



# Evidence of non-Markovian behavior in the process of bank rating migrations\*

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July 17, 2007

## Abstract

This paper estimates transition matrices for the ratings on financial institutions, using an unusually informative data set. We show that the process of rating migration exhibits significant non-Markovian behavior, in the sense that the transition intensities are affected by macroeconomic and bank specific variables. We illustrate how the use of a continuous time framework may improve the estimation of the transition probabilities. However, the time homogeneity assumption, frequently done in economic applications, does not hold, even for short time intervals. Thus, the information provided by migrations alone is not enough to forecast the future behavior of ratings. The stage of the business cycle should be taken into account, and individual characteristics of banks must be considered as well.

**JEL Classification:** *C4, E44, G21, G23, G38.*

**Keywords:** *Financial institutions; macroeconomic variables; capitalization; supervision; transition intensities.*

## 1 Introduction

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\**Disclaimer: The views herein are those of the authors and do not necessarily represent the views of the Banco de la Republica or the Office of the Comptroller of the Currency. Acknowledgement: We thank seminar participants at Cornell University for helpful comments.*

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External ratings provide important information for managers, investors and supervisors about a firm's default risk. They summarize the firm's overall financial health by placing it into a specific category according to the perception of the risk of default, and therefore complement the information available through financial markets. In emerging market economies, where financial markets are not well developed, external ratings constitute a fundamental piece of information in the process of investment allocation. It is a regular practice for firms to pay a fee to rating companies in order to receive a grading, which will be used by investors (and also by supervisors) to make decisions that will affect the firm's future.

In the Basel Accord (Basel Committee on Banking Supervision, 2004), external ratings promote market discipline in the financial intermediation industry, in the sense that by signalling a bank's default probability to other economic agents, ratings give the bank incentives to adopt more conservative risk taking policies. If ratings are accurate, and therefore reflect closely the default probability of an institution, a bank taking higher risks is more likely to be downgraded, because higher risks imply a greater default probability. Therefore, the rating will provide a signal to investors and supervisors, and banks will be more inclined toward sound financial practices.

In order to be accurate, rating agencies need to have an adequate knowledge of the firm and the environment in which it operates. When issuing a credit rating, rating agencies use qualitative and quantitative information obtained both from public and private sources (see, for instance, Grey et al (2006)). Several studies have argued that the methodologies used by international rating agencies such as Standard & Poor's and Moody's to evaluate the risk of default of firms on emerging market economies are not completely adequate, in the sense that in order to provide "uniformity" in the rating policies across countries they sacrifice precision, because they do not take into account idiosyncratic effects properly (see, for instance, Rojas-Suarez (2001), and Ferri and Liu (2003)). Those studies argue that there exist a high positive correlation between the grade given to sovereign debt of a developing country and the grading received by firms in that country, which does not appear in the case of developed economies. Therefore, in the case of emerging economies, although ratings from international agencies are important because they provide foreign investors information about domestic firms, alternative external ratings by domestic agencies provide important complementary information for decision makers. This complementary external ratings are particularly important

for financial firms' supervisors, who benefit from tools that provide a signal about possible threats to the stability of the financial system.

Rating transition matrices are at the core of risk modelling and are a standard starting point for risk dynamics. In their application to banks, migration matrices are particularly attractive for supervisors in the sense that they are in the set of available early warning tools. The main objective is to use current ratings and the past history of rating migrations to predict future downgrades and defaults. Usually, transition dynamics are analyzed using Markov chains. In many important economic applications (e.g. J.P. Morgan's Credit Metrics), transition matrices are estimated in a discrete-time setting using a cohort method under the assumption of time-homogeneity; in a discrete and finite space setting, the probability of migrating from state  $i$  to state  $j$  is estimated by dividing the number of observed migrations from  $i$  to  $j$  in a given time period by the total number of firms in state  $i$  at the beginning of the period. One implication of this cohort method is that if no firm migrates from state  $i$  to  $j$  during the observation period, the estimate of the corresponding probability is zero. This is a not desirable feature, specially when dealing with the estimation of rare event probabilities which, in case of occurring, may have a deep impact.

Various studies have proposed using continuous time methodologies as an alternative to the cohort approach, which not only overcomes the problem of the zero estimates for rare event probabilities, but also offer additional advantages such as allowing simple tests for non-Markovian behavior. Lando and Skodeberg (2002) present the way of estimating transition probabilities in a continuous time framework, both with and without the assumption of time homogeneity. With a data set covering several years of rating history of Standard and Poor's, and using survival analysis techniques, they study two deviations from the Markov assumption: the dependence on previous rating, and waiting time effects, and find evidence that supports the hypothesis of non-Markovian behavior of migration dynamics. Other studies have reported different types of non-Markovian behavior. For instance, Kavvathas (2000) finds dependence of rating migrations on macroeconomic variables, while Jonker (2002), using a data set of ratings of banks in Europe, USA and Japan, finds that the country of origin of the bank matters in the downgrading process.

