

Determinants of the Exchange Rate in  
Colombia under Inflation Targeting

Por:  
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# Determinants of the Exchange Rate in Colombia under Inflation Targeting<sup>1</sup>

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## Abstract

This research studies the forecasting performance of conventional and more recent exchange rate models in Colombia. The purpose is to explain which have been the main exchange rate determinants under an Inflation Targeting regime and a completely floating exchange rate scheme. Compared to similar studies, this paper includes conventional specifications and Taylor rule approaches that assume exogenous and endogenous monetary policy respectively. Based on the Johansen multivariate cointegration methodology, the results provide evidence for the existence of cointegration in all specifications except in the Sticky-Price Monetary Model and the Taylor Rule model that includes the real exchange rate. In addition, out of sample forecasting performance is analyzed in order to compare if all specifications outperform the drift less random walk model. All models outperform the random walk at one month horizon. However, the Flexible Price Monetary Model and the Uncovered Interest Parity Condition have superior predictive power for longer horizons.

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**Keywords:** Exchange rate determination, Inflation Targeting, Out of sample performance, Johansen multivariate cointegration.

**JEL Classification:** C32, C53, E58, F31

# **Determinantes de la Tasa de Cambio en Colombia bajo el Régimen de Inflación**

## **Objetivo**

## **Resumen**

El presente trabajo estudia la capacidad predictiva tanto de los modelos convencionales como de las recientes teorías de tasa de cambio en Colombia. El objetivo es explicar cuáles han sido los principales determinantes de la tasa de cambio bajo el régimen de Inflación Objetivo y un régimen de flexibilidad cambiaria. En comparación con trabajos similares, este documento incluye especificaciones convencionales y reglas de Taylor, las cuales asumen una política monetaria exógena y endógena respectivamente. Con base en la metodología de cointegración multivariada de Johansen, los resultados muestran evidencia de la existencia de cointegración en todas las especificaciones excepto en el modelo Monetario de Precios Rígidos y el modelo de regla de Taylor que incluye la tasa de cambio real. Adicionalmente, se realizan predicciones fuera de muestra con el propósito de comparar si todas las especificaciones tienen una capacidad predictiva superior al modelo de caminata aleatoria. Todos los modelos son superiores al modelo de caminata aleatoria en horizontes de un mes. Sin embargo, el modelo Monetario de Precios Flexibles y la Condición de Paridad de Interés no cubierta tienen poder de predicción superior en horizontes más largos.

**Palabras Clave:** Determinantes de la tasa de cambio, Inflación Objetivo, Predicción fuera de muestra, Cointegración multivariada de Johansen.

**Clasificación JEL:** C32, C53, E58, F31

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## 1. Introduction

Inflation Targeting (IT) is a monetary scheme in which the main goal of monetary policy is price stability. Colombia converged to a pure IT regime after September 1999 when the Central Bank (Banco de la República) decided to eliminate a system of exchange rate bands and adopted a scheme which allows floating the exchange rate (Vargas 2005). The evidence suggests that Colombia completely adopted an IT system under a difficult economic situation. According to statistics its economy experienced huge fiscal and external imbalances and deeper economic slowdown. These factors together with the international economic situation, specially the Russian crisis and Brazilian devaluation forced the adoption of a floating regime (Chang 2008). For instance, one of the reasons which explain this transition was the deep real depreciation of the Colombian Peso by close to 22% between 1998 and 1999 (Vargas 2005).

Despite the fact that economic stability is the main precondition of implementing IT regimes, the Colombian monetary policy converged to a complete IT regime besides a floating exchange rate scheme (Vargas 2005). After the adoption of IT, Colombia's real exchange rate has experienced periods of strong depreciation and appreciation. Although the Banco de la República (BR) does not have a specific target for the exchange rate (Uribe and Toro 2005), it has two mechanisms of intervention in order to avoid high exchange rate volatility. Firstly, put and call options can be classified such as instruments created for accumulating or decumulating international reserves or for reducing the high volatility of the exchange rate (Uribe and Toro 2005)<sup>3</sup>.

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<sup>3</sup> Put and call options for accumulating or decreasing the international reserves "give the holder the right to sell (buy) foreign exchange to (from) the central bank" (Uribe and Toro, 2005, p. 140). In addition, volatility options



Secondly, direct intervention in the foreign exchange market. This mechanism was created in September 2004 motivated by the strong appreciation of Colombian Peso between 2003 and 2004. According to these facts, recent evidence (Kamil 2008) demonstrates that under high exchange rate volatility, the BR could face conflict between the exchange rate policy and the monetary policy and this tradeoff could arise when the exchange rate deviates notably from its fundamentals (Zampolli 2006).

During the last decade, there has been an important concern in Colombia about the strong volatility of the exchange rate under an IT regime. In this sense, it is really important to study its determinants in order to improve the implementation and design of the monetary and exchange rate policies. Recently, studies in this subject have found that capital controls (Clements and Kamil 2009) and the official intervention (Kamil 2008) by the BR have not affected the level of the exchange rate, but have increased its volatility.

Thus, based on the literature about Monetary Models of the Exchange Rate the purpose of this dissertation is to evaluate the adequacy of different models and to explain which have been the main determinants of the exchange rate in Colombia during IT regime (2000 – 2009). The analysis is based on conventional models such as the Flexible Monetary Model (FPMM), the Sticky Price Monetary Model (SPMM), the Balassa-Samuelson approach (BSA), the Uncovered Interest Parity Condition (UIP) and more recent theories such as Taylor Rule Models. Studying these different approaches, the hypothesis is that conventional models and specifications that

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can be used when “the nominal exchange rate (TRM) is 4% or more below (above) its last 20 working day moving average” (Ibid).

assume endogenous monetary policy have more out of sample forecasting power than the drift less random walk specification. In other words, alternative specifications to the random walk have more prediction ability of the exchange rate behaviour. Furthermore, the inclusion of endogenous models tries to explain that there is a strong link between the exchange rate movements and an interest rate rule in Colombia.

This research is based on the paper elaborated by Moura, Lima and Mendonca (2008). That study assessed different conventional models and a Taylor Rule model in Brazil under an IT regime. They concluded that Taylor Rule models and approaches that combine productivity differentials with portfolio balance effects outperform the prediction capability of the random walk. The relevance of this dissertation is that besides the concern about the exchange rate volatility, there are no previous studies of Colombia that examine the out of sample performance of exchange rate models such as Taylor Rule specifications. In addition, the studies performed before only include conventional exchange rate models until 2002 which means that they do not take into account a long sample of IT scheme. Furthermore, Colombia has been one of the most successful countries in the adoption of IT regimes in South America. However, the conflict between monetary and exchange rate policies has generated studies which try to explain the factors that determine the exchange rate behaviour. And this is one of the main objectives of this research.

This paper does not include the Portfolio Balance Model and the Behavioural Equilibrium Exchange Rate Model<sup>4</sup> as result of the absence of monthly data for some variables. In addition, the UIP condition is only estimated for evaluating the out of sample forecasting in long horizons. As Cheung, Chinn and Pascual (2005) state this specification is not a model, rather it is a useful condition that supports the exchange rate behaviour in the long run.

In order to assess the adequacy of each functional form, this paper determines if there is a cointegration relation in each of the different approaches and then examines the out of sample performance of the exchange rate compared to the random walk model. The former is performed using the Johansen Cointegration framework and the latter using rolling regressions. Out of sample evaluation is carried out using different forecast error statistics such as the Root Mean Square Error (RMSE), the Mean Absolute Error (MAE) and a Sign Test in order to compare the forecasting power between different models and the drift less random walk.

The results obtained show evidence of cointegration in all exchange rate specifications with the exception of the SPMM and the Taylor Rule model that includes the real exchange rate as explanatory variable. In addition, the out of sample forecasting results demonstrate that all specifications outperform the drift less random walk model at one month horizon. The FPMM and the UIP condition outperform the random walk at one and twelve horizons and one, three and six

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<sup>4</sup> The Portfolio Balance Model includes the monetary aggregates, interest rates, net government debt, Embi+ and public sector dollar denominated net foreign assets. In addition to those variables, the Behavioural Equilibrium Exchange Rate Model includes the price of nontradable goods and the terms of trade. However, the net government debt and net foreign assets data are only available quarterly for Colombia.

horizons respectively. Although the Taylor Rule model has only the best out of sample predictability at one month horizon, the inclusion of expectations in all the variables and lagged interest rate could improve its forecasting power. However, this issue is out of the main purpose of this study.

