The Fan Chart: The Technical Details of the New Implementation

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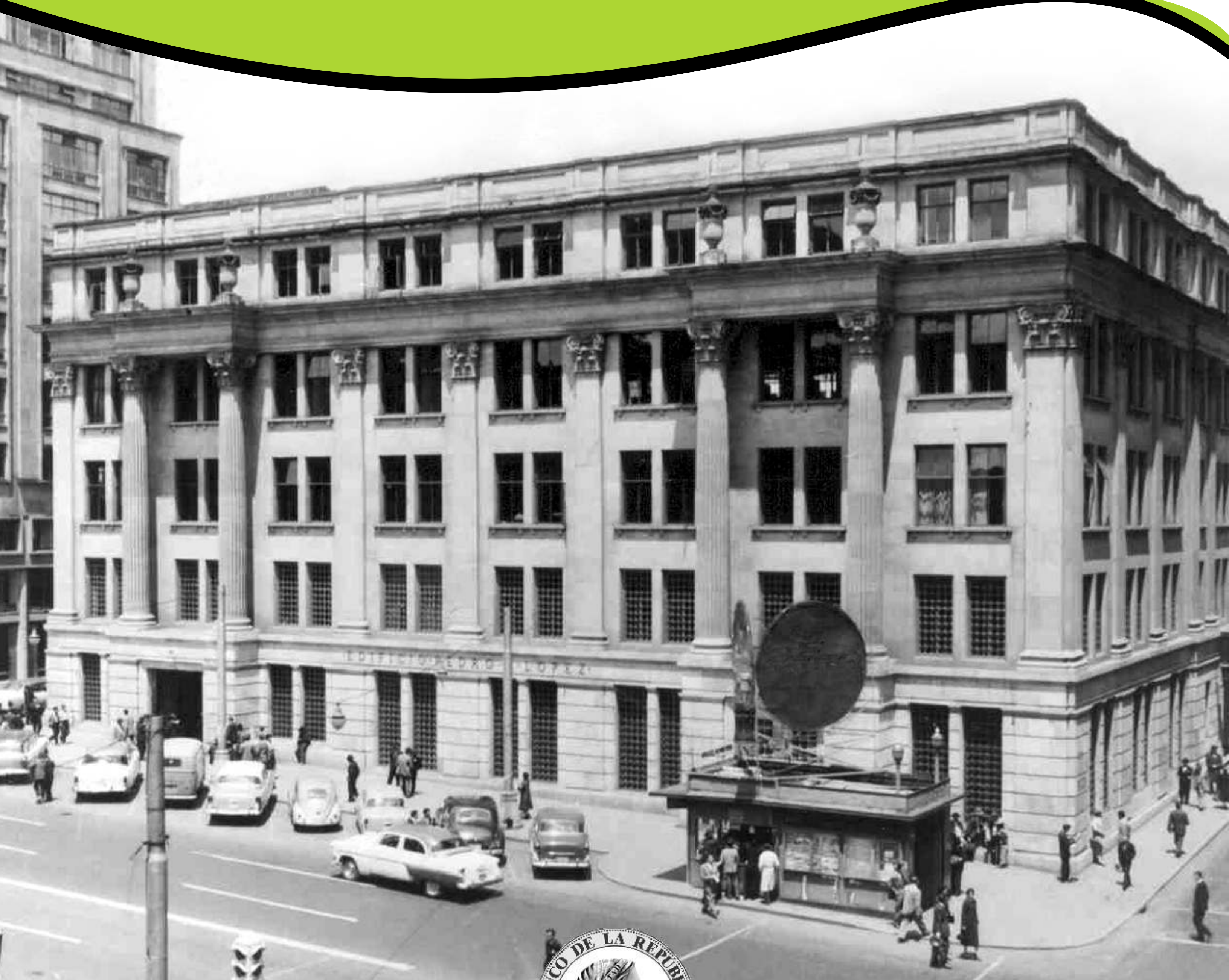

