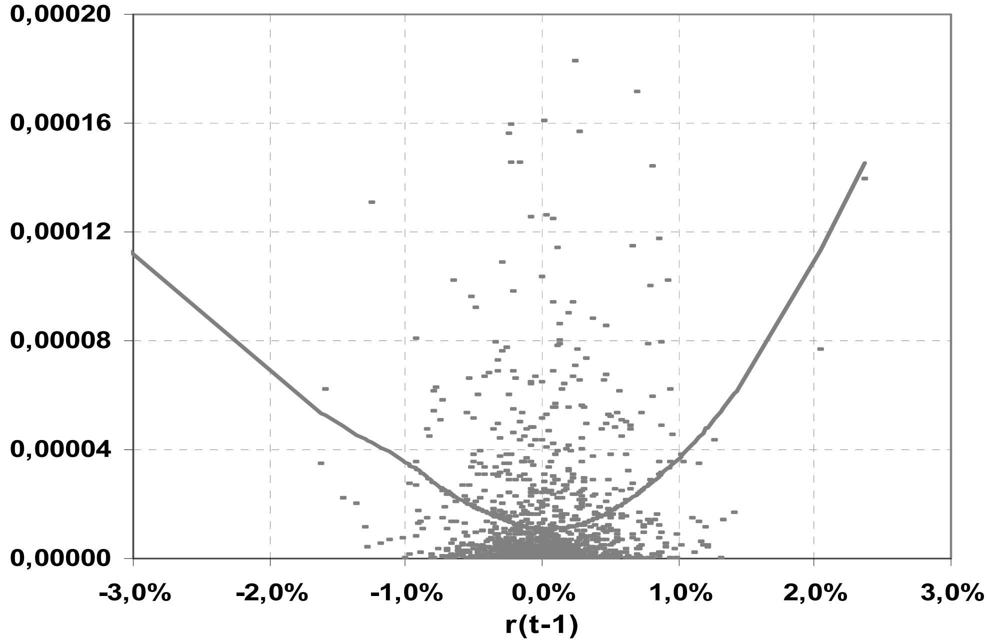

Figure 6. Conditional Variance, "Volatility Smile" Under no Intervention.

We conclude that for the relevant interval of the sample, that is, where the mean and "volatility smile" are efficiently estimated, the mean response function is linear, with a slope of 0,23, which indicates a small degree of inefficiency. For this interval the "volatility smile" surprisingly lacks symmetry. Results on the rest of the returns support are highly unreliable because of the lack of sample information.

4.2. Whole Sample Estimation Results

Table 4 contains the estimation results for the whole sample including the "discretional" interventions. From this table we can observe similar results to the ones found in the sample before the "discretional" interventions started.

Name	Expected Returns	Volatility Smile
Fit Method	Direct	Direct
Number Obs	1288	1288
Degree Local Poly	2	1
Smoothing Parameter	0,9243	0,7302
Points Local Neighborhood	1190	940
RSS	0,0217	0,0000
Trace[L]	6,0793	5,4344
GCV	1,33E-08	1,63E-12
AICC	-9,9759	-18,9813
AICC1	-12849	-24448
Delta1	1281,3660	1281.6678
Delta2	1281,0425	1280.9733
Equiv. Number Parameters	5,5247	4.53675
Lookup DF	1281,6895	1282.3627
RSE	0,0041	0.0000457

Table 4. Estimation Summary Whole Database

Figure 7 displays the scatter-plot of the observed returns and estimated conditional mean returns function for given lagged returns based on the whole sample. This figure is quite similar to the one estimated with the pre intervention sample. However, it is worth noticing that the third line segment does not clearly differ from the second as the derivative of the two segments seems to be continuous. The second line segment now looks as a nonlinear function with a decreasing slope. The first segment maintains its clear difference with the second, which in turn seems to have an increasing slope. In order to determine how big is this change, we rely on figure 8 which displays the estimated mean response for the pre intervention sample (dark line) and the estimate using the whole sample (light line). This figure reveals that the difference between the two lines is apparent.