The question is not whether the ratings are in fact Markovian. With an absorbing state of default the Markovian assumption essentially implies all assets

will eventually default. The question is rather whether the Markovian specification, which provides simplicity, is adequate, and if so on what time scale.

This study contributes to the literature on rating transition dynamics by presenting evidence of non-Markovian behavior in the process of rating transition, using a rich data set on ratings of financial institutions in Colombia. Using monthly data covering the period December 1996 to November 2005, we find that macroeconomic variables, as well as bank specific variables (summarized in the capitalization ratio) affect significantly the probability of migrating from one rating category to another. The paper shows how moving from a discrete time to a continuous time framework improves the estimation of transition probabilities in the sense that the number of zero estimates is reduced, but still non-homogeneities remain. By introducing macroeconomic variables using survival analysis techniques we show that upgrades are procyclical while downgradings are countercyclical. This fact, together with evidence on the influence of bank specific factors on the migration process indicates that a simple Markov chain is not adequate for explaining the bank rating migration process in Colombia.

The dataset used in this paper is unique, in the sense that in contrast with traditional datasets from external rating agencies in which the frequency of the data is annual, the frequency of the data used here for estimation is monthly. This allows to identify with more precision the moment in which a transition occurs, and also increases the number of observed transitions, which permits a finer estimation of migration probabilities. We expect that our qualitative results hold more generally. Certainly this is worth investigating as other data become available.

Section 2 describes the data. Section 3 presents the estimation of a Markov chain using the data, both under a discrete time and a continuous time framework. It shows how the results differ whether estimation is done assuming time homogeneity or without that assumption. Section 4 presents the results of tests for the dependence of rating migration on macroeconomic and bank specific variables, and Section 5 presents conclusions.

## **2 Description of the data**

In 1994, the Department of Financial Stability (DFS) of the Banco de la República (the Central Bank of Colombia) began grading financial institutions in Colombia.

Based on financial indicators derived from their balance sheets and on expert opinion, each institution is rated into one of four non-default categories, denoted I, II, III and IV. Category I corresponds to the highest rating, while category IV corresponds to the lowest one. All institutions that are in operation at the moment in which the rating is done are rated. During the first two years, ratings were computed only once a year. However, since December 1996 the DFS decided to produce monthly ratings in order to have a tool to evaluate frequently potential risks to the soundness of the financial system in Colombia. Several different financial indicators are taken into account in the grading process. Taking into account these indicators and also considering expert opinion, a number is given to each bank. Then, the number is compared to predetermined threshold values, and each bank is assigned to one of the four categories.

Because few institutions were ever rated I, categories I and II were combined. The new categories are denoted A, B, C and the default category is D.

In this study, we consider all ratings of commercial banks and financial companies from December 1996 to November 2005<sup>1</sup>. Table 1 presents a summary of the data, showing the number of financial institutions at the beginning and the end of the observation period, as well as the number of transitions observed among the different categories.

Table 1: Summary of the dataset on rating transitions				
Number of institutions				
	Banks	F.C.'s	Total	
December 1996	42	57	99	
November 2005	29	21	50	
Average annual transitions among categories Dec 96 - Dec 04				
	A	B	C	D
A	0.7158	0.2216	0.0537	0.0089
B	0.2659	0.5630	0.1439	0.0272
C	0.0218	0.2958	0.3956	0.2868

<sup>1</sup>Financial companies specialized in commercial leasing are not included, because they are quite different, in the sense that they have different purposes than the other intermediaries mentioned before, and their activities and portfolio composition are also very different. Therefore, for the purpose of this paper, data are collected only from commercial banks and financial companies

The reduction in the number of institutions during the observation period obeys to consequences of the financial crisis that took place during the late 1990s, leading to several bank failures, and to merges and acquisitions (for details on the effects of the crisis on bank failure see Gomez-Gonzalez and Kiefer (2006)). Regarding the fraction of average annual transitions, Table 1 shows that better rated institutions are more likely to remain in the same category. For instance, on average 72% of institutions rated A at the beginning of a year are rated A at the end of the year, while only 56% of those rated B at the beginning of a year are in the same rating at the end of the year. Migrations outside of a given category concentrate on neighbor categories. For example, migrations from A to B are more frequent than migrations from A to C or D. It is important to keep in mind that Table 1 considers annual migrations only, i.e. changes in rating comparing December of a given year with December of the next year. Therefore, it does not take into account migrations occurring within the year. For example, a bank rated C in December 1996 that was rated B in June 1997 but went back to category C in December 1997 will be considered as a transition from C to C.<sup>2</sup> Therefore, the diagonal elements of this matrix tend to be higher than those of a transition matrix that considers transitions within the year.