The following section presents a review of empirical studies about out of sample performance of exchange rate models since the classical paper introduced by Meese and Rogoff (1983). The conventional and Taylor Rule models of exchange rate are described in section 3. Section 4 discusses the data set, the current exchange rate scheme in Colombia and the Johansen multivariate cointegration framework with the corresponding results. Section 5 evaluates the out of sample forecasting performance, and conclusions are considered in Section 6.

## **2. Review of Empirical Studies**

The first attempt to evaluate the out of sample performance of exchange rate models was introduced by Meese and Rogoff (1983). This paper concluded that exchange rate models during the 1970's do not explain the behaviour of the exchange rate and that random walk model outperforms the ability of prediction of conventional models.

Different studies have been developed after this research. For instance, Sarno and Taylor (2002) analyze diverse theories and outline many empirical works that have been performed since the 1980's. One conclusion is that researches have not found robust and reliable models that outperform a random walk using in sample or out of sample forecasting techniques. Although the introduction of dynamic equations into out of sample forecasting methodology could outperform the random walk, the main

problem is that sometimes these methods are difficult to replicate using different horizons and different currencies.

In this attempt to include dynamic equations in the exchange rate models, Engel and West (2006) study an asset market approach to exchange rates and conclude that even including rational expectations, it is very difficult for conventional models to outperform the random walk.

Cheung, Chinn and Pascual (2005) assess the predictive ability of conventional models during 1990's. They added other factor such as the productivity differential in the tradable sector between countries. They concluded that the forecasting performance of any model does not explain completely the exchange behaviour because some models do better at specific horizons than others. It means that there is not enough consistency when outperform forecasting is tested (Ibid).

During the last years, out of sample performance studies have been focused on studying endogenous monetary policy models. For instance, Molodtsova and Papell (2009) studied the short term predictability for different currencies (OECD countries) using Taylor Rule Models and comparing with conventional forms such as the Purchasing Power Parity (PPP) and monetary models. The authors concluded that there is stronger evidence of out of sample predictability in endogenous models than conventional ones at one month horizon.

Different techniques have been developed in order to study Taylor Rule Models. Mark (2005) modelled a method where market participants are not familiar with the

coefficients of the models and try to get that information in a learning environment. He found evidence that the dollar-deutschmark exchange rate behaviour is well explained by Taylor rule fundamentals and they are more accurate than conventional models between 1976 and 2003.

In Colombia there are few studies that attempt to compare out of sample forecasting performance between the exchange rate models and the random walk specification. Rowland and Oliveros (2003a) analyzed the PPP using quarterly data from 1980 to 2002. The authors took into account the Balassa-Samuelson effect in the model specification. Under Johansen framework of multivariate cointegration, they found that this model outperforms the random walk specially for long forecasting horizons of 12 and 24 months. However, the results are far away from the parity relationship stated by the theory probably as result of the short data used in the research.

Another study developed in Colombia was assessed by Rowland (2003b). He attempted to study three different conventional exchange rate models and a simple random walk form in line with Meese and Rogoff (1983). Two of the models outperform the random walk in long horizons but not in short ones. The technique used for assessing forecasting performance was rolling regressions and they used different statistics such as the RMSE and the MAE for forecast evaluation. The study covered quarterly data from 1970 to 2002. During this period there were two notable changes in the exchange rate regimes. Firstly, the crawling exchange rate peg regime was abandoned in 1991 and the exchange rate was floated in 1999. In this sense, the authors found that Johansen multivariate cointegration could have failed in one of the specifications as result of the lack of modelling those structural breaks.

However, this is not the problem of the present study because the period covered in the analysis only includes the IT regime and there is no need to model structural breaks.

### 3. Exchange Rate Models

This section describes the models used in this study. The FPMM and the SPMM models are discussed in section 3.1 and section 3.2 respectively. The BSA model and the UIP condition are covered in section 3.3 and 3.4 respectively. Finally, the Taylor rule model is introduced in section 3.5.

#### 3.1. The Flexible Price Monetary Model (FPMM)

As Sarno and Taylor (2002) state, this economic approach of the exchange rate was predominant in the early 1970's. Its main characteristics are that output is at its natural level and prices are completely flexible. In addition, prices adjust instantaneously if there is any excess demand in the economy. Furthermore, the interest rate is exogenous in the long run according to the assumption of perfect capital mobility. Finally, this model assumes that PPP holds and requires the UIP condition.

In this sense given the following monetary approach for the domestic and foreign<sup>5</sup> countries it is possible to derive the fundamental equation of the FPMM:

$$m_t = p_t + \kappa y_t - \theta i_t \quad (3.1)$$

$$m_t^* = p_t^* + \kappa^* y_t^* - \theta^* i_t^* \quad (3.2)$$

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<sup>5</sup> Asterisks denote foreign variables and parameters

where  $m$  and  $m^*$  are the money aggregates,  $p$  and  $p^*$  are the price levels,  $y$  and  $y^*$  are the national and foreign income, and  $i$  and  $i^*$  are the interest rates. In addition, taking into account that PPP holds:

$$s_t = p_t - p_t^* \quad (3.3)$$

The final specification of the FPMM expressed in terms of the nominal exchange rate comes after subtract equation 3.2 from equation 3.1 and using equation 3.3:

$$s_t = (m_t - m_t^*) - \kappa (y_t - y_t^*) + \theta (i_t - i_t^*) \quad (3.4)$$

In econometrics terms as Moura, Lima and Mendoca (2008) stated this model could be estimated by:

$$s_t = \beta_0 + \beta_1(m_t - m_t^*) + \beta_2(y_t - y_t^*) + \beta_3(i_t - i_t^*) + v_t \quad (3.5)$$

In this specification  $s_t$  corresponds to the nominal exchange rate logarithm (COP\$/US\$)<sup>6</sup>,  $m_t$  and  $m_t^*$  are the M1<sup>7</sup> logarithms in Colombia and United States respectively;  $y_t$  and  $y_t^*$  are the logarithm of the industrial production in both countries and  $i_t$  and  $i_t^*$  corresponds to the short term interest rates in Colombia and United States respectively. Finally,  $v_t$  is the error term in the econometric specification.

### 3.2. The Sticky Price Monetary Model (SPMM)

This model was introduced by Dornbusch in 1976 and allows for the overshooting of the exchange rates above their long run equilibrium (Sarno and Taylor 2002). As in

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<sup>6</sup> (COP\$/US\$) means Colombian pesos for one American dollar

<sup>7</sup> M1 includes currency, traveller's checks, demand deposits, and other checkable deposits in United States. However, this money stock only includes currency and demand deposits in Colombia.



the FPMM, output is at its natural level but the prices adjust slowly to any demand shock. The main result of this specification is that variables such as exchange rates and interest rates compensate the stickiness in variables as good prices (Ibid).

As Cheung, Chinn and Pascual (2005) state this model is considered as an extension of the PPP regarding the theory explained before. As a result of this consideration, the SPMM is able to explain the neutrality of money. In other words any change in money aggregates results in a proportional change in prices rather than in output in the long run. In terms of the exchange rate behaviour, if there is a cut in monetary aggregates and prices are sticky in the short run, at the beginning real money balances decrease and interest rates increase in the short run (Sarno and Taylor 2002). Although the exchange rate will appreciate, during the transition to the long run prices start to fall as consequence of the initial fall in real money balances and then interest rates start to decline. It implies that in the long run the exchange rate will depreciate and this could be one explanation of why countries with higher interest rates could induce to higher expected devaluation (Ibid). In this sense, this study uses the following specification:

$$s_t = \beta_0 + \beta_1(m_t - m_t^*) + \beta_2(y_t - y_t^*) + \beta_3(i_t - i_t^*) + \beta_4(\pi_t - \pi_t^*) + v_t \quad (3.6)$$

where  $\pi_t$  and  $\pi_t^*$  are the logarithms for the annual inflation rates at each month in Colombia and United States respectively.

### 3.3. The Balassa-Samuelson Model (BSA)

This approach is based on Portfolio Balance Models where the exchange rate is completely determined by supply and demand of financial assets (Binici 2007). There are three differences between this specification and the last two models. Firstly, the

absence of the PPP; secondly that it does not impose the UIP condition and finally, that it includes a productivity differential component which determines the path of the exchange rate.