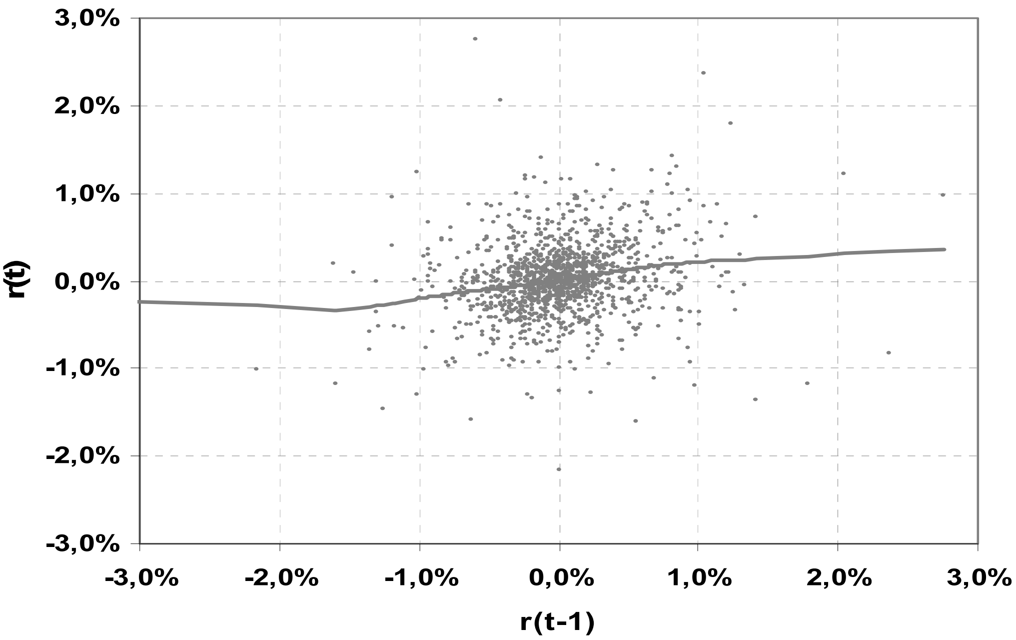

Figure 7. Conditional Mean Returns Whole Sample.

This result is consistent with the view that, for the relevant interval of lagged returns, "discretional" interventions do not affect the conditional expected returns function, but moves the returns along the line towards desired levels of return.

Figure 9 displays the estimated conditional variance response for the whole sample along with actual residuals from the estimated mean equation. For the relevant interval on lagged returns, -1,5% to 1,5%, figure 9 suggests an important degree of non-symmetry related to a concavity change beyond a moderate revaluation of 0,2%. This concavity change reveals that after a revaluation between 0,2% and 0,9% the risk on future returns is higher than the risk after a devaluation of the same size. Moreover, due to this concavity change, the risk on future returns after a revaluation between 0,9% and 1,5% is lower than the risk after a devaluation of the same size. The estimated risk on future returns after a devaluation/revaluation equals at an absolute return close to 0,9%. This result suggests that "discretional" interventions affected risk on future returns in a non-symmetric way, which induces non-symmetry in subintervals where it was not present, as can be confirmed in figure 10.

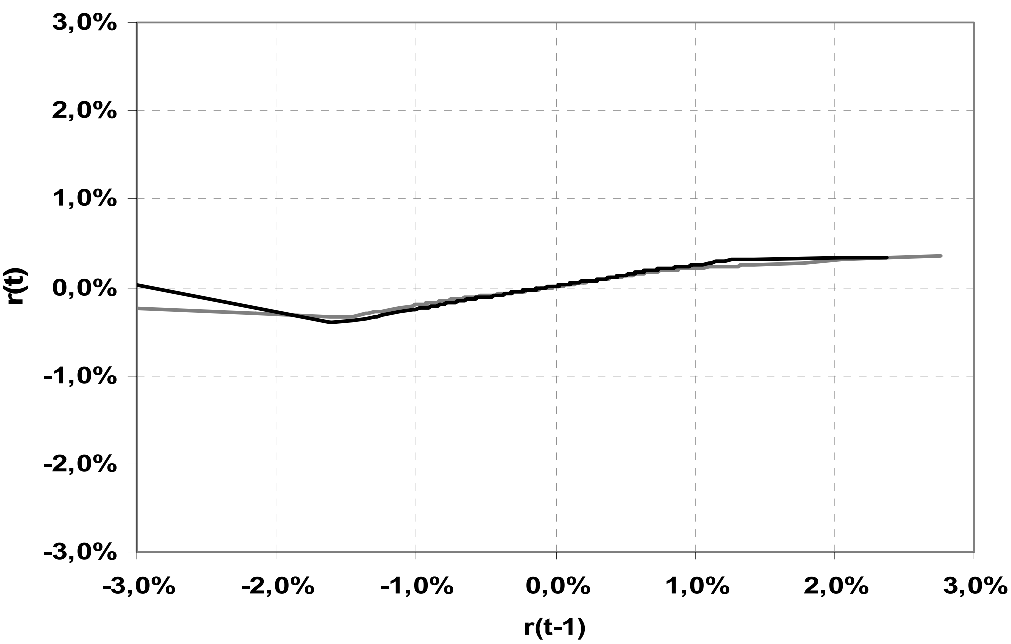

Figure 8. Estimated Mean Response Under no Intervention and Including Intervention.

This result supports the claim that central bank interventions aimed at an implicit or explicit target, band or wall on nominal exchange rates or devaluation rates induce non-symmetry in the "volatility smile".