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THE FAN CHART: THE TECHNICAL DETAILS OF THE NEW IMPLEMENTATION

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Abstract. Our current implementation of the Fan Chart follows the original Britton Fisher and Whitley [2] and Blix and Sellin [1] proposal in which the inputs enter at the fourth and ninth quarters and are distributed within the forecasting horizon according to pre established weights. This procedure does not allow enough flexibility to control the shape of the distribution at the shorter end of the horizon when more reliable information is available. On the other hand, no published material presents the details of the Fan Chart computation. In this note all the technical details of the new implementation are described. This implementation provides more flexibility than the previous one since it permits the inputs to be entered on a quarterly basis instead of on a yearly basis. Click here to obtain a Visual Basic for Excel implementation and save the file as FanChart.xls.

1. Introduction

The *Fan Chart* represents the forecasting distribution of a variable based on the information available at present. In comparison to the traditional expected forecast path and its corresponding symmetrical bands, the Fan Chart has two important advantages: First, it depicts the whole marginal forecasting distribution at each period of time in the forecasting horizon. And second, this marginal distribution may be non symmetric. If the forecasting distribution is non symmetric, the probability that the variable takes on values below the more likely point forecast differs from the probability that it takes on values above of it, which makes it a desirable tool to show the risks of not fulfilling pre established targets on future values of the variable.

The term *Fan Chart* was proposed for the first time by the *Bank of England*, who has been publishing it as a part of its *Inflation Report* since 1997. See Britton, Fisher and Whitley [2] and Blix and Sellin [1].

For an inflation targeting Central Bank the Fan Chart satisfies two objectives; First, the Fan Chart reveals to the public opinion the Central Bank's inflation foresight based on its *best knowledge* of the economy, a goal related to the transparency

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Key words and phrases. Fan Chart, Forecasting Distribution, Balance of Risks.

JEL: E31, E37, E59.

The author indebts very much Matteo Ciccarelli from the European Central Bank for pointing out a typo in the previous implementation. However, any remaining errors as well as the results and opinions contained in this paper are the sole responsibility of its author and do not compromise Banco de la República, its board of governors or the Universidad Nacional de Colombia.

of the inflation targeting regime and the credibility of the policies to reach these targets. And second, the Fan Chart serves as a guide to produce the inflation forecast, which has to do with the production and organization of the inflation report.

The current Fan Chart implementation used by the Colombian Central Bank follows the lines of the Britton, Fisher and Whitley [2] and Blix and Sellin [1], where the inputs enter at the fourth and ninth quarters of the forecasting horizon, and are distributed within the horizon according to pre established weights. This procedure does not allow enough flexibility to control the shape of the distribution at the very short end, one to two quarters, when more reliable information is available. See Julio [5].

On the other hand, no published material shows the Fan Chart computation in detail.

In this note the technical details of the new implementation are described. For this implementation inputs can be entered on a quarterly basis. A Visual Basic for Excel program is available.

This note contains three sections apart from this short introduction. The split normal distribution, the workhorse of Britton, Fisher and Whitley's chart, is described in the first. In the second, the Fan Chart computation is described. In the third and last, we conclude.

2. The Split Normal Density

There are several equivalent parameterizations for the split normal density, and the relationships between them are used in this implementation. In this section the more intuitive parametrizations are studied in detail and the relationships between them are established.

In order to describe the alternative parameterizations we will adopt the notation of table 1.

Table 1. Split Normal Density Notation.

Symbol	**Meaning**
$-\infty < \mu < \infty$	Mode
$\sigma_1 > 0$	Left side standard deviation
$\sigma_2 > 0$	Right side standard deviation
$\sigma > 0$	Uncertainty indicator
$-1 < \gamma < 1$	An inverse skewness indicator
$0 < p < 1$	Balance of risks $P[X \leq \mu]$
$-\infty < \widetilde{\mu} < \infty$	Mean
$-\infty < \mu_e < \infty$	Median
$-\infty < \xi < \infty$	Skewness indicator