In econometrics terms, the specification could be estimated by:

$$s_t = \beta_0 + \beta_1(m_t - m_t^*) + \beta_2(y_t - y_t^*) + \beta_3(i_t - i_t^*) + \beta_5(z_t - z_t^*) + v_t \quad (3.7)$$

As in Moura, Lima and Mendonca (2008),  $z_t$  is the new variable that corresponds to the logarithm of productivity ratio between tradable and no tradable sectors. This variable is measured as the ratio of the inverse price level in each sector.

### 3.4. The Uncovered Interest Parity Condition (UIP)

The basic definition of this economic relationship is that the difference in nominal interest rates between two countries determines the expected change of the exchange rate (Molodtsova and Papell 2009). If this condition holds it implies that investors do not have the chance to make a profit. As in Moura, Lima and Mendoca (2008), this dissertation studies two UIP specifications. The first one assumes that the exchange rate is only a function of the interest rate differentials. The second one tries to model the exchange rate under other fundamentals that incorporate many risk factors.

The functional forms used in the empirical exercise are given by the following equations:

$$s_t = \beta_0 + \beta_1(i_t - i_t^*) + v_t \quad (3.8)$$

$$s_t = \beta_0 + \beta_1(i_t - i_t^*) + \beta_2 embi_t + v_t \quad (3.9)$$

In the last functional form, the variable *embi* corresponds to the logarithm of the EMBI+index in order to incorporate a risk factor in the original specification.

### 3.5. The Taylor Rule Model

Recent studies have focused their attention on exchange rate models that incorporate monetary rules within their specifications. Molodtsova and Papell (2009) define the Taylor Rule as the framework that the central banks use to adjust their interest rate policies in order to diminish the impact of changes in inflation and the output gap on the economy.

There are different specifications of exchange rate models under Taylor Rule functional forms. According to Molodtsova and Papell (2009), monetary policy rules in the domestic country can be described by the following equation:

$$i_t = \mu + \lambda \pi_t + \gamma y_t \quad (3.10)$$

where  $i_t$  is the nominal interest rate,  $\pi_t$  is the inflation rate and  $y_t$  is the output gap. In this specification the term  $\mu$  incorporates the equilibrium level of the real interest rate and the inflation target. The terms  $\lambda$  and  $\gamma$  correspond to the elasticity of the interest rate with respect to inflation and output gap respectively. The main assumption is that  $\lambda > 1$  and  $\gamma > 0$ . Clarida, Gali and Gertler (1998) include the real exchange rate within this specification. The main idea is to model the exchange rate behaviour in countries where the central banks target this variable. Although the BR does not target the exchange rate, it controls its volatility through different intervention mechanisms in order to avoid that the exchange rate deviates notably

from its recent trend. Then, this may be one reason to include the real exchange rate in the basic form. In this sense the new functional form is modelled by:

$$i_t = \mu + \lambda\pi_t + \gamma y_t + \delta q_t \quad (3.11)$$

where  $q_t$  is the real exchange rate. In this case the term  $\delta$  is the elasticity of the interest rate with respect to the real exchange rate and it is assumed to be greater than zero.

In addition, if the interest rate adjusts gradually, following Clarida, Gali and Gertler (1998), the new Taylor Rule specification is described by:

$$i_t = \mu + \lambda\pi_t + \gamma y_t + \delta q_t + \rho i_{t-1} \quad (3.12)$$

The last equation for the foreign country is the same without including the real exchange rate. In this sense, the basic Taylor Rule model is derived subtracting the foreign country specification from the equation 3.12. Following this procedure stated in Molodtsova and Papell (2009) the model is described by<sup>8</sup>:

$$i_t - i_t^* = \alpha + \lambda_{d\pi}\pi_t - \lambda_{f\pi}\pi_t^* + \gamma_{dy}y_t - \gamma_{fy}y_t^* + \delta_q q_t + \rho_d i_{t-1} - \rho_d^* i_{t-1}^* + \eta_t \quad (3.13)$$

Based on the economic theory, there is a negative relationship between the exchange rate behaviour and the inflation rate (Molodtsova and Papell 2009). It means that if there is an increase in the domestic inflation level, the Central Bank will raise the short interest rates and the exchange rate will appreciate. Furthermore, an increase in the domestic output gap will produce an increment in the interest rates

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<sup>8</sup>  $d$  and  $f$  mean domestic and foreign country respectively.

causing an appreciation. In addition, if the real exchange rate depreciates, it will cause the Central Bank to increase the interest rates, causing an appreciation of the nominal domestic currency. Finally, if there is a gradual adjustment in interest rates, an increase in lagged interest rates implies a rise in the current ones with the immediately effect of currency appreciation and expected depreciation during the transition to the long run (Ibid).

Considering these arguments, this study use two specifications for the exchange rate:

$$s_t = \beta_0 + \beta_1(\pi_t - \pi_t^*) + \beta_2(y_t - y_t^*) + \beta_3(i_t - i_t^*) + v_t \quad (3.14)$$

$$s_t = \beta_0 + \beta_1(\pi_t - \pi_t^*) + \beta_2(y_t - y_t^*) + \beta_3(i_t - i_t^*) + \beta_4q_t + v_t \quad (3.15)$$

There are two important considerations within these models. First, there is no partial adjustment in any specification due to this hypothesis is not tested in this study and it is not the main purpose of this research. Second, recent Taylor Rule models incorporate expectations in the right hand side variables. However, the complex methodology to model them as result of the absence of historical data is outside of the objectives of this piece of work.

#### **4. Empirical Analysis**

This chapter presents the methodology used for testing the existence of multivariate cointegration. In the first section the data is discussed. The second section discusses the actual exchange rate regime in Colombia during the IT scheme and its

implications on the analysis. The third section describes the Johansen multivariate cointegration framework and discusses the econometric results.

#### **4.1. The Data Set**

The data used for the empirical analysis cover monthly information from January 2000 to May 2009. The data used and their sources are listed in Table 4.1. Average monthly data is used for the nominal exchange rate and M1 is used as proxy for monetary aggregates in Colombia and United States. The Industrial Production Index (IPI) is used in each country as result of the absence of monthly data of the Gross Domestic Product (GDP). These series were seasonally adjusted by the X(11) methodology. The IPI gap is calculated using the Hodrick-Prescott Filter over seasonally adjusted series. The short interest rates for both countries correspond to the average rate on 3 month negotiable certificate of deposit.

The consumer price index (CPI) was used to calculate the annual inflation rate for each month in both countries. In order to calculate the productivity differential in the BSA, the CPI and the CPI Less Energy Services are used as proxy for the non tradable price indexes in Colombia and United States respectively. Furthermore, the Producer Price Index (PPI) is used as proxy for tradable price index in both countries.

The real exchange rate corresponds to the real exchange rate index for the total trade that use the PPI as deflator. Finally, the risk factor incorporated in the UIP condition corresponds to the EMBI+Colombia (Emerging Market Bond Index – Colombia). This variable is calculated as the monthly average of daily data.

**Table 4.1** Data and sources

<b>Data series</b>	<b>Source</b>
Nominal exchange rate COP/US\$	Banco de la República
Money Supply M1 (CO)	Banco de la República
Money Supply M1 (US)	The Federal Reserve
Industrial Production Index (CO)	Departamento Nacional de Estadística (DANE) and Banco de la República calculations
Industrial Production Index (US)	The Federal Reserve
Interest Rates (CO)	Banco de la República
Interest Rates (US)	The Federal Reserve
CPI (CO)	DANE
CPI (US)	Bureau of Labour Statistics
PPI (CO)	Banco de la República
PPI (US)	Bureau of Labour Statistics
EMBI+Colombia	JP Morgan
Real exchange rate index	Banco de la República

#### **4.2. The Exchange Rate Regime in Colombia under IT**

After September 1999 the BR eliminated the exchange rate band system and adopted a new scheme where the exchange rate was allowed to float. This change implied a structural adjustment in the exchange rate policy linked to IT framework adopted in 2000. According to the BR the main goal of the monetary policy in Colombia is to reach and maintain a low and stable inflation rate and to achieve a long term GDP growth trend. Every year the Board of Directors of the BR defines inflation targets and its monetary policy is implemented by changing the interest rates

as the main monetary tool. This is one of the reasons to include the Taylor Rule model in this study in order to determine if endogenous monetary policy models have stronger power of prediction of the exchange rate behaviour.