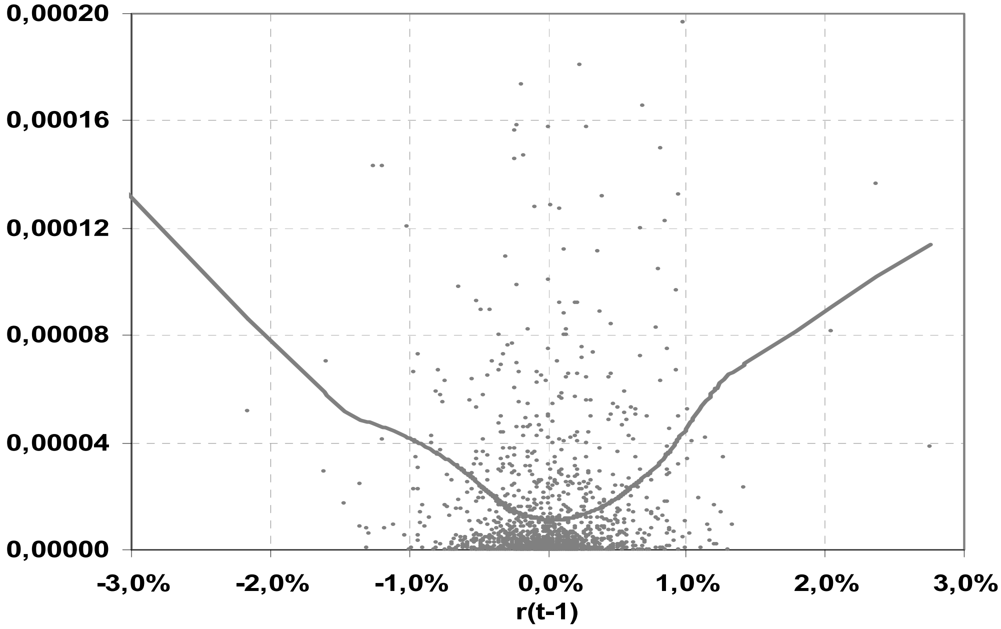

Figure 9. Squared Residuals From Mean Equation and "Volatility Smile" Whole Sample Including Interventions.

Figure 10 displays the estimated "volatility smile", for the pre intervention sample, dark line, in comparison with the estimated smile based on the whole sample. In the relevant interval, -1,5% to 1.5%, we can observe that interventions tend to increase the volatility smile in a non-uniform way. The closer the actual return is to zero, the smaller is the conditional variance increase due to interventions. For absolute lagged returns below 1.0% the conditional risk increases are faster for revaluations than for devaluations of the same level, but for absolute returns between 1.0% and 1.5% risk increases are higher for devaluations than for devaluations of the same level, with a higher increase at a lagged devaluation around 1,4%.

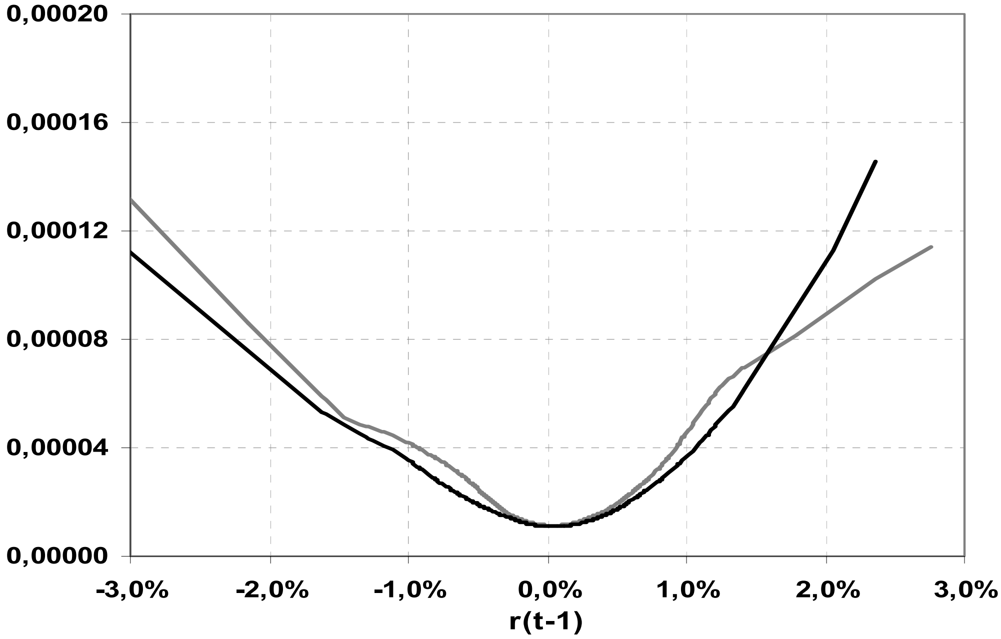

Figure 10. Estimated Volatility Smile Comparison.

Figure 11 displays the standardized residuals time series, which show that most of the relevant conditional heteroskedasticity was removed from the original data, but there still remains some persistency probably related to GARCH effects which this model is not able to capture. See Bossaerts *et al* (1996).