2.1. The split normal density and its properties.

A random variable X is said to have a split normal or two piece normal density if its probability density function can be written as

$$f_X(x;\mu,\sigma_1,\sigma_2) = \begin{cases} C\exp\left\{-\frac{1}{2\sigma_1^2}(x-\mu)^2\right\}, & \text{for} \quad -\infty \leq x \leq \mu \\ C\exp\left\{-\frac{1}{2\sigma_2^2}(x-\mu)^2\right\}, & \text{for} \quad \mu < x < \infty \end{cases} \tag{2.1}$$

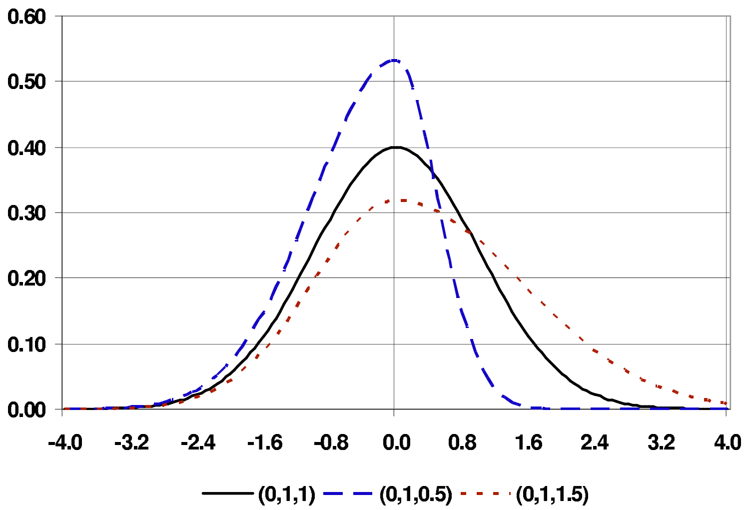
where

- $-\infty < \mu < \infty$ is the mode or more likely value of the variable.
- $\sigma_1 > 0$ and $\sigma_2 > 0$ are the left and right hand side standard deviations.
- $C = \sqrt{\frac{2}{\pi}}(\sigma_1 + \sigma_2)^{-1}$ is a normalizing constant such that (2.1) integrates to unity.
- If $\sigma_1 = \sigma_2$, the normal density arises.
- The density is completely specified if the triplet $(\mu, \sigma_1, \sigma_2)$ is known.

See John [4].

Figure 1 displays three split normal densities for $(\mu, \sigma_1, \sigma_2) = (0, 1, 1)$, $(\mu, \sigma_1, \sigma_2) = (0, 1, 0.5)$ and $(\mu, \sigma_1, \sigma_2) = (0, 1, 1.5)$. From this figure we can notice that if $\sigma_1 > \sigma_2$ the density biases to the left and the *balance of risks*, $p = P[X \le \mu]$, is higher than 0.5. If $\sigma_2 > \sigma_1$ the opposite happens, and if $\sigma_1 = \sigma_2$ the density reduces to the normal which has balanced risks.

FIGURE 1. Three split normal densities



Probability computations may be carried out by using equation (2.1) as follows,

$$P[L_1 \le X \le L_2] = \int_{L_1}^{L_2} f_X(x)\,dx \tag{2.2}$$

$$= \begin{cases} \frac{2\sigma_1}{\sigma_1+\sigma_2}\left[\Phi\left(\frac{L_2-\mu}{\sigma_1}\right) - \Phi\left(\frac{L_1-\mu}{\sigma_1}\right)\right], & \text{if} \quad L_1 < L_2 \le \mu \\ \frac{2\sigma_2}{\sigma_1+\sigma_2}\left[\Phi\left(\frac{L_2-\mu}{\sigma_2}\right) - \Phi\left(\frac{L_1-\mu}{\sigma_2}\right)\right], & \text{if} \quad \mu \le L_1 < L_2 \\ \frac{2}{\sigma_1+\sigma_2}\left[\sigma_2\Phi\left(\frac{L_2-\mu}{\sigma_2}\right) - \sigma_1\Phi\left(\frac{L_1-\mu}{\sigma_1}\right) + \frac{\sigma_1-\sigma_2}{2}\right], & \text{if} \quad L_1 \le \mu < L_2 \end{cases}$$