### 3 Markov chain estimation

Markov chains are widely used to estimate migration probabilities. This section shows the results of estimating the probabilities of bank rating transitions assuming that the stochastic process underlying the observed migration dynamics can be represented adequately by a Markov chain. We present the results of estimations in a discrete time setting and in a continuous time setting. Within each of these two settings, we presents results when time-homogeneity is assumed, and when such assumption is not made.

#### 3.1 Estimation of discrete time Markov chains

Suppose we have a sample of  $N$  banks, which are observed during  $T+1$  (discrete) periods of time. At every moment of time, each bank is given a particular rating. The number of ratings is finite, and transitions from one rating to another, which

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<sup>2</sup>Table 1 was constructed this way to allow comparisons with the transition matrices provided by rating agencies like Standard and Poor's and Moody's.

are assumed to be independent across banks, are observed. Let  $n_i(t)$  denote the number of banks in category  $i$  at time  $t$ , and  $n_{ij}(t)$  the number of banks migrating from category  $i$  to  $j$  between dates  $t-1$  and  $t$ . The total number of banks exposed to migration from category  $i$ , during the whole period of observation, is given by  $N_i(T) = \sum_{t=0}^{T-1} n_i(t)$ , while the total number of transitions from rating  $i$  to  $j$  is given by  $N_{ij}(T) = \sum_{t=1}^T n_{ij}(t)$ . The rating of the banks in the first period of time observed ( $t=0$ ) is given.

If time-homogeneity is assumed, then  $p_{ij}(t) = p_{ij}$  for all  $t$ , and the log-likelihood function is given by

$$\sum_{(i,j)} N_{ij}(T) \log p_{ij}$$

Maximizing the log-likelihood function above, subject to the constraint  $\sum_{j=1}^S p_{ij} = 1$ , which indicates that every bank is rated in only one of the  $S$  possible categories at every date, we get that the maximum likelihood estimator for the probability of migrating from one category to another at time  $t$  is given by

$$\hat{p}_{ij} = \frac{N_{ij}(T)}{N_i(T)} \quad (1)$$

If the time-homogeneity assumption is removed, the maximum likelihood estimator is given by

$$\hat{p}_{ij}(t) = \frac{n_{ij}(t)}{n_i(t)} \quad (2)$$

Although time-homogeneity has the inconvenience that it is hard to justify for long periods of time, it is a very convenient assumption, specially for forecast purposes, and therefore many Markov chain applications rely on this assumption. In credit rating applications, it is frequently assumed that the process can be represented by a discrete time-homogeneous Markov chain for a one year period. Using our dataset, we performed a likelihood-ratio test, to check whether the hypothesis of time-homogeneity in a discrete time setup is adequate. Suppose we are interested in testing whether the  $i$ -th row of the transition matrix for different periods are statistically equal. We can test that with a likelihood-ratio test where



$$L_i = -2 \log \prod_{t=t'}^{T'-1} \prod_{j=1}^S \left( \frac{\widehat{p}_{ij}}{\widehat{p}_{ij}(t)} \right)^{n_{ij}(t)} \quad (3)$$

is a  $\chi^2$  random variable with  $(S-1) \times (T'-t')$  degrees of freedom. Note that  $t'$  and  $T'$  determine the period for which time homogeneity wants to be tested. It is clear that by setting  $t' = 0$  and  $T' = T - 1$ , the hypothesis is tested for the whole sample period. A test of time-homogeneity for the whole transition matrix can be done by noting that  $\sum_{i=1}^S L_i$  has a  $\chi^2$  distribution with  $S \times (S-1) \times (T'-t')$  degrees of freedom (see Thomas, Edelman and Crook (2002))<sup>3</sup>.

We performed tests of the time-homogeneity assumption for different time periods, using a roll over technique. To avoid problems with zeros in the division or  $\log(0)$ , we bounded each transition probability below by  $10^{-7}$ . This appears to be a sufficiently low bound to avoid changing the results of the test, and trying different bounds did not change the results significantly. We performed this test for different periodicity, ranging from four months to two years, using a roll over technique. We calculated the  $\chi^2$  statistic, together with the corresponding p-value, and found that the hypothesis of time-homogeneity can be rejected at very low significance levels (even for four months, in most cases). In all cases the null hypothesis can be rejected at the 5 percent level for eleven months or more.