There is a strong relationship between monetary policy and foreign exchange policy. For instance, when the BR changes its intervention rates, this affects the market's interest rates, the inflation rate expectations, the exchange rate and the demand and growth. These transmission mechanisms are explained as follows: if the projected inflation lies below (above) the target range, the BR reduces (increases) its intervention rates and could induce that the exchange rate devaluates (appreciates). However, these effects are not always predictable because they depend on future expectations of the main macroeconomic variables and monetary policy transmission may take long time. According to the BR it could take between one and two years.

How are linked the monetary and exchange rate policies in Colombia? As was explained before, monetary policy under the IT regime has been implemented besides a floating exchange rate scheme. This monetary policy is based on intervention rules. In these sense, maintaining an optimal level of international reserves, limiting the excessive volatility and the strong appreciation or depreciation of the exchange rate are the main objectives of the foreign exchange rate policy.

Under IT regime, exchange rate policy needs to be coherent with monetary policy. It means that the intervention mechanisms used in the foreign exchange market have to go in line with the achievement of inflation objectives. This consistency between monetary and exchange rate policies can be explained through the following



example. In case that the BR intervenes in the foreign exchange market, if it reduces (increases) its intervention rates in order to achieve the inflation target, the policies are consistent if the BR purchases (sells) foreign exchange in order to reduce the appreciation (depreciation) of the currency.

There are two important issues that can be briefly discussed. Firstly, the different exchange intervention mechanisms used during the IT regime. Second, if there has been consistency between monetary and exchange rate policies in the same period.

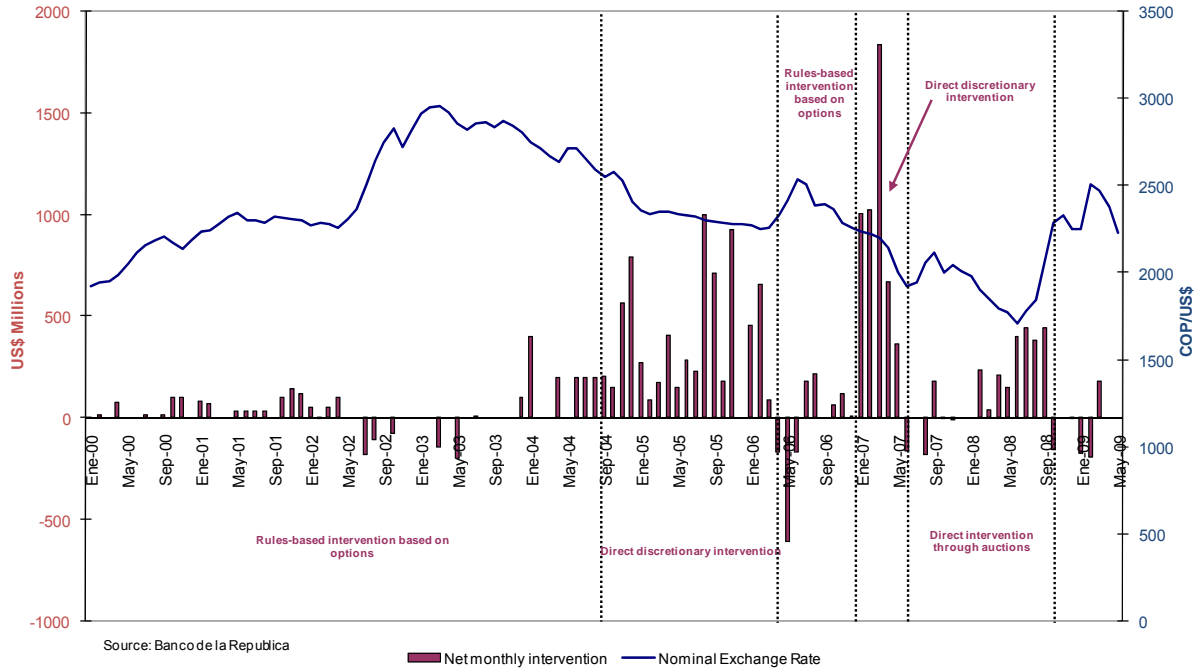
In this sense, between January 2000 and August 2004 the BR used put and call options mechanisms as instruments of exchange rate intervention. However, in September 2004 the BR created a new mechanism of intervention performed in a discretionary way as result of the strong appreciation occurred from April 2003<sup>9</sup> (Uribe and Toro 2005). Discretionary means that the Central Bank can intervene directly in the exchange market and that instrument can be used as an alternative to the existent options mechanism. In addition, given the strong appreciation of the exchange rate during the first semester of 2008, from June 2008 the BR used other mechanism of direct intervention managed through direct auctions of dollar purchases<sup>10</sup>. This instrument has not been used since October 2008. Graph 4.1 shows the different mechanisms used during the IT regime. This graph was initially created by Kamil (2008) and it is updated until May 2009.

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<sup>9</sup> From April 2006 until mid January 2007 the BR decided to intervene in the foreign exchange market using foreign exchange rate options. It means that direct intervention mechanism was stopped temporarily during this period.

<sup>10</sup> From June 2008 to October 2008 the BR auctioned purchases of dollars for US\$20 million daily.

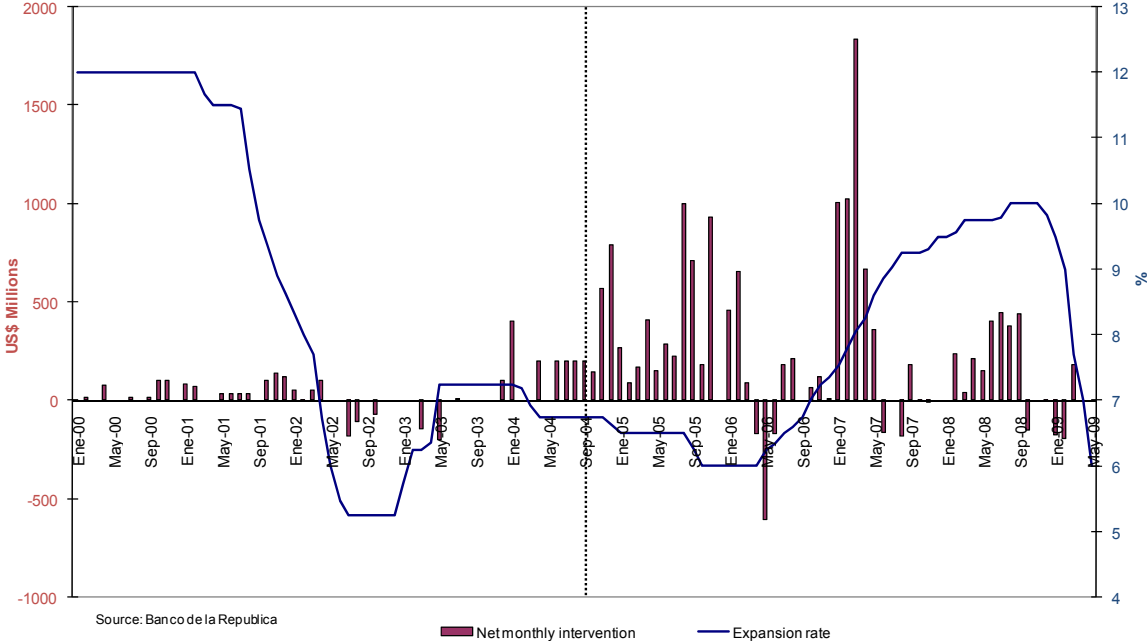
**Graph 4.1** Intervention Operations and the Nominal Exchange Rate



According to Vargas (2005), IT regime and intervention in the exchange market were successful and effective between 2000 and 2004. The main reason is the absence of conflict between monetary and exchange rate policies. From graph 4.2 can be concluded that in this period purchase of dollars correspond to periods of decreasing or stable short interest rate while sales of dollars correspond to periods of increasing or stable interest rates (Ibid). As Kamil states (2008), between September 2004 and March 2006 there is evidence that direct intervention affected in the desired way the exchange rate trend, moderating the appreciation of the currency and being consistent with the inflation targets. However, during the first half of 2007 the impact of intervention on exchange rate was diminished and ineffective. In addition there was an important concern about the increasing inflation level besides the strong exchange rate appreciation which resulted in the conflict between achieving the inflation target and reducing the appreciation. Graph 4.2 shows that during the first

half of 2007 purchases of dollars were not consistent with the increase of the intervention interest rates.