where $\Phi(x)$ is the standard normal distribution, and the balance of risks becomes

$$p = P[X \le \mu] = \frac{\sigma_1}{\sigma_1 + \sigma_2} \tag{2.3}$$

Moreover, the mean and standard deviation are:

$$\widetilde{\mu} = E[X] = \sqrt{\frac{2}{\pi}}(\sigma_2 - \sigma_1) + \mu \tag{2.4}$$

and

$$V[X] = \left(1 - \frac{2}{\pi}\right)(\sigma_2 - \sigma_1)^2 + \sigma_1\sigma_2 \tag{2.5}$$

and the bias or non symmetry coefficient becomes

$$E\big[(X - \widetilde{\mu})^3\big] = \sqrt{\frac{2}{\pi}}(\sigma_2 - \sigma_1)\left[\left(\frac{4}{\pi} - 1\right)(\sigma_2 - \sigma_1)^2 + \sigma_1\sigma_2\right] \tag{2.6}$$

which is proportional to

$$\xi = \widetilde{\mu} - \mu = \sqrt{\frac{2}{\pi}}(\sigma_2 - \sigma_1) \tag{2.7}$$

another skewness indicator such that $\xi > 0$ when the distribution is biased to the right and $\xi < 0$ when the distribution is biased to the left.

On the other hand, for $0 < \alpha < 1$, the α-th percentile of the distribution is a constant k such that $P[X \leq k] = \alpha$. From equation (2.2) it is easily shown that

$$P[X \leq k] = \begin{cases} C\sqrt{2\pi}\sigma_1 \Phi(\frac{k-\mu}{\sigma_1}), & \text{for} \quad k \leq \mu \\ 1 - C\sqrt{2\pi}\sigma_2\Big[1 - \Phi(\frac{k-\mu}{\sigma_2})\Big], & \text{for} \quad k > \mu \end{cases} \tag{2.8}$$

and hence, the distribution quantiles may be computed from

$$k = \begin{cases} \mu + \sigma_1\Phi^{-1}\Big(\frac{\alpha}{C\sqrt{2\pi}\sigma_1}\Big), & \text{for} \quad \alpha \leq p = P[x \leq \mu] \\ \mu + \sigma_2\Phi^{-1}\Big(\frac{\alpha + C\sqrt{2\pi}\sigma_2 - 1}{C\sqrt{2\pi}\sigma_2}\Big), & \text{for} \quad \alpha > p = P[x \leq \mu] \end{cases} \tag{2.9}$$

2.2. **A second parametrization.** An alternative parametrization of the split normal density may be found in Johnson, Kotz and Balakrishnan [3], which depends on the parameters (μ, σ, γ) as

$$f_X(x; \mu, \sigma, \gamma) = \frac{A}{\sqrt{2\pi}\sigma} \begin{cases} \exp\Big\{ -\frac{1-\gamma}{2\sigma^2}\Big[(x-\mu)^2\Big]\Big\}, & \text{for} \quad x \leq \mu \\ \exp\Big\{ -\frac{1+\gamma}{2\sigma^2}\Big[(x-\mu)^2\Big]\Big\}, & \text{for} \quad x > \mu \end{cases} \tag{2.10}$$

where $A = \frac{2}{(1/\sqrt{1-\gamma}) + (1/\sqrt{1+\gamma})}$ is a normalization constant and $-1 < \gamma < 1$ is a skewness indicator.

Comparing equation 2.10 to the original parametrization 2.1, we obtain

$$\sigma_1 = +\sqrt{\frac{\sigma^2}{1-\gamma}}; \qquad \sigma_2 = +\sqrt{\frac{\sigma^2}{1+\gamma}} \tag{2.11}$$

which helps us see that if $\gamma > 0$, $\sigma_1 > \sigma_2$ and then the density is biased to the left and if $\gamma < 0$, $\sigma_1 < \sigma_2$ and then the density is biased to the right. If $\gamma = 0$ the density reduces again to the normal. Then, γ is an inverse indicator of the skewness.