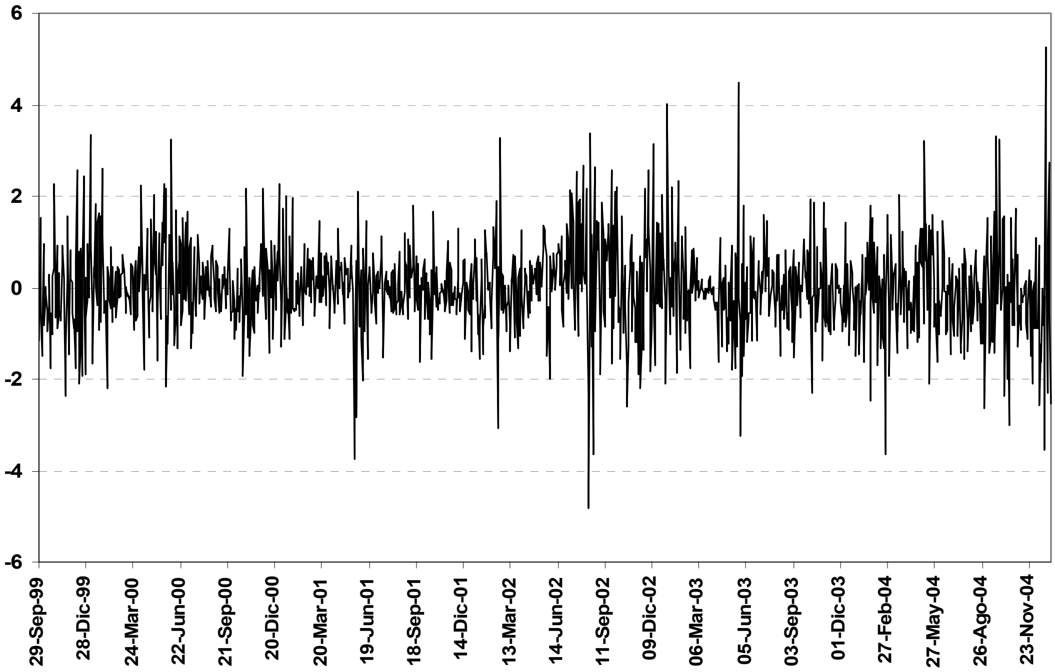

Figure 11. Standardized Residuals.

4.3. Market Effect of Central Bank Announcements

In this section we study, in a graphical way, the market response to the two alternative announcements on "discretionary interventions", Sep/29/2004 and Dec/17/2004. The Sep/29/2004 announcement informed the market about the intent of the Colombian central to buy USD from the market in a total amount of 1 billions USD up to the end of the year. This announcement was characterized for fixing the amount and time to perform this intervention. The Dec/17/2004 announcement, on the other hand, dropped the amount and time restriction on the interventions.

Due to the amount and time constraint, the September announcement was not perceived as a strong signal by the market. In fact, without the need of sophisticated tools, market agents knew exactly the USD amount bought by the bank and its effect on the market, the following day after each intervention. Since the bank announced a fixed amount and time to perform interventions, and agents knew this information, as the time went and the amount was spent, the signal sent by the central bank became weaker. In fact, according to central bank press releases, the central bank bought 1325,3 millions USD during the fourth quarter 2004, 325,3 millions above the pre announced target of one billion.

The Dec/17/2004 announcement dropped the amount and time restriction on the interventions, which was taken by the market as a stronger signal. In fact, by restricting this information the central bank announces that it will do whatever it takes in order to reach its objective of slowing down the revaluation escalade. The credibility of this announcement may be seen in the USD amount bought by the bank during the first quarter of 2004, which was just 773,8 million USD.

Figure 12 displays the estimated expected returns and volatility smile functions, along with the scatter plot of data-points before the "discretional interventions" started, light dots, the scatter plot for the dates between the two intervention announcements, Sep/29-Dec/17 2004, dark dots, and the scatter plot for the dates after the Dec/17 2004 announcement, dark stars.

From this figure it is evident that black dots tend to locate on the negative side, and are fairly concentrated on a few cells of the figure. These data points do not seem to have an important effect on the expected conditional returns or the "volatility smile". In fact, these points concentrate on places where these estimates did not change with respect to the results obtained with the pre intervention sample. Moreover, these data points tend to move along the original expected returns line instead of affecting it.

The last result contrast with the evidence of interventions after the Dec/17/2004 announcement, the dark stars. These data points look evenly dispersed for the expected returns figure, and a little less evenly dispersed for the "volatility smile" figure. From this figure there does not seem to be an important effect of these interventions on the expected returns function, but they seem to affect the volatility smile, explaining the concavity change for moderate revaluations.