**Graph 4.2** Intervention Operations and the Expansion Repo Rate<sup>11</sup>



**4.3. The Johansen multivariate cointegration framework and results**

Non stationary time series analysis has been developed from the result that many macroeconomic variables may contain a unit root. For example as Engel and Granger (1987) state, the linear combination between two or more variables with the presence of unit root may be stationary. For instance, if there are two non stationary variables  $y_{1t}$  and  $y_{2t}$  with one unit root  $I(1)$ , it is possible that their linear combination  $y_{1t} = \zeta y_{2t} + u_{1t}$  is stationary  $I(0)$ . In this case these variables are said to be cointegrated. The cointegration equation corresponds to this linear combination and it is usually associated with the long relationship among variables.

<sup>11</sup> This interest rate corresponds to the BR monetary policy tool.

This study uses the methodology developed by Johansen (1991, 1995). First, a Vector Autoregressive system (VAR) of order  $p$  is considered:

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + Bx_t + \varepsilon_t \quad (3.16)$$

where  $y_t$  is a  $n$ -vector of non stationary  $I(1)$  variables,  $x_t$  is a  $k$ -vector of deterministic variables, and  $\varepsilon_t$  is a random term. The Vector Error Correction (VEC) specification of the last VAR can be written as:

$$\Delta y_t = \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + Bx_t + \varepsilon_t \quad (3.17)$$

$$\text{where } \Pi = \sum_{i=1}^p A_i - I \quad \text{and} \quad \Gamma_i = - \sum_{j=i+1}^p A_j \quad (3.18)$$

According to Granger's representation theorem if there are  $0 < r < n$  cointegration relationships then there exist a  $(n \times r)$  matrices  $\alpha$  and  $\beta$  such that  $\Pi = \alpha\beta'$  and  $\beta'y_t$  is stationary  $I(0)$ . In this case  $r$  is the number of cointegration relations and each column of matrix  $\beta$  correspond to a cointegration vector. In addition, the elements of  $\alpha$  corresponds to the speed of adjustment of each of the variables.

The procedure for estimating the presence of cointegration relations is as follows:

- Within the Engle –Granger framework is necessary that all variables are integrated of order one in order to find a cointegration relationship. In this sense, unit roots test are evaluated to prove that all variables are  $I(1)$  as the first step in this econometric analysis.
- The maximum lag length is selected according to different information criteria. For instance, the sequential modified likelihood ratio (LR) test, the final prediction error (FPE) and Akaike (AIC), Schwarz (SC) and Hannan-Quinn

(HQ) information criterion are used to select the optimal lag. Although sometimes these tests report different results, the lag selected is consistent with the assumption of normality and absence of residual correlation.

- In addition, likelihood ratio test is used for determining the cointegration rank and the optimal model for the deterministic components. The criteria used are the Trace and the Max-Eigenvalue tests with MacKinnon-Haug-Michelis critical values. After deciding the appropriate deterministic trend specification and the number of cointegration vectors, the cointegration equations are estimated.
- At the same time, tests for exclusion, stationarity and weak exogeneity are used in addition to the residuals tests of autocorrelation and normality.

The null hypothesis that the variables are integrated of order one is evaluated with the Augmented Dickey-Fuller test. These tests are summarised in table 4.2.

**Table 4.2** Unit root test for the time series (monthly data from 2000:01 to 2009:05)

Variable	Level	First Difference
<i>S</i>	ADF(1) = -2.22	ADF(0) = -6.37
<i>(m - m*)</i>	ADF(0) = -1.31	ADF(2) = -8.65
<i>(y - y*) IPI</i>	ADF(2) = 0.05	ADF(1) = -16.19
<i>(i - i*)</i>	ADF(1) = -1.40	ADF(0) = -8.90
<i>(π - π*)</i>	ADF(2) = -1.65	ADF(1) = -8.51
<i>(z - z*)</i>	ADF(1) = -1.09	ADF(0) = -7.27
<i>EMBI+</i>	ADF(1) = -1.99	ADF(1) = -7.28
<i>(y - y*) IPI Gap</i>	ADF(2) = -3.13	
<i>q</i>	ADF(0) = -3.51	

Note: The value in parenthesis is the order of the lag used which is selected by default in Eviews. The 5 percent rejection for unit root is  $ADF < -2.89$ , and the 10 percent rejection is  $ADF < -2.58$ .

It can be concluded from table 4.2 that all variables except the IPI Gap and the real exchange rate are integrated of order 1. Although the theory states that all variables should be I(1) in order to find cointegration relationships, this study evaluates the existence of such relations in each of the exchange rate models described before.

The cointegration results for each specification and the different tests for stationarity, exclusion, weak exogeneity and residual analysis are presented in the next tables.

**Table 4.3** Likelihood ratio test of the number of cointegration vectors (FPMM)

Hypothesized Number of Cointegration Equations	Eigenvalue	Statistic	5 percent critical value	Probability**
<i>Trace Statistic</i>				
None *	0.4270	119.0107	63.8761	0.0000
At most 1*	0.2933	62.2012	42.9152	0.0002
At most 2	0.1459	24.7875	25.8721	0.0584
At most 3	0.0995	10.6923	12.5179	0.0991
<i>Max-Eigen Statistic</i>				
None *	0.4270	56.8094	32.1183	0.0000
At most 1*	0.2933	35.4137	25.8232	0.0002
At most 2	0.1459	16.0952	19.3870	0.1412
At most 3	0.0995	10.6923	12.5179	0.0991

\* denotes rejection of the hypothesis at the 0.05 level

\*\*MacKinnon-Haug-Michelis (1999) p-values

**Table 4.4** Test for exclusion, stationarity and weak exogeneity (FPMM)

Test	Critical Value	$s$	$(m - m^*)$	$(y - y^*)$	$(i - i^*)$
Test under 2 cointegration vectors					
Exclusion	5.99	12.22	6.92	7.61	11.70
Stationarity	7.81	10.66	15.53	16.77	13.34
Weak exogeneity	5.99	6.73	7.56	7.94	22.64

**Table 4.5** Estimation of the model (FPMM)

Model	Cidrift
Variables	$s, (m - m^*), (y - y^*), (i - i^*)$
Cointegration vectors	$\beta' = 1.00 \quad 0.64 \quad 0.47 \quad -0.09$ <b>1.00 -7.79 9.12 -0.16</b>
Speed of adjustment	$\alpha' = -0.05 \quad -0.03 \quad 0.07 \quad 1.27$ -0.03 0.04 -0.04 0.33

**Table 4.6** Residual tests (FPMM)

Test	Test Statistic	P-value
<i>Autocorrelation</i>		
LM test	LM(4) = 14.74	0.54
<i>Multivariate Normality</i>		
Lutkepohl test	$\chi^2(8) = 11.95$	0.15

According to table 4.3, the trace and the Max-Eigen statistics show the existence of two cointegration vectors in the FPMM. Table 4.4 presents the initial tests under that

hypothesis and it can be inferred that none of the variables within the system are excluded and they are not stationary.

Tables 4.5 and 4.6 present the estimation of the model and the corresponding residual tests. The second cointegration vector has the expected signs and it shows that the exchange rate is well explained by all the variables, especially monetary aggregates and the difference in the IPI. In addition, the speed of adjustment of the exchange rate is close to three percent in one month. It means that any disequilibrium of this variable is corrected at that rate of adjustment. Finally, the residual tests present evidence of no serial correlation and the existence of multivariate normal residuals.

The results corresponding to the SPMM are presented in the next table:

**Table 4.7** Likelihood ratio test of the number of cointegration vectors (SPMM)

Hypothesized Number of Cointegration Equations	Eigenvalue	Statistic	5 percent critical value	Probability**
<i>Trace Statistic</i>				
None *	0.5705	224.2454	88.8038	0.0000
At most 1*	0.4279	138.0362	63.8761	0.0000
At most 2*	0.3856	81.0703	42.9152	0.0000
At most 3*	0.1608	31.3822	25.8721	0.0093
At most 4*	0.1238	13.4896	12.5179	0.0343
<i>Max-Eigen Statistic</i>				
None *	0.5705	86.2091	38.3310	0.0000
At most 1*	0.4279	56.9659	32.1183	0.0000
At most 2*	0.3856	49.6881	25.8232	0.0000
At most 3*	0.1608	19.8925	19.3870	0.0413
At most 4*	0.1238	13.4896	12.5179	0.0343

\* denotes rejection of the hypothesis at the 0.05 level

\*\*MacKinnon-Haug-Michelis (1999) p-values



Table 4.7 presents the cointegration test results for the SPMM. There is no evidence of cointegration in this model. The trace and the Max-Eigen statistics reject the null hypothesis of cointegration in all cases. In this sense, the exchange rate behaviour is better explained by the FPMM, rather than the SPMM in the long run.