On the other hand, combining equation 2.3 with the last equation we obtain that

$$\gamma = \frac{2p - 1}{1 - 2p + 2p^2} \tag{2.12}$$

which gives us γ when σ_1 and σ_2 are known.

Finally, in order to compute the Fan Chart a last relationship between ξ and γ must be established when σ is known. By replacing equation 2.11 into equation 2.7 we obtain

$$\xi = \widetilde{\mu} - \mu = \sqrt{\frac{2}{\pi}}(\sigma_2 - \sigma_1) = \sqrt{\frac{2}{\pi}}\left(\sqrt{\frac{\sigma^2}{1+\gamma}} - \sqrt{\frac{\sigma^2}{1-\gamma}}\right) \tag{2.13}$$

provided we know σ, this equation may be solved for γ as

$$\gamma = \begin{cases} -\sqrt{1-\left(\frac{\sqrt{1+2\beta}-1}{\beta}\right)^2}, & \text{if} \quad \xi > 0 \\ \sqrt{1-\left(\frac{\sqrt{1+2\beta}-1}{\beta}\right)^2}, & \text{if} \quad \xi < 0 \end{cases} \tag{2.14}$$

where $\beta = \frac{\pi}{2\sigma^2}\xi^2$

3. The Fan Chart

In order to describe the Fan Char computation from the equations derived in the las section, we will follow the notation established in table 2.

Table 2. Fan Chart Notation

Symbol	Meaning
$h = 9$	Forecasting horizon
X_t^i	Factor or variable that affects π_t $i = 1, 2, 3, \ldots, n = n_1 + n_2$
μ_t^i	More likely value of the i-th factor at t
p_t^i	Balance of risks for i-th factor at t
σ_t^i	Forecast error standard deviation i-th factor at t
n_2	Number of factors X_t^i such that $p_t^i = 0.5$
π_t	Inflation rate at t
μ_t^π	More likely inflation rate at t
p_t^π	Balance of risks of the inflation rate at t
σ_t^π	Forecast error standard deviation of inflation at t
ϕ_j^i	Response of π_{t+j} to an impulse in X_t^i
ξ_t^i	Bias indicator of i-th factor at t
ξ_t^π	Bias indicator of π at t
γ_t^i	Inverse bias indicator of i-th factor at t
γ_t^π	Inverse bias indicator of π at t

3.1. **Computing the Fan Chart.** Computing the Fan Chart entails computing the triplet $(\mu^\pi_{t+h|t}, \sigma^\pi_{1,t+h|t}, \sigma^\pi_{2,t+h|t})$ for every $h = 1, 2, 3, ..., 9$. This process is performed by following two separate steps, where the second is the Fan Chart computation. In the first step the more likely inflation forecast path, $\mu^\pi_{t+h|t}$, is found, and in the second $\left(\sigma^\pi_{1,t+h|t}, \sigma^\pi_{2,t+h|t}\right)$ are determined.

- Forecasting the inflation rate:

TABLE 3. Transforming the factor bias to inflation bias

	Response of the bias indicator of π_t to bias indicator of:				
t	Factor 1	Factor 2	$\cdots$	Factor n_1	Todos ξ_t^π
1	$\phi_0^1\xi_1^1$	$\phi_0^2\xi_1^2$	$\cdots$	$\phi_0^{n_1}\xi_1^{n_1}$	$\sum_{i=1}^{n_1}\phi_0^i\xi_1^i$
2	$\phi_0^1\xi_2^1+\phi_1^1\xi_1^1$	$\phi_0^2\xi_2^2+\phi_1^2\xi_1^2$	$\cdots$	$\phi_0^{n_1}\xi_2^{n_1}+\phi_1^{n_1}\xi_1^{n_1}$	$\sum_{i=1}^{n_1}\phi_0^i\xi_2^i+\phi_1^i\xi_1^i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
t	$\sum_{j=0}^{t-1}\phi_j^1\xi_{t-j}^1$	$\sum_{j=0}^{t-1}\phi_j^2\xi_{t-j}^2$	$\cdots$	$\sum_{j=0}^{t-1}\phi_j^{n_1}\xi_{t-j}^{n_1}$	$\sum_{i=1}^{n_1}\sum_{j=0}^{t-1}\phi_j^i\xi_{t-j}^i$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
9	$\sum_{j=0}^{8}\phi_j^1\xi_{t-j}^1$	$\sum_{j=0}^{8}\phi_j^2\xi_{t-j}^2$	$\cdots$	$\sum_{j=0}^{8}\phi_j^{n_1}\xi_{t-j}^{n_1}$	$\sum_{i=1}^{n_1}\sum_{j=0}^{8}\phi_j^i\xi_{t-j}^i$