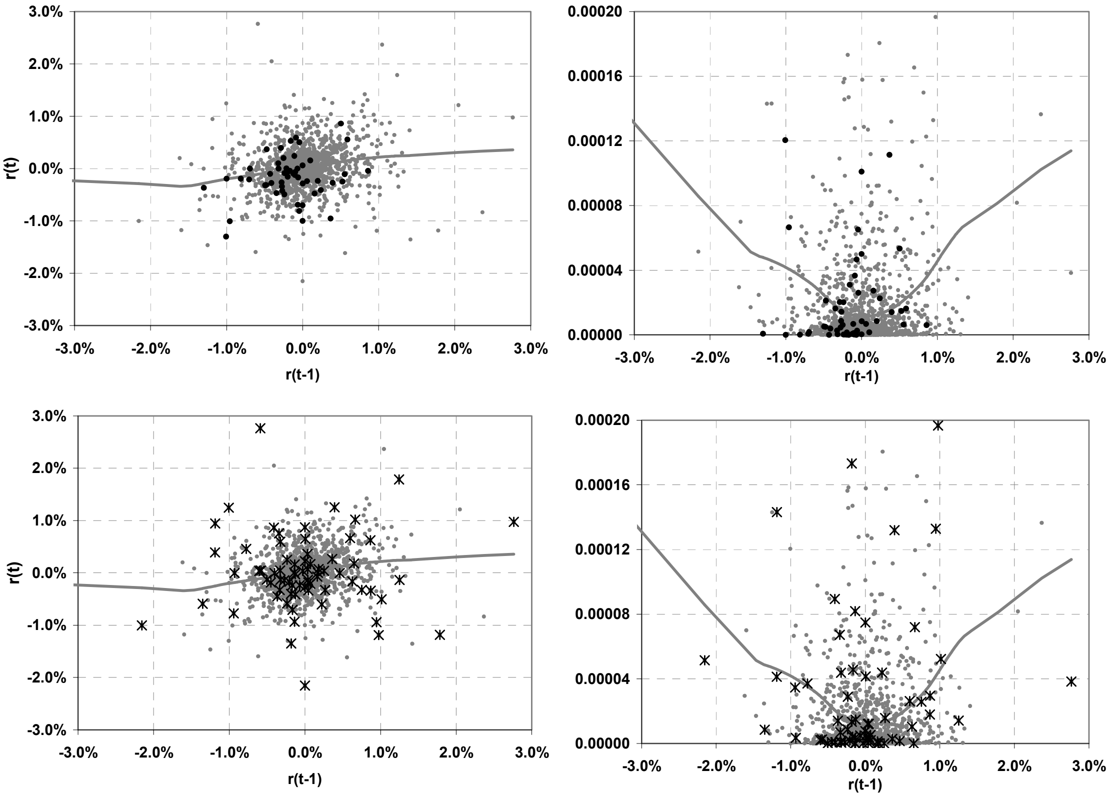

Figure 12. Estimated Expected Returns and Volatility Smile Functions and Scatter Plot Two Moments.

These results suggest that the September-2004 announcement was not as credible as the December-2004 announcement. In fact, the data points between announcements are fairly concentrated on the slight revaluation zone, but do not seem to have an important effect on the conditional expected returns line or the "volatility smile". The December-2004 announcement was more successful in spreading returns more evenly on the positive and negative intervals, shifting the location of expected returns towards zero. However, the "volatility smile" shape was affected in a non-symmetric way as described above.

5. Conclusion

Conventional models to evaluate the effectiveness of central bank interventions in the FOREX market rely on simple parametric or nonparametric assumptions built upon a symmetric "volatility smile". If the central bank does not intervene in the FOREX market this assumption may be valid provided that the market is fully efficient. However, since central bank intervention and/or market inefficiency may be important sources of "volatility smile" non-symmetry, the use of simple or rigid models may hide relevant information on the estimated risk perception and the market effect of interventions. In this paper we used a novel nonparametric approach to estimate the conditional mean and "volatility smile" functions, that is, the conditional mean response and conditional variance response to actual returns. Under this setup the conditional mean and conditional variance equations are unknown but assumed to be smooth and continuous. In each case, the estimated function derives from a flexible kernel smoother, which produce unrestricted shapes. In particular, the estimated conditional mean function is not necessarily linear and the estimated "volatility smile" is not necessarily symmetric. In this way, we are able to determine the interesting features of the "volatility smile" and mean response functions, and the market effect of central bank interventions on those functions. By using nonparametric techniques we not only avoid making explicit assumptions on the conditional mean and variance response functions behavior, but also free the results of any distributional assumption.

We estimated two CHARN models by LPE methods, one with the pre intervention sample, and the other using the whole sample. By comparing these two estimates, we derived the effect of "discretional" interventions on the conditional mean and "volatility smile" functions. In addition to that, we studied the effect of the two announcements regarding the "discretional" interventions.