After testing the FPMM and the SPMM, the productivity differential is considered in the BSA specification and the results are:

**Table 4.8** Likelihood ratio test of the number of cointegration vectors (BSA)

Hypothesized Number of Cointegration Equations	Eigenvalue	Statistic	5 percent critical value	Probability**
<i>Trace Statistic</i>				
None *	0.5876	157.3740	69.8188	0.0000
At most 1*	0.2684	67.0258	47.8561	0.0003
At most 2*	0.1892	35.1410	29.7970	0.0110
At most 3	0.1079	13.7401	15.4947	0.0903
At most 4	0.0202	2.0863	3.8414	0.1486
<i>Max-Eigen Statistic</i>				
None *	0.5876	90.3482	33.8768	0.0000
At most 1*	0.2684	31.8848	27.5843	0.0131
At most 2*	0.1892	21.4009	21.1316	0.0458
At most 3	0.1079	11.6538	14.2646	0.1243
At most 4	0.0202	2.0863	3.8414	0.1486

\* denotes rejection of the hypothesis at the 0.05 level

\*\*MacKinnon-Haug-Michelis (1999) p-values

**Table 4.9** Test for exclusion, stationarity and weak exogeneity (BSA)

Test	Critical Value	$s$	$(m - m^*)$	$(y - y^*)$	$(i - i^*)$	$(z - z^*)$
Test under 3 cointegration vectors						
Exclusion	7.81	21.48	19.05	15.98	50.25	20.21
Stationarity	5.99	11.17	17.62	18.30	14.98	16.96
Weak exogeneity	7.81	26.28	13.11	16.47	25.39	38.18

**Table 4.10** Estimation of the model (BSA)

Model	Drift
Variables	$s, (m - m^*), (y - y^*), (i - i^*), (z - z^*)$
Cointegration vectors	$\beta' =$ 1.00 -0.52 1.86 -0.09 -0.95 1.00 -1.01 1.69 -0.03 2.57 <b>1.00 -7.31 2.74 -0.16 2.95</b>
Speed of adjustment	$\alpha' =$ -0.16 -0.04 0.09 1.22 0.06 -0.24 0.09 0.25 1.78 -0.12 -0.06 0.01 -0.01 0.15 0.01

**Table 4.11** Residual tests (BSA)

Test	Test Statistic	P-value
<i>Autocorrelation</i>		
LM test	LM(4) = 23.61	0.54
<i>Multivariate Normality</i>		
Lutkepohl test	$\chi^2(10) = 10.53$	0.40

Table 4.8 presents the results of the likelihood ratio test for the BSA model. Considering the trace and the Max-Eigen statistics there is evidence of cointegration. Under the assumption of three cointegrating vectors, it is inferred from table 4.9 that none of the variables are excluded and the hypothesis of non stationarity is accepted in all of them. Tables 4.10 and 4.11 present the estimation of the model and the corresponding residual tests. The second and third cointegration vectors have the expected signs. However, the third one has higher coefficients than the second one. Furthermore, the speed of adjustment of the exchange rate is close to six percent in one month. In this sense, it seems that the rate at which exchange rate corrects any disequilibrium is higher in the BSA model than in the FPMM. This result could arise from the inclusion of productivity differentials.

The cointegration results for the basic UIP condition are described in the following tables.

**Table 4.12** Likelihood ratio test of the number of cointegration vectors (UIP)

Hypothesized Number of Cointegration Equations	Eigenvalue	Statistic	5 percent critical value	Probability**
<i>Trace Statistic</i>				
None *	0.2729	41.0223	25.8721	0.0003
At most 1	0.0799	8.5019	12.5179	0.2134
<i>Max-Eigen Statistic</i>				
None *	0.2729	32.5204	19.3870	0.0004
At most 1	0.0799	8.5019	12.5179	0.2134

\* denotes rejection of the hypothesis at the 0.05 level

\*\*MacKinnon-Haug-Michelis (1999) p-values

**Table 4.13** Test for exclusion, stationarity and weak exogeneity (UIP)

Test	Critical Value	$s$	$(i - i^*)$
Test under 1 cointegration vector			
Exclusion	3.84	7.26	22.28
Stationarity	5.99	23.90	22.92
Weak exogeneity	3.84	6.41	20.93

**Table 4.14** Estimation of the model (UIP)

Model	Cidrift
Variables	$s, (i - i^*)$
Cointegration vectors	$\beta' = 1.00 \ -0.11$
Speed of adjustment	$\alpha' = -0.04 \ 1.06$

**Table 4.15** Residual tests (UIP)

Test	Test Statistic	P-value
<i>Autocorrelation</i>		
LM test	LM(4) = 5.19	0.27
<i>Multivariate Normality</i>		
Lutkepohl test	$\chi^2(4) = 11.06$	0.03

The existence of cointegration is present under the UIP condition. According to the likelihood ratio test there is one cointegration vector under the IT regime. Although none of the variables are excluded and all of them are non stationary, the residual

tests accept the hypothesis of multivariate normality only at one percent of significance.

In addition, the UIP condition that includes the EMBI+Colombia as a proxy for the risk in an emerging country reports the following results:

**Table 4.16** Likelihood ratio test of the number of cointegration vectors (UIP+EMBI)

Hypothesized Number of Cointegration Equations	Eigenvalue	Statistic	5 percent critical value	Probability**
<i>Trace Statistic</i>				
None *	0.3551	72.5920	42.9152	0.0000
At most 1*	0.1931	27.8461	25.8721	0.0281
At most 2	0.0566	5.9533	12.5179	0.4663
<i>Max-Eigen Statistic</i>				
None *	0.3551	44.7459	25.8232	0.0001
At most 1*	0.1931	21.8927	19.3870	0.0212
At most 2	0.0566	5.9533	12.5179	0.4663

\* denotes rejection of the hypothesis at the 0.05 level

\*\*MacKinnon-Haug-Michelis (1999) p-values

**Table 4.17** Test for exclusion, stationarity and weak exogeneity (UIP+EMBI)

Test	Critical Value	$s$	$(i - i^*)$	$embi$
Test under 2 cointegration vectors				
Exclusion	5.99	20.92	37.07	21.85
Stationarity	5.99	19.23	23.76	23.78
Weak exogeneity	5.99	13.85	32.00	9.61

**Table 4.18** Estimation of the model (UIP+EMBI)

Model	Cidrift
Variables	$s, (i - i^*), embi$
Cointegration vectors	$\beta' = 1.00 \ -1.25 \ 5.40$ <b>1.00 -0.02 -0.49</b>
Speed of adjustment	$\alpha' = -0.01 \ 0.18 \ -0.03$ -0.09 0.52 0.33

**Table 4.19** Residual tests (UIP+EMBI)

Test	Test Statistic	P-value
<i>Autocorrelation</i>		
LM test	LM(4) = 7.74	0.56
<i>Multivariate Normality</i>		
Lutkepohl test	$\chi^2(6) = 6.45$	0.37

As in Moura, Lima and Mendonca (2008), there are risk factors not captured in the basic UIP which can explain the behaviour of the exchange rate, specially in emergings countries such as Colombia. In this sense, tables 4.16, 4.17 and 4.18 show the existence of two cointegration equations and that none of the variables are excluded and all of them are non stationary. However, only the second cointegration relation has the correct signs and the elasticity of the exchange rate to the interest rate differential is diminished. In addition, the speed of adjustment of the exchange rate is increased from four percent to nine percent in the new model including the EMBI+ index.