  - All the factors that may affect inflation rate X_t^i over the forecasting horizon are determined and their more likely path, $\mu_{t+h|t}^i$, is determined.
  - Conditional on these paths and the whole historical information up to time t the more likely short term inflation forecast is computed, $\mu_{t+h|t}^\pi$, for $h = 1, 2$. This step is performed by using the suite of models.
  - By using the Transmission Mechanisms Model, MMT, the more likely medium term inflation forecast is computed, $\mu_{t+h|t}^\pi$, for $h = 3, 4, ..., 9$, restricted to cross $\mu_{t+h|t}^\pi$ at $h = 1, 2$.
  - The forecast error standard deviation of the inflation rate is computed, $\sigma_{t+h|t}^\pi$, for $h = 1, 2, 3, ..., 9$. These measures are composed of two terms, the historical forecast error standard deviation estimation and an uncertainty multiplier to adjust the historical figures to the actual situation.
- Computing the Fan Chart:
  - Factors are classified according to their corresponding balance of risks. For the Fan Chart computation, only the factors that have $p_{t+h|t}^i \neq 0.5$ for at least one point of time in the horizon affect the forecasting distribution.
  - For these factors, the forecast error standard deviations are computed, $\sigma_{t+h|t}^i$, and the response of the inflation rate to a one unit impulse in the factor, ϕ_{h-1}^i, are estimated from either of the models in the suite, for all horizons $h = 1, 2, 3, ..., 9$. Again, the forecast error standard deviations are composed of two terms, the historical measurements and the uncertainty multipliers.
  - The factors balance of risks, $p_{t+h|t}^i$, and forecast error standard deviations, $\sigma_{t+h|t}^i$ are transformed into the skewness indicators, $\xi_{t+h|t}^i$ for each $h = 1, 2, 3, ..., 9$ by using equations 2.12, 2.11 and 2.7.
  - Transform these factor's skewness indicators, $\xi_{t+h|t}^i$, into the skewness indicators of the inflation rate, ξ_t^π, by using the impulse response function, ϕ_{h-1}^i. See table 4
  - Transform the skewness indicators of the inflation rate, ξ_t^π, and the forecast error standard deviations of the inflation rate, $\sigma_{t+h|t}^\pi$, into the

inflation's left and right standard deviations, $\sigma^{\pi}_{1,t+h|t}$ and $\sigma^{\pi}_{2,t+h|t}$, by using equations 2.14 and 2.11.

– Compute the percentiles of the forecast inflation rate by using equation 2.9.
– Compute the probability table for given presentation ranks using equation 2.2, and the median and expected forecasts by using equations 2.9 and 2.4.

By following this procedure we arrive to figure 2 and table 4.