These results provide strong evidence that the rating migration process underlying our data set is not time-homogeneous. In fact, misleading conclusions can be derived from imposing this assumption on the data.

We now turn to continuous-time estimation, avoiding the awkward question of the definition of the period<sup>4</sup>.

### 3.2 Estimation of continuous time Markov chains

External rating systems may have trouble when estimating continuous time Markov chains using migrations data, if they do not have sufficiently frequent information to update the rating of a firm as soon as a change occurs that takes the firm to a different risk profile. Internal rating systems do not have this problem, because

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<sup>3</sup>A likelihood ratio test for the assumption of time homogeneity when data are not observed in regular intervals is developed in Kiefer and Larson (2007).

<sup>4</sup>One important advantage of continuous time estimation is that it avoids the problem of defining the period: should data be observed monthly, quarterly, annually? Does the frequency with which data is observed correspond to the frequency with which transitions occur?

they have the required information at every moment of time; particularly, with internal information the exact moment at which a transition occurs can be recorded. Our data is somewhere in between. We do not have information about the exact moment in which the migration occurred, but we have a good approximation to it due to the relatively high frequency of the data; the fact that we have ratings available on a monthly basis allows us to estimate continuous time Markov chains for different time periods.

### 3.2.1 Estimation under time-homogeneity assumption

A starting point for estimating continuous time Markov chains is the assumption of time-homogeneity. Above we showed that this assumption does not seem adequate in the discrete-time specification and therefore is unlikely to hold in continuous time; however, it provides a good starting point, a benchmark to compare the results obtained when this assumption is removed, and when covariates are included in the estimation.

Suppose we observe the ratings of  $N$  banks between time 0 and time  $T$ . Assume that the state space is finite, being 1 the highest category and  $S$  the lowest one. For a given time period, let  $P(t)$  be the transition matrix. This matrix can be expressed in terms of transition intensities, which appears to be a more natural way to formulate statistical hypotheses (Lando (2004)), by noting that

$$P(t) = \exp(\Lambda t), \quad t \geq 0$$

where  $\Lambda$  represents the generator matrix. Given that for any  $t$ , the transition matrix is a function of the generator matrix, we can obtain maximum likelihood estimates of the elements of the transition matrix by obtaining first maximum likelihood estimators of the elements of the generator matrix, and then applying the exponential matrix function to this estimates, after scaling appropriately by  $t$ . The element of the generator matrix are the transition intensities, whose maximum likelihood estimator (Kuchler and Sorensen, 1997) is given by

$$\hat{\lambda}_{ij} = \frac{N_{ij}(T)}{\int_0^T Y_i(s) ds}, \quad \text{for } i \neq j \quad (4)$$

where  $Y_i(s)$  is the number of banks rated  $i$  at time  $s$ . The diagonal elements

are  $\hat{\lambda}_{ii} = - \sum_{j \neq i} \hat{\lambda}_{ij}$ . The key point here is that the denominator takes into account every bank that has been rated  $i$  during some time during the observation period. Therefore, this method uses information differently than the cohort method.

The advantage of this method is that it takes into account not only direct transitions from one rating class to another, but also "indirect" transitions. In particular, the estimation of a transition will be strictly positive if during the observation period there was a sequence of migrations between intermediate rating classes, even if there is no single bank that experienced all those migrations. For example, if we are interested in estimating the probability of a rare event, say the one year transition from category A to default, but no bank experienced this transition directly, we can still obtain a positive estimate if there was at least one bank which migrated from A to B, at least one which migrated from B to C, and at least one which migrated from C to default, even if the migrating banks are different, during the observation period. Using our dataset we still had some zero estimates for some probabilities in some time intervals, because some periods of time presented very few transitions.

For illustration purposes only, we present the average one year transition matrix of the data set:

$$\hat{P}(1) = \begin{pmatrix} 0.5353 & 0.3291 & 0.1117 & 0.0238 \\ 0.3065 & 0.4254 & 0.2114 & 0.0567 \\ 0.1845 & 0.3294 & 0.3434 & 0.1427 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Note that all probabilities are strictly positive, except for transitions out of default, which is assumed to be an absorbing state.