After considering different conventional models, it is convenient to prove if there is evidence of cointegration in Taylor Rule models. In this sense, the first specification reports the following results:

**Table 4.20** Likelihood ratio test of the number of cointegration vectors (Taylor Rule)

Hypothesized Number of Cointegration Equations	Eigenvalue	Statistic	5 percent critical value	Probability**
<i>Trace Statistic</i>				
None *	0.4244	130.1720	63.8761	0.0000
At most 1*	0.3681	73.8202	42.9152	0.0000
At most 2*	0.1641	26.9920	25.8721	0.0362
At most 3	0.0817	8.7021	12.5179	0.1995
<i>Max-Eigen Statistic</i>				
None *	0.4244	56.3517	32.1183	0.0000
At most 1*	0.3681	46.8282	25.8232	0.0000
At most 2	0.1641	18.2898	19.3870	0.0716
At most 3	0.0817	8.7021	12.5179	0.1995

\* denotes rejection of the hypothesis at the 0.05 level

\*\*MacKinnon-Haug-Michelis (1999) p-values

**Table 4.21** Test for exclusion, stationarity and weak exogeneity (Taylor Rule)

Test	Critical Value	$s$	$(\pi - \pi^*)$	$(y - y^*)$	$(i - i^*)$
Test under 2 cointegration vectors					
Exclusion	5.99	26.07	36.82	9.01	34.31
Stationarity	7.81	38.02	29.10	30.05	28.65
Weak exogeneity	5.99	5.32	33.68	2.34	30.44

**Table 4.22** Estimation of the model (Taylor Rule)

Model	Cidrift
Variables	$s, (\pi - \pi^*), (y - y^*), (i - i^*)$
Cointegration vectors	$\beta' = \mathbf{1.00 \ -0.82 \ 4.41 \ 0.44}$ 1.00 -0.05 3.81 -0.06
Speed of adjustment	$\alpha' = -0.01 \ 0.55 \ 0.01 \ -0.24$ -0.05 2.16 0.05 1.48

**Table 4.23** Residual tests (Taylor Rule)

Test	Test Statistic	P-value
<i>Autocorrelation</i>		
LM test	LM(4) = 22.02	0.14
<i>Multivariate Normality</i>		
Lutkepohl test	$\chi^2(8) = 9.21$	0.33

The Taylor rule model indicates at least two cointegration vectors. All the variables are non stationary and none of them are excluded from the system. The two cointegrated vectors have different sign in the interest differential. However, the first vector is more consistent with the empirical and theoretical evidence. As Molodtsova and Papell (2008) states it is more reliable that an increase in the interest rate will produce an immediate currency appreciation, rather than depreciation. This assumption goes against the UIP condition which predicts the reverse relationship. Under Taylor rule models an increase in inflation level is associated to an increase in interest rates by central banks and it will appreciate the currency. In this case, the



sign is opposite probably because this mechanism of transmission on the exchange rate is not contemporaneous.

Although Colombia does not target the exchange rate under its floating regime, the following specification tries to explain if the real exchange rate incorporates new information that explains the long run exchange rate behaviour.

**Table 4.24** Likelihood ratio test of the number of cointegration vectors (Taylor with  $q_t$ )

Hypothesized Number of Cointegration Equations	Eigenvalue	Statistic	5 percent critical value	Probability**
<i>Trace Statistic</i>				
None *	0.5285	223.3232	88.8038	0.0000
At most 1*	0.4566	146.6242	63.8761	0.0000
At most 2*	0.3337	84.3979	42.9152	0.0000
At most 3*	0.2458	42.9774	25.8721	0.0002
At most 4*	0.1299	14.1952	12.5179	0.0259
<i>Max-Eigen Statistic</i>				
None *	0.5285	76.6990	38.3310	0.0000
At most 1*	0.4566	62.2262	32.1183	0.0000
At most 2*	0.3337	41.4204	25.8232	0.0002
At most 3*	0.2458	28.7822	19.3870	0.0016
At most 4*	0.1299	14.1952	12.5179	0.0259

\* denotes rejection of the hypothesis at the 0.05 level

\*\*MacKinnon-Haug-Michelis (1999) p-values

According to the results in table 4.24 there is no cointegration relationship in the Taylor rule model when the real exchange rate is included. The main reason could be

that under IT Colombia does not target the exchange rate and only intervene directly in the foreign market when the exchange rate exhibit high volatility.

## **5. Out of sample forecasting performance**

This chapter evaluates the out of sample forecasting performance of the models described before. The first section explains the methodology used and the second one analyzes the results.

### **5.1. The methodology**

In order to test the forecasting power of the conventional and Taylor rule models, this study relies on the methodology used by Meese and Rogoff (1983). In that study they compared exchange rate models with a simple drift less random walk.

According to the results obtained in the last section, this study only evaluates the forecast performance for the models that exhibit cointegration. As stated before, monthly data set from January 2000 to May 2009 is used for testing cointegration. If there is evidence of cointegration then future values of the exchange rate are forecasted at different horizons based on the VEC model identified before. The exchange rate is forecasted at  $k = 1, 3, 6$  and 12 months ahead. The idea is to generate a series of out of sample forecasted exchange rate based on rolling regressions methodology. Initially, the data used is from January 2000 to December 2007. It means that for testing forecasting power the estimations are calculated over a window of eight years. In other words, the first estimation is run from January 2000 to December 2007. Then, the second regression uses data from February 2000 to January 2008; the third regression uses data from March 2000 to February 2008

moving the sample one period ahead and so on. This procedure is repeated until reach the last date May 2009 but keeping the same size of the initial sample. This out of sample period is selected because after the half of 2007 there was more stability in the Colombian exchange rate after the strong appreciation and the huge direct intervention in the exchange market. The methodology used in Eviews is static forecasting which means that forecasted values of the exchange rate are not used to generate the subsequent forecasts such as dynamic forecasting.

After estimating the forecasts, the results are compared with the drift less random walk model. As in Moura, Lima and Mendonca (2008) this model is specified as:

$$S_{t+k} = S_t \quad (3.19)$$

Two statistics and one test are used to evaluate the out of sample forecast performance. Firstly, the Root Mean Squared Error (RMSE) and the Mean Absolute Error (MAE) are statistics used frequently in order to compare forecasts for the same series but considering different models. In addition, one sign test is used in order to analyze if there is any difference between the forecasts from different specifications.

The ratio between the RMSE of each model and the RMSE of the random walk is used in order to determine if the exchange rate models outperform the drift less random walk. The same methodology is used with the MAE. Furthermore, the sign test was introduced in Diebold and Mariano (1995) in order to prove the accuracy of forecasts when there are small numbers of projections within any specification. This test is based on the use of a loss function.

As in Rowland (2003b) The RMSE and MAE statistics are defined as follows:

$$RMSE = \sqrt{\sum_{s=0}^{N_k-1} (F_{(t+s+k)} - A(t+s+k))^2} / N_k \quad (3.20)$$

$$MAE = \sum_{s=0}^{N_k-1} |F(t+s+k) - A(t+s+k)| / N_k \quad (3.21)$$

where  $F(t)$  and  $A(t)$  are the forecasted and actual value of the exchange rate in period  $t$ . In addition  $k$  corresponds to the forecasting horizon and  $N_k$  is the number of total forecasts within the projection period. If the RMSE or MAE ratios are greater than one it means that the random walk outperforms the exchange rate model analysed.

The loss function used to calculate the sign test is the squared predicted error (SPE)<sup>12</sup> for each exchange rate specification and the random walk model. The next step is calculating a new variable called “ $d$ ” that is the difference between these loss functions:

$$d = (SPE_{random\ walk} - SPE_{exchange\ rate\ model}) \quad (3.22)$$

The null hypothesis is that the median of the loss differential (“ $d$ ”) is expected to be equal to zero. According to Diebold and Mariano (1995) the Sign Test is calculated based on a binomial distribution with parameters  $N_{k-1}$  and 1/2 as follows:

$$S = \sum_{s=0}^{N_k-1} I(d_{t+s+k})$$

$$\text{where } I(d_{t+s+k}) = \begin{cases} 1 & \text{if } d_{t+s+k} > 0 \\ 0 & \text{otherwise} \end{cases}$$

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<sup>12</sup> The SPE is calculated as  $(F_t - A_t)^2$  at each date of the out of sample range.

The significance of this test is assessed using the cumulative binomial distribution. This cumulative distribution function is the probability that there is at most certain number of successes that in this case correspond to the number of positive loss-differential observations (Diebold and Mariano 1995).

## 5.2. The results

Table 4.25 displays the RMSE and MAE ratios only for the models that exhibit cointegration.