For the pre intervention sample, we found that for the sub interval of the returns support where the expected return and volatility smile functions are efficiently estimated, the expected returns function conditional on the last observed return is linear with a small positive slope, which reflect some degree of non-efficiency in the market. For this sub interval we found a surprising non-symmetric "volatility smile" behavior related to absolute returns in the interval 1,0% to 1,5%, but found symmetry for absolute returns below 1,0%.

We also found that, for the -1,5% to 1,5% interval, "discretional interventions" did not change the shape of the expected returns function, but moves the points along the line in order to achieve the required level of expected returns. However, "discretional interventions" do have an important effect on the "volatility smile". In fact, "discretional interventions" tend to increase the market risk perception in a non-symmetric way. Given a positive lagged return discretional intervention increase risk on future returns in a proportional way. However, the concavity of the "volatility smile" changes after a revaluation, producing a clear lack of symmetry. Moderate lagged revaluations are perceived to have higher risk on future returns in comparison with devaluations of the same level, and revaluations beyond 0,9% seems to be related to less risk that perceived after a devaluation of the same size.

We finally found that because it included a restriction on time and amount of the intervention and agents easily knew the ex-post daily intervention amount, the Sep/29/2004 announcement was not as credible as the December/17/2004. In fact, the average daily devaluation rate between announcements was -0,17%. The data points after this announcement up to Dec/17/2004 locate on the negative side of the figure and do not seem to be related to expected returns or conditional risk shifts. The Dec/17/2004 announcement was more successful in terms of moving mean returns to the required levels but increased the market's risk perception. As a matter of fact, the daily devaluation after this announcement was 0,006%.

Our results suggest that in order to be successful, central bank announcements on discretional foreign exchange rate interventions should not set explicit deadline and amount of intervention.

References

Aguilar, J. and Nydalh, S. (2000) "Central bank intervention and exchange rates: the case of Sweden", Journal of International Financial Markets, Institutions and Money, 10, 303-322.

Andersen, T. and Bollerslev, T. (1998) "Deutsche Mark-Dollar Volatility: Intraday Activity Patterns, Macroeconomic Announcements, and Longer Run Dependencies", The Journal Of Finance, 53, # 1, 219-265.

Andersen, T., Bollerslev, T., and Diebold, F. (2002) "Parametric and No Parametric Volatility Measurement", Handbook of Financial Econometrics.

Beine, M. and Laurent, S. (2003) "Central Bank Interventions and Jumps in Double Long Memory Models of Daily Exchange Rates", Journal of Empirical Finance, 10, 641-660.

Black, F. (1976) "Studies in Stock Price Volatility Change", Proceedings of the 1976 meeting of the Business and Economics Statistics Section, American Statistical Association, pp 177-181.

Bossaerts, P., Hardle, W. and Hafner, Ch. (1996) "Foreign Exchange Rates have Surprising Volatility", Athens Conference on Applied Probability and Time Series, Robinson and Rosenblatt Editors, Springer-Verlag, New York.

Bossaerts, P. and Hillion, P. (1991) "Market Microstructure Effects of Government Intervention in the Foreign Exchange Market", Review of Financial Studies, 4, 513-541.

Cleveland, W (1979) "Robust Locally Weighted Regression and Smoothing scatter-plots". Journal of the American Statistical Association 74, 829-836.

Dominguez, K. M. (1998) "Central Bank Intervention and the Exchange Rate Volatility", Journal of International Money and Finance, 18, 161-190.

Edison, H., Cashin, P. and Liang, H. (2003) "Foreign Exchange Intervention and the Australian Dollar: Has It Mattered?" International Monetary Fund Working Paper, (May).

Fan, J. and Gijbels, I.(1996) "Local Polynomial Modeling and Its Applications" Chapman and Hall, London.

Frenkel, M., Pierdzioch, M. and Stadtmann, G. (2003) "The effects of Japanese Foreign Exchange Market Interventions on the Yen/US Dollar Exchange Rate Volatility", Kiel Working Paper No. 1165.

Gourieroux, Ch. and Alian Manford (1992) "Qualitative Threshold ARCH Models" Journal of Econometrics, Vol 52, No 1-2, pp 129-158.

Hali, J. E., Cashin, P. A. and Liang, H. (2003) "Foreign Exchange Intervention and the Australian Dollar: Has it Mattered" International Monetary Found Working Paper No. 03-99

Hardle, W. (1990) "Applied Nonparametric Regression", Cambridge University Press, U. K.

Hardle, W. and Tsibakov, A. (1997) "Local Polynomial Estimators of the Volatility function in nonparametric Auto-regression", Journal of Econometrics, 81, 223-242.

Hung, J. (1997) "Intervention strategies and Exchange Rate Volatility: A Noise Trading perspective", Journal of International Money and Finance, Vol. 16, # 5, 779-793.

Hurvich, Clifford M.; Jeffrey S. Simonoff; Chih-Ling Tsai, (1998), "Smoothing Parameter Selection in Nonparametric Regression Using an Improved Akaike Information Criterion", Journal of the Royal Statistical Society. Series B (Statistical Methodology), Vol. 60, No. 2., pp. 271-293.