One may be tempted to assume that rating dynamics can be modeled adequately by using a continuous time homogeneous Markov chain. This would indeed be very conveniently, as using current data one could calculate the aggregate number of transitions between any two categories; this would in turn be very useful for supervisors. However, using a rollover estimation technique, it is clear that non homogeneities appear. Figures 1, 2 and 3 show time series for one year transition intensities away from categories A, B and C, respectively, estimated under the time homogeneity assumption. From these figures it can be seen clearly that transition intensities vary a lot in time. This holds true when the estimation period of the Markov chains is modified. Therefore, even though it would be useful to assume

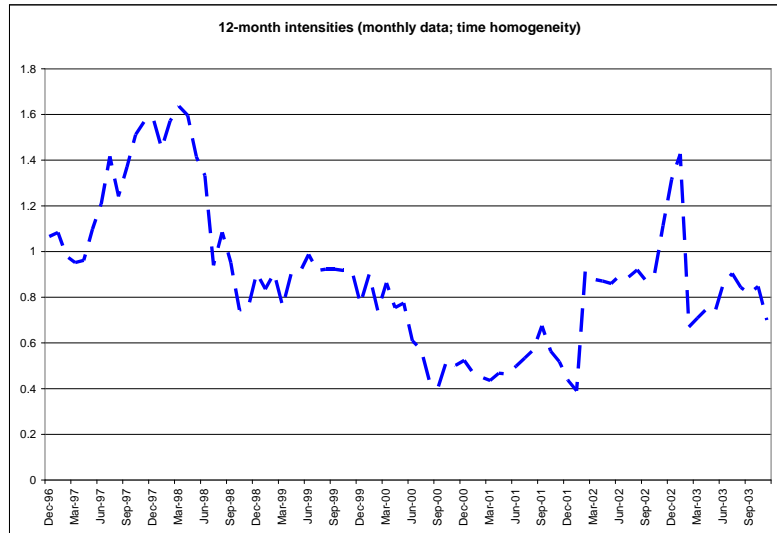


Figure 3.1: Transitions from category A to B

time homogeneity in rating migrations estimation, this assumption does not seem to be adequate.

### 3.2.2 Estimation without the time homogeneity assumption

An alternative non-parametric method exists to estimate continuous time Markov chains without assuming time homogeneity. The method is based in the Aalen-Johansen estimator (for a discussion see Lando and Skodeberg (2002)). Suppose  $m$  transitions are observed during a period of time  $s$ . The transition matrix,  $P(s)$ , is consistently estimated by

$$\hat{P}(s) = \prod_{i=1}^m \left( I + \Delta \hat{A}(T_i) \right) \quad (5)$$

where  $I$  is the identity matrix, and  $T_i$  is a jump time occurring in the observation period;  $\Delta \hat{A}(T_i)$  is a matrix in which the non-diagonal entry  $ij$  is given by the ratio of the number of transitions observed from state  $i$  to state  $j$  at date  $T_i$  and the total number of banks in state  $i$  at the instant right before the time of the jump. The diagonal entries are given by the negative of the summation of the non-diagonal entries of the row, so each row in the matrix adds up to zero. The last row of this matrix is a zero vector, as there are no transitions out of default. As it can be seen, this method also allows censoring properly.