**Table 4.25** RMSE and MAE ratios

Model	1-month	3-month	6-month	12-month
<i>FPMM</i>				
RMSE	0.2569*	1.1500	1.0020	0.5110*
MAE	0.2511*	1.1687	1.0827	0.4749*
<i>BSA</i>				
RMSE	0.1516*	0.9235*	1.1456	2.1016
MAE	0.1618*	0.8851*	1.0636	2.0316
<i>UIP</i>				
RMSE	0.3642*	0.9325*	0.9989*	1.1511
MAE	0.3773*	0.8993*	0.9433*	1.1662
<i>UIP with EMBI+</i>				
RMSE	0.2881*	0.9279*	1.1000	1.6762
MAE	0.3101*	0.8664*	1.1032	1.6874
<i>Taylor (first specification)</i>				
RMSE	0.2497*	1.2361	1.1871	1.7323
MAE	0.2481*	1.1948	1.1951	1.4120

According to the results, all the exchange rate specifications outperform the drift less random walk at one month forecast. The FPMM presents the better out of sample predictability at short and long horizons. Although the UIP condition outperforms the

drift less random walk for three and six months forecast, the UIP ratios for these horizons are close to one. It could mean that although there is evidence of cointegration under the UIP condition, there is not qualitative difference between the UIP and random walk predictability.

Taylor rule model only outperforms random walk forecasts at one month horizon. The lack of forecasting power at other horizons could be explained by the absence of interest rate smoothing. In other words, the model used does not include lagged interest rates and only incorporate a contemporaneous relationship with the exchange rate. According to the theory the effect of interest rates on different macroeconomic variables is gradual. However, as it was explained before the purpose of this study is to explain the exchange rate behaviour under endogenous monetary policy and not determine the time taken for interest rate changes to affect the exchange rate.

Other reason that supports the lack of power prediction is the absence of expectations in the Taylor rule specification. As New Keynesian models state, the expectations are important for modelling interest rate rules. In this sense this issue is object of future studies but it is out of the main purpose of this research that is not more that explain the capability of prediction of different exchange rate models.

Table 4.26 displays the cumulative binomial distribution of the Sign Test for each exchange rate specification.

**Table 4.26** Sign Test significance

Model	1-month	3-month	6-month	12-month
<i>FPMM</i>	0.9988	0.3036	0.1132	1.0000
<i>BSA</i>	0.9999	0.9407	0.7255	0.1093
<i>UIP</i>	0.9999	0.8491	0.7256	0.1093
<i>UIP with EMBI+</i>	0.9998	0.8491	0.2744	0.0156
<i>Taylor (first specification)</i>	0.9988	0.8491	0.5000	0.6562

These results confirm the analysis of the RMSE and MAE ratios. As explained before, the FPMM has the better predictive power within all the models while UIP with EMBI+ and Taylor specification loose capability of prediction especially at six and twelve horizons.

## 6. Conclusions

Meese and Rogoff (1983) introduced the discussion about the out of sample predictability of exchange rate models. Since that time many studies have been developed in order to asses the accuracy of conventional and new Taylor Rule models. Following earlier studies in Colombia and in other emerging countries such as Brazil, the cointegration results show that the economics fundamentals may explain the exchange rate behaviour during the last nine years in Colombia.

All specifications exhibit cointegration relationships with the exception of the SPMM and the second specification of the Taylor Rule model that includes the real exchange rate. Firstly, it could mean that prices adjust faster under any excess

demand such as the FPMM states. Second, there is not cointegration in the second Taylor Rule specification because in Colombia there is not a specific target of the exchange rate and only there are intervention instruments that control its volatility.

In terms of out of sample predictability there is evidence that the FPMM which requires the UIP condition and assumes the PPP is the best model that outperforms the random walk specification at short and long term horizons. The BSA model that does not impose the PPP and UIP performs well at one and three month periods ahead. In addition the simple specification of the UIP condition outperforms the random walk at the same horizons that the BSA model and additionally at six months ahead. Surprisingly, the Taylor rule model only has better predictability at one month horizon. According to the theory it was expected that this exchange rate specification outperforms the random walk at short and long horizons. However, future studies could model expectations in all variables in order to improve the predictability power of this type of models. In addition, the high exchange rate volatility during the last years has made more difficult to recognize other factors that could improve the predictability of the exchange rate within the Taylor Rule framework. Nevertheless, the results show that there is a strong link between economic fundamentals and the exchange rate and that interest rate rules explain the exchange rate in Colombia at least in the short run.



## References

- Binici, M. 2007. Taylor Rule and Exchange Rate Dynamics. PhD diss. proposal, University of California. <http://www.ucsc.edu> (accessed July 10, 2009).
- Chang, R. 2008. Inflation Targeting, Reserves Accumulation, and Exchange Rate Management in Latin America. *Working Papers Banco de la República de Colombia* 487. <http://www.banrep.gov.co> (accessed June 15, 2009).
- Clements, B., H. Kamil. 2009. Are Capital Controls Effective in the 21<sup>st</sup> Century? The Recent Experience of Colombia. *IMF Country Report No. 09/24*. <http://www.imf.org> (accessed February 20, 2009).
- Clarida, R., J. Gali, and M. Gertler. 1998. Monetary Rules in Practice: Some International Evidence. *European Economic Review* 42: 1033-1067.
- Cheung, Y., M. Chinn and A. Pascual. 2005. Empirical exchange rate models of the nineties: are any fit to survive? *Journal of International Money and Finance* 24: 1150-1175.
- Diebold, F., R. Mariano. 1995. Comparing Predictive Accuracy. *Journal of Business and Economics Statistics* 13: 253-265
- Engel, C., K. West. 2006. Taylor Rules and the Deutschemark-Dollar Real Exchange Rate. *Journal of Money, Credit and Banking* 38 (5): 1175-1194.
- Engel, R., C. Granger. 1987. Co integration and error correction: representation, estimation and testing. *Econometrica* 55: 251:276.

- Johansen, S. 1991. Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. *Econometrica* 59 (6): 1551-1580.
- Johansen, S. 1995. *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford: Oxford University Press.
- Kamil, H. 2008. Is Central Bank Intervention Effective Under Inflation Targeting Regimes? The Case of Colombia. *IMF Working Papers 08/88*. <http://www.imf.org> (accessed June 20, 2009).
- MacKinnon, J., A. Haug, and L. Michelis. 1999. Numerical Distribution Functions of Likelihood Ratio Tests for Cointegration. *Journal of Applied Econometrics* 14: 563-577.
- Mark, N. 2005. Changing Monetary Policy Rules, Learning, and Real Exchange Rate Dynamics. *NBER Working Paper Series 11061*. <http://www.imf.org> (accessed June 25, 2009).
- Meese, R., K. Rogoff. 1983. The out of sample failure of empirical exchange rate models: sampling error or misspecification? In *Exchange Rates and International Macroeconomics*, ed. J. Frenkel, 67-112. Chicago: University of Chicago Press.
- Molodtsova, T., D. Papell. 2009. Out of sample exchange rate predictability with Taylor Rule Models. *Journal of International Economics* 77 (2): 167-180.
- Moura, M., A. Lima and R. Mendonca. 2008. Exchange Rate and Fundamentals: The Case of Brazil. *IBMEC Working Paper 66*. <http://www.ibmec.br> (accessed February 10, 2009).

- Rowland, P., H. Oliveros. 2003a. Colombian Purchasing Power Parity Analysed Using a Framework of Multivariate Cointegration. *Working Papers Banco de la República de Colombia* 252. <http://www.banrep.gov.co> (accessed July 1, 2009).
- Rowland, P. 2003b. Forecasting the USD/COP Exchange Rate: A Random Walk with a Variable Drift. *Working Papers Banco de la República de Colombia* 253. <http://www.banrep.gov.co> (accessed July 1, 2009).
- Sarno, L., and M. Taylor. 2002. *The economics of exchange rates*. Cambridge: Cambridge University Press, chapter 4.
- Taylor, L., and M. Taylor. 2002. *The economics of exchange rates*. Cambridge: Cambridge University Press, chapter 4.
- Uribe, J., J. Toro. 2005. Foreign Exchange Market Intervention in Colombia. This chapter was published in *Bank for International Settlements Papers 24: 139-49*. <http://www.bis.org> (accessed June 20, 2009).
- Vargas, H. 2005. Exchange Rate Policy and Inflation Targeting in Colombia. *Research Department Working Papers Inter-American Development Bank* 539. <http://www.iadb.org> (accessed February 20, 2009).
- Zampolli, F. 2006. Optimal monetary policy in a regime-switching economy: the response to abrupt shifts in exchange rate dynamics. *Journal of Economics Dynamics and Control* 30 (9-10): 1527 -1567