Kim, S. J., Kortian, T. and Sheen, J. (2000) “Central Bank Intervention and Exchange Rate Volatility- Australian Evidence”, Journal of International Financial Markets. Institutions and Money, Sept-Dec, 381-405.

Neuman, M. (1984) “Intervention in the Mark/Dollar Market: The Authorities Reaction Function”, Journal of International Money and Finance, 3, 233-239.

Rodríguez N, N. and Siado C., P. (2003) "Un Pronóstico No-Paramétrico de la Inflación Colombiana”. Borradores de Economía, 248, Banco de la República, Bogotá. Junio de 2003; and: Revista Colombiana de Estadística, Universidad Nacional, Bogotá, 2003, Vol. 26, No. 2, pp. 89-128.

Svensson, L. (2002) “Open Economy Inflation Targeting”, Journal of International Economics, 50, 1, 155-184.

Stone, C. (1977) “Consistent Nonparametric Regression”, Annals of Statistics, 5, 595-645.

Schaechter, A., Stone, M. and Zelmer, M. (2000) “Adopting Inflation Targeting: Practical Issues for Emerging Market Countries”, International Monetary Found Occasional Papers Series # 202

Toro, J. and Julio, J. (2005) “Efectividad De La Intervención Discrecional Del Banco De La República En El Mercado Cambiario”, Borradores Semanales de Economía # 336, Banco de la República, Bogotá.

Uribe, J. and Toro, J. (2004) “Foreign Exchange Market Intervention. In: Colombia”, mimeo, Banco de la República

Appendix A
Mathematical Details

In this section we present a short review of the Local Polynomial Approach, whose theoretical ground has been developed since the work of Cleveland (1979). Empirical applications to volatility function using this framework could be found in Bossaerts *et al* (1996)

Local Regression Models: Suppose, for each t from 1 to T, that r_t is a measurement of the response and x_t is a corresponding vector of measurements of q factors. In a regression model the response and factors are related by $r_t = m(x_t) + \varepsilon_t$; where m is the regression surface and the ε_t are random errors. If x is any point in the space of the factors, $m(x)$ is the value of the surface at x; for example, $m(x_t)$ is the expected value of r_t In the fitting of local regression models we specify properties of the regression surface and the errors; that is, we make assumptions about them. More details in Fan and Gijbels (1992).

Specification of the Errors: In all cases, it is assumed that ε_t are mean 0, uncorrelated random variables, formally in the nonparametric setup the requirement are satisfied by mixing conditioning, which means locally independency. Distribution assumptions are not done in this context, which lead us to robust methods of estimation. We can specify properties of the variances of the ε_t in one of two ways. The first is simply that they have a constant variance, σ^2.

Specification of the Surface: For each x in the space of the factors, we suppose that in a certain neighborhood of x, the regression surface is well approximated by appropriate polynomial functions. The overall sizes of the neighborhoods are controlled by the value of h, which will be defined later. Size, implies a metric, and the Euclidean distance is commonly used. For two or more factors, the shapes of the neighborhoods are specified by deciding whether to normalize the scale of the factors.

Fitting is done locally. That is, for the fit at point x, the fit is made using points in a neighborhood of x, weighted by their distance from x. The size of the neighborhood is controlled by h. For h, the neighborhood includes a proportion k of the points, and the weighting shape is governed by the kernel function selected. For $h \to \infty$, all points are used, with the 'maximum distance' assumed to be h times the actual maximum distance for q explanatory variables.

The local linear estimator is unbiased when m is linear, while the Nadaraya-Watson (local mean) estimator may be biased depending on the marginal density of the design. We note here that fitting higher order polynomials can result in bias reduction, see Fan and Gijbels (1992) and Ruppert and Wand (1992) - who also extend the analysis to multidimensional explanatory variables.

By the Taylor Series expansion arguments:

$$m(r) \approx m(x_0) + m^{(1)}(x_0)(r - x_0) + \frac{m^{(2)}(x_0)}{2!}(r - x_0)^2 + \ldots + \frac{m^{(p)}(x_0)}{p!}(r - x_0)^p,$$

or $\hat{m}(r) \approx \sum_{j=0}^{p} \beta_j (r - x_0)^j$, where $m^{(j)}$ refers to the j-th derivate of m. This polynomial is fitted by a weighted least squared regression problem:

$$\min_{\beta_j} \sum_{i=1}^{T} \left\{ r_t - \sum_{j=0}^{p} \beta_j \left(r - x_0\right)^j \right\}^2 K_h\left(r - x_0\right), \qquad (*)$$

where $K_h\left(\cdot\right) = \frac{K(\cdot/h)}{h}$, h is the bandwidth controlling the size of the local neighborhood and K is a kernel function assigning weights to each data point. K should obey some restrictions. K should be symmetric around cero, continuous inside its support, integrable and usually non-negative.