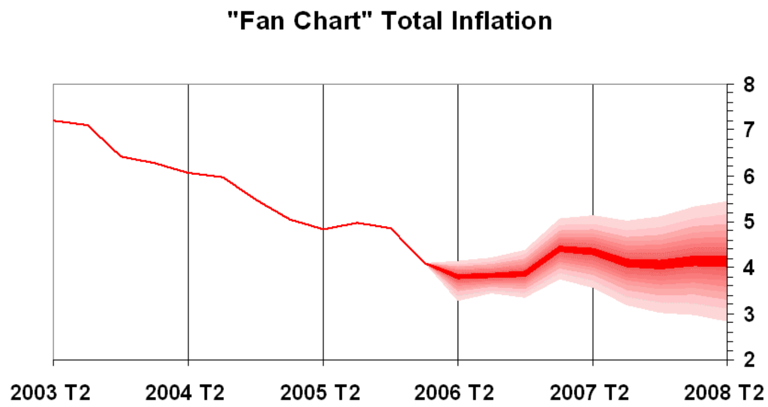


Figure 2. The Fan Chart

In order to produce table 4 and figure 2, only one factor was introduced, which had a balance of risks higher than 0.5 in the first quarter with unit response of inflation on impact and zero at higher lags. All the other parameters were set in a sensible way in order to isolate the response of the Chart to this effect.

Table 4 and figure 2, show the results we were looking for. The forecast distribution is affected only during the first quarter, when the balance of risks of the inflation rate increases. By changing the remaining parameters the response of the chart may be determined.

Table 4. probability Distribution Total Inflation for Selected Ranges

	2006Q2	2006Q3	2006Q4	2007Q1	2007Q2	2007Q3	2007Q4	2008Q1	2008Q2
Pr.[<3%]	0.76%	0.03%	0.33%	0.02%	0.26%	2.55%	4.85%	5.40%	7.83%
Pr.[3%-3.5%]	15.88%	8.72%	12.01%	1.15%	3.73%	11.93%	14.13%	12.67%	13.60%
Pr.[3.5%-4%]	63.75%	67.88%	53.36%	14.33%	19.89%	28.92%	27.14%	23.29%	21.95%
Pr.[4%-4.5%]	19.61%	23.12%	31.84%	43.79%	39.11%	33.23%	29.18%	26.94%	24.28%
Pr.[4.5%-5%]	0.01%	0.25%	2.44%	33.83%	28.52%	18.11%	17.56%	19.61%	18.40%
Pr.[5%-5.5%]	0.00%	0.00%	0.02%	6.56%	7.70%	4.67%	5.91%	8.98%	9.55%
Pr.[>5.5%]	0.00%	0.00%	0.00%	0.31%	0.79%	0.60%	1.23%	3.11%	4.38%
Pr. [<Mode]	70.46%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%

3.2. **Notes on the program's usage.** The procedure described above was programmed in Visual Basic for Excel in the attached file. The following recommendations should be taken into account when using it:

- The workbook file name as well as the names of the worksheets it contains should not be changed since the program refers to them at several steps.

- You can include any number of factors in the corresponding worksheets. Once the number of factors is input, the program determines the worksheets ranges to read beginning at the c6 cell in the first four worksheets and at b6 in the last two, `Inflacion` and `Parametros`. That is, rows can be inserted only after the sixth in the first four worksheets only. Do not insert columns to any worksheet.
- The input ranges in the last two worksheets are fixed as determined in the original file. Any information outside these ranges is not read.
- Be sure to have the file `FanCharResults.xls` closed before you run the program.
- The program is executed just by clicking the tiger button in the `FactSD` worksheet.
- Please be sure to adjust Excel's security to medium in order to allow the macro to be loaded.
- This program was developed under MS Office Excel 2003, and may not run under some other versions. However, please report any bugs to my e-mail account.
- This program is delivered without any warranty and its usage and results are not the responsibility of the author of the program or the institutions he belongs to.

4. Conclusion

The current Fan Chart implementation permits introducing the relevant parameters only at the fourth and ninth quarter of the forecasting horizon. The parameters are distributed within the horizon according to pre established weights. See Britton, Fisher and Whitley [2]. This procedure does not permit enough flexibility to control the shape of the distribution at the very short end, one to two quarters, when more reliable information is available.

Moreover, no published material shows the Fan Chart computation in detail.

In this note all the technical details of the new implementation are described. This implementation provides more flexibility than the previous one since it permits the inputs to be entered on a quarterly basis instead of on a yearly basis, thus providing the desired flexibility. A Visual Basic for Excel program is available.

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