The average one year continuous time transition matrix estimated without

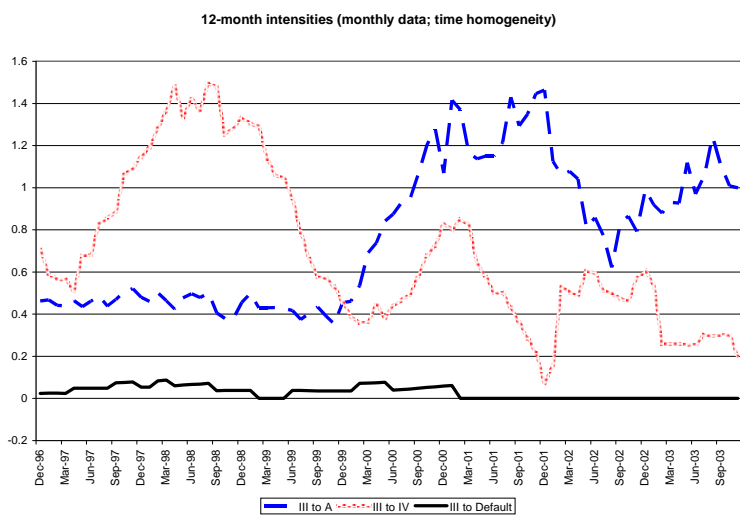


Figure 3.2: Transitions out of category B

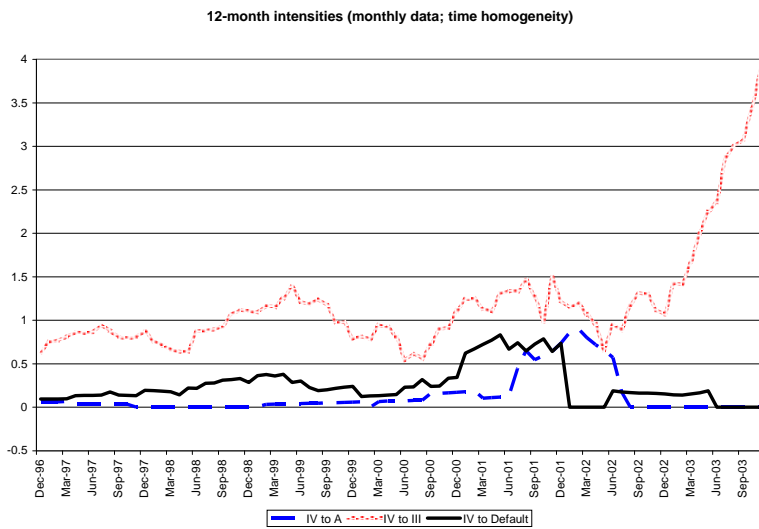


Figure 3.3: Transitions out of category C

using the time homogeneity assumption is given by:

$$\hat{P}(1) = \begin{pmatrix} 0.5489 & 0.3364 & 0.0953 & 0.0193 \\ 0.3196 & 0.4247 & 0.1992 & 0.0565 \\ 0.1911 & 0.3522 & 0.3017 & 0.1550 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Note that the average one year continuous time transition matrix estimated without using the time homogeneity assumption looks similar to the one estimated under the time homogeneity assumption. Therefore, if the rollover method were not used, one may be tempted to conclude that the homogeneity assumption seems appropriate. However, as it was discussed above, the huge variations over time of the transition matrix estimated under the time homogeneity assumption show clearly that this assumption is not adequate in this context.

## 4 Introducing covariates to explain migration dynamics

Above we showed that rating dynamics vary over time. It is not clear why. Different studies have shown that different covariates influence significantly the transition probabilities. Jonker (2002), using a data set of ratings of banks in Europe, USA and Japan, finds that the country of origin of the bank matters in the downgrading process. Bangia et al (2002), using data from the Standard & Poor's CreditPro 3.0 database, show that the business cycle influences significantly credit migration matrices, by separating the economy into two states (contraction and expansion) and computing transition matrices for these states separately. Lando and Skodeberg (2002), and Kavvathas (2000) use survival analysis techniques to show the influence of migration matrices on previous rating and waiting time effects, and on macroeconomic variables, respectively.

This study introduces macroeconomic variables and bank specific variables (summarized by the capitalization ratio) to explain bank rating dynamics. Covariates are introduced using survival analysis techniques, which appears to be a very convenient way of doing so (for an introduction to these methods in general see Klein and Moeschberger (2003), and for an introduction to the application of these methods in economics see Kiefer (1988)), because censoring is handled, and the time a bank spends in a given category provides useful information for

estimating the transition probabilities.

Given the frequency of the data, the set of macroeconomic variables that can be used effectively is limited<sup>5</sup>. Two different macroeconomic variables were used: the monthly average interest rate on deposits (RIR), computed by the Banco de la Republica, and the real production index (RPI) provided by the Department of National Statistics of Colombia (DANE). Monthly information for these two variables was collected from November 1996 to November 2005. Both macroeconomic variables are included in the regressions with one lag. Additionally, the capitalization ratio (CAP), given by the ratio of equity and assets, was used as a proxy for the financial institutions' overall financial health. Although other financial variables are also important bank specific indicators, CAP is a special indicator determining the probability of bank failure in Colombia (see Gomez-Gonzalez and Kiefer, 2006), and therefore it seems to be a variable which summarizes compactly the overall financial performance of a bank<sup>6</sup>.

Let  $\lambda_{ij}^n(t)$  denote the transition intensity from category  $i$  to category  $j$  of bank  $n$ . Then,

$$\lambda_{ij}^n(t) = Y_i^n(t)\alpha_{ij}^n(\beta_{ij}, t, X^n(t)) \quad (6)$$

where  $Y_i^n(t)$  is an indicator function which takes the value 1 if the firm is rated in category  $i$  at time  $t$  and 0 otherwise;  $\alpha_{ij}^n(\beta_{ij}, t, X^n(t))$  is a function both of time and of a vector of covariates of bank  $n$  at time  $t$ , denoted  $X^n(t)$ . In this study, we use time varying covariates; however, if time varying covariates are not available or if the covariates to be included do not vary during the observation period, a vector of fixed covariates can be used. It is assumed that the function  $\alpha_{ij}^n(\beta_{ij}, t, X^n(t))$  has the multiplicative (proportional hazards) form, as in Cox (1972):

$$\alpha_{ij}^n(\beta_{ij}, t, X^n(t)) = \alpha_{ij}^0(t)\phi(\beta_{ij}, X^n(t)) \quad (7)$$

where  $\alpha_{ij}^0(t)$  represents the baseline intensity, common to all banks, which captures the direct effect of time on the transition intensity. For estimation purposes, a functional form is specified for  $\phi(\beta_{ij}, X^n(t))$ , while the baseline intensity is let

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<sup>5</sup>We expect that one or two variables would be enough to capture the major macroeconomic effects. Recall that the theoretical model underlying the Basel II regulation is a single-factor model.