Notational and computationally, it is more convenient to work with matrix notation. Denote by X the design matrix of problem in (*)

$$X = X(x_0) = \begin{pmatrix} 1 & (x_1 - x_0) & \dots & (x_1 - x_0)^p \\ 1 & (x_2 - x_0) & \dots & (x_2 - x_0)^p \\ : & : & & : \\ : & : & & : \\ 1 & (x_T - x_0) & \dots & (x_T - x_0)^p \end{pmatrix}$$

and put $Y = \begin{pmatrix} r_1 \\ : \\ : \\ r_T \end{pmatrix}$ and $\beta = \begin{pmatrix} \beta_0 \\ : \\ : \\ \beta_p \end{pmatrix}$

let W be a diagonal $T \times T$ matrix of weights: $W = diag\left\{K_h\left(x_t - x_0\right)\right\}$. Then the weighted least squares estimator can be written as $\min_{\beta}\left(Y - X\beta\right)' W\left(Y - X\beta\right)$.

The solution vector is provided by weighted least squared theory and is obtained by

$$\hat{\beta} = (X'WX)^{-1} X'WY$$

where $\hat{\beta}_j$, $j = 0,1,...,p$ denotes the solution to the least squares problem (*), and $\hat{m}(r) = e_1'\hat{\beta} = e_1'(X'WX)^{-1}X'WY = \hat{\beta}_0$, where $e_1' = (1,0,...,0)$ the first unit vector of dimension $p+1$.

With $p = 1$, the estimator $\hat{m}(r)$ is termed as local linear regression smoother or local linear fit. This estimator can be explicitly expressed as $\hat{m}(r) = \hat{\beta}_0 = \overline{r}_K - \hat{\beta}_1(\overline{x}_K - x)$ and

$$\hat{\beta}_1 = \hat{\beta}_1(r) = \frac{\sum(x_t - \overline{x}_K)(r_t - \overline{r}_K)K_t}{\sum(x_t - \overline{x}_K)K_t}.$$

where $\overline{r}_K = \left(\sum K_t\right)^{-1} \sum K_t r_t$ and $\overline{x}_K = \left(\sum K_t\right)^{-1} \sum K_t x_t$, with $K_t = K_h(X_t - x)$. These is the familiar expression for regression estimator but locally weighted according to $K(\cdot)$.

Some comments are in demand. First: the choice of h, the bandwidth, plays a crucial role. Selecting an appropriate bandwidth is one of the most important choices in practice. In this regard, two ways of selecting it are used in practice: Plug-in methods and Cross-Validation methods. Second, the order of the polynomial: not too large not too small; Akaike Information Criteria was used here in order to select between p=1 or 2. Third, the choice of the Kernel function it is not so transcendental; see Rodríguez and Siado for a list of these functions.

Appendix B
STATISTICAL RESULTS

Each Fit Summary Table (as 3 and 4 in text) displays the following characteristics of model: The fit method used for estimation may be Direct or kd tree fitting. The former implementation of the model means that fitting is done at every data point in the sample. The latter is a faster method which is done by doing the fitting at vertices of a partition of the predictor space followed by blending of the local polynomial to obtain the regression surface. The degree of the choice of the local polynomial fitting can be either linear (1) or quadratic (2). The smoothing parameter refers to the fraction of points in the neighborhood that is used to obtain the local fitting, i.e. Points in Local Neighborhood/Number of Observations. L is named the smoothing matrix and defines a linear relationship between the fitted and observed values. Thus, this matrix satisfies the identity $\hat{Y} = LY$. The summary table also reports the lookup degrees of freedom computed by: $\rho = \frac{\delta_1^2}{\delta_2}$ where $\delta_1 = tr[(I-L)'(I-L)]$ and $\delta_2 = tr[(I-L)'(I-L)]^2$. The equivalent number of parameters of the fit is found by: $tr(L'L)$. This is a measure of the amount of smoothing done by the local fitting procedure from which large values of this parameter means a reduction of the neighborhood size, which in turn, causes less smoothness of the regression surface. Finally, the residual standard error is computed in order to do statistical inference through confidence limits on the predicted value. The choice of the degree of the local polynomial, p, is done by the minimization of either an information criterion generalized by cross-validation GCV or the Akaike criterion AIC. Where $AICC$ is $ln(\hat{\sigma}^2) + n + n\frac{2(v_1+1)}{n-v_1-2}$, which is the analogous to the usual computation for parametric models, and $AICC1$ is $n\ln(\hat{\sigma}^2) + n\frac{\delta_1/\delta_2(n+v_1)}{\delta_1^2/\delta_2-2}$ from Hurvich *et al* (1998) is an Improved Akaike Information Criterion, which is more valid for nonparametric adjustments. The tri-cube weight function is used to define the weightening scheme, that is $K(h) = \frac{70}{81}(1-|h|^3)^3 I_{[-1,1]}(h)$, which has the advantage of being differentiable in all its support.