<sup>6</sup>Regarding practical matters, CAP performed better in terms of fit than other bank-specific financial variables such as profitability, non-performing loans and different measures of efficiency and asset composition.

unspecified (the only restriction is that it is non-negative). A functional form which is frequently chosen for  $\phi()$  is an exponential form,  $\phi(\beta_{ij}, X^n(t)) = \exp(\beta_{ij}X^n(t))$ , which has the advantage of guaranteeing non-negativity without imposing any restrictions on the values of the parameters of interest ( $\beta'_{ij}s$ ). The model is estimated by the method of partial likelihood estimation, developed by Cox (1972).

Tables 2 to 4 present the results of the estimation when only macroeconomic variables are included as covariates. Note that RIR affects significantly all transition intensities, except for that from category C to default. The sign of the coefficient corresponding to this covariate is the expected one in all regressions in which it is significant: when the transition implies a downgrading, the sign of RIR is positive, indicating that increases in the real interest rate lead to increases in the probability of a downgrading. When the transition implies an upgrading, the sign of RIR is negative, indicating that increases in the real interest rate lead to decreases in the probability of an upgrading. Taking into account that the interest rate is countercyclical, this implies that migrations depend on the business cycle.

Meanwhile, the impact of the RPI on the transition intensities is non-significantly different from zero in most of the cases (at a 5 percent level of significance). However, the sign of the coefficient corresponding to this variable is always the expected one: positive when the transition implies an upgrading and negative when the transition implies a downgrading. Additionally, RPI and RIR are jointly significant at the 5 percent level in all regressions except on those from category A to category B (they are significant at the 10 percent level in this case) and from category C to default.

It is interesting to note that, contrary to what occurs with all other transitions, no macroeconomic variable is significant in explaining migrations from category C to default. A possible reason is that few transitions from C to default are observed (relative to the number of banks exposed in category C).

Note RIR is a better explanatory variable than RPI in terms of fit and when both are included RPI is typically insignificant.



Table 2: Transitions away from category A

1. Transition from A to B

	Model 1		Model 2		Model 3	
Covariate	Coeff.	Std.Dev.	Coeff.	Std.Dev.	Coeff.	Std.Dev.
RIR	0.0348	0.1447			0.0363	0.0147
RPI			-0.0002	0.0106	0.0053	0.0109
Log-likelihood	-596.91		-599.58		-596.79	
LR $\chi^2(d.f.)$	5.34 (1)		0.00 (1)		5.57 (2)	
Prob $>\chi^2$	0.0209		0.984		0.0616	

Table 3: Transitions away from category B

1. Transition from B to A

	Model 1		Model 2		Model 3	
Covariate	Coeff.	Std.Dev.	Coeff.	Std.Dev.	Coeff.	Std.Dev.
RIR	-0.0448	0.0163			-0.0427	0.0164
RPI			0.0187	0.0108	0.0158	0.0110
Log-likelihood	-683.09		-685.74		-682.04	
LR $\chi^2(d.f.)$	8.32 (1)		3.03 (1)		10.42 (2)	
Prob $>\chi^2$	0.0039		0.0820		0.0055	

2. Transitions from B to C

Covariates	Model 1		Model 2		Model 3	
RIR	0.0730	0.0120			0.0719	0.0121
RPI			-0.0210	0.0104	-0.0163	0.0103
Log-likelihood	-703.79		-718.98		-702.55	
LR $\chi^2(d.f.)$	34.41 (1)		4.05 (1)		36.90 (2)	
Prob $>\chi^2$	0.0000		0.0442		0.0000	

Table 4: Transitions away from category C

## 1. Transition from C to B

	Model 1		Model 2		Model 3	
Covariate	Coeff.	Std.Dev.	Coeff.	Std.Dev.	Coeff.	Std.Dev.
RIR	-0.0331	0.0149			-0.0330	0.0149
RPI			0.0055	0.01021	0.0053	0.0103
Log-likelihood	-585.10		-587.53		-584.96	
LR $\chi^2(d.f.)$	5.16 (1)		0.29 (1)		5.43 (2)	
Prob> $\chi^2$	0.0231		0.5879		0.0662	

## 2. Transitions from C to Default

Covariates	Model 1		Model 2		Model 3	
RIR	-0.0503	0.0352			-0.0532	0.0356
RPI			-0.0503	0.0352	-0.0199	0.0243
Log-likelihood	-97.34		-98.18		-97.00	
LR $\chi^2(d.f.)$	2.16 (1)		0.48 (1)		2.84 (2)	
Prob> $\chi^2$	0.1416		0.4874		0.2419	

Tables 5 to 7 present the results of the estimation when the capitalization ratio is included as a covariate. Two different models are presented for each rating migration: one in which CAP is the only covariate included, and another in which CAP and RIR are included. It is interesting to note that, similar to the case in which only macro variables were included, neither CAP nor RIR appear to affect significantly the transition intensity from category C to default. This can probably be explained by the low proportion of defaults out of bank exposures in category C, together with the fact that banks that spend a long time in category C are already in bad financial health, independently on whether they default or not.

Another important feature is that in all other regressions the two covariates included result jointly significant at the 5 percent level. The signs of the coefficients of these two variables are the expected in all cases (the coefficient of CAP is positive when the transition implies an upgrading and negative when the transition implies a downgrading, while the coefficient of RIR is negative when the transition implies an upgrading and negative when the transition implies a downgrading), except

for the sign of CAP in the migration from A to B, which is the opposite to the expected one.

Table 5: Transitions away from category A				
1. Transition from A to B				
Covariate	Model 1		Model 2	
CAP	0.0068	0.0052	0.0065	0.0052
RIR			0.0349	0.0146
Log-likelihood	-595.07		-592.42	
LR $\chi^2(d.f.)$	1.56 (1)		6.84 (2)	
Prob> $\chi^2$	0.2115		0.0327	

Table 6: Transitions away from category B				
1. Transition from B to A				
Covariate	Model 1		Model 2	
CAP	0.0086	0.0050	0.01055	0.0051
RIR			-0.0483	0.0165
Log-likelihood	-685.64		-680.88	
LR $\chi^2(d.f.)$	2.55 (1)		12.07 (2)	
Prob> $\chi^2$	0.1104		0.0024	
2. Transition from B to C				
Covariates	Model 1		Model 2	
CAP	-0.0108	0.0074	-0.0158	0.0079
RIR			0.0771	0.121
Log-likelihood	-708.19		-689.34	
LR $\chi^2(d.f.)$	2.45 (1)		40.14 (2)	
Prob> $\chi^2$	0.1177		0.0000	

Table 7: Transitions away from category C

1. Transition from C to B

Covariate	Model 1		Model 2	
CAP	0.0054	0.0069	0.0054	0.0069
RIR			-0.0334	0.0148
Log-likelihood	-587.24		-584.60	
LR $\chi^2(d.f.)$	0.57 (1)		5.85 (2)	
Prob $>\chi^2$	0.4499		0.0537	

2. Transition from C to Default

Covariates	Model 1		Model 2	
CAP	0.0192	0.0148	0.0191	0.0148
RIR			-0.0502	0.0350
Log-likelihood	-97.71		-96.62	
LR $\chi^2(d.f.)$	1.39 (1)		3.57 (2)	
Prob $>\chi^2$	0.2385		0.1675	

Altogether, the results of the regressions indicate that the process of rating dynamics depends on external covariates related to the business cycle and on bank specific ratios. This provides evidence that supports the idea that a simple Markov model is not adequate to represent this process. The evidence reported here complements evidence of non-Markovian behavior on rating migrations reported in other studies that use datasets with different time scales, and test for dependence in different sets of covariates.

## 5 Conclusions

This paper estimates transition matrices for the ratings on financial institutions in Colombia. Using an unusually informative data set, we show that the process of rating migration exhibits significant non-Markovian behavior, in the sense that the transition intensities are affected by macroeconomic and bank specific variables. The monthly real interest rate influences significantly all transition migrations to neighboring ratings, except for the migration from category C to default. The same conclusion holds when the capitalization ratio is included in the regression.

The use of a continuous time framework may improve the estimation of the transition probabilities, in the sense that problems related to zero probability estimates of rare events can be avoided, but, as well as in other studies, this study finds that the time homogeneity assumption, commonly assumed in important economic applications, does not hold, not even for short periods of time. Therefore, the information provided by migrations alone is not enough to forecast the future behavior of ratings. The stage of the business cycle should be taken into account, and individual characteristics of banks must be considered as well